Recent results on nucleon form factors

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- G_{En}: “triggering conjecture”
- Phenomenological ansatz for nucleon form factors
- Pion-cloud model
- Summary

Introduction

Elastic form factors of the nucleon are of fundamental importance for an understanding of the nucleon's internal structure in the non-perturbative region.

They parametrise the nucleon's ability as a composite object to scatter a lepton coherently and without excitation of internal degrees of freedom.

First evidence for electromagnetic structure of the nucleon: Frisch and Stern, 1932 [Z. Phys. **88** (1933) 4]

$$\mu_p = 2.5\,\mu_N$$

contrary to expectation $\mu_p = \mu_N \cdot 2\,S = \mu_N$ for Dirac particle

Form factors are still a very active field of research after 70 years:

- Proton G_E^p/G_M^p ratio at high Q^2
- Neutron form factors G_M^n and G_E^n
- Charge radii $\langle r_p^2 \rangle$ and $\langle r_n^2 \rangle$

Neutron electric form factor G_E^n is experimentally most challenging.

Nucleon form factors

Scattering of unpolarised leptons at point-like Dirac particles:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \cdot \left(1 - \tau \tan^2 \frac{\theta}{2}\right)$$

where $\tau = Q^2/4M^2$ and

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{\alpha^2 \cos^2(\theta/2)}{4E^2 \sin^4(\theta/2)} \cdot \underbrace{\frac{1}{1 + \frac{2E}{M} \sin^2(\theta/2)}}_{\text{recoil term} = E'/E}$$

Structure of the nucleon is parametrised by form factors. Matrix element of e. m. current:

$$\langle N | J^\mu(0) | N \rangle = \bar{u} \left[\gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu} q^\nu}{2M} F_2(Q^2) \right] u$$

For the cross section follows

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \cdot \left[F_1^2 - \tau F_2^2 + 2\tau (F_1 + F_2)^2 \tan^2 \frac{\theta}{2} \right]$$

Sachs form factors

$F_1(Q^2)$ Dirac form factor: charge and Dirac magn. moment
$F_2(Q^2)$ Pauli form factor: anomalous magnetic moment

Sachs form factors G_E and G_M are defined as linear combinations of F_1 and F_2:

$$G_E = F_1 - \tau F_2 \qquad F_1 = \frac{G_E + \tau G_M}{1+\tau}$$

$$G_M = F_1 + F_2 \qquad F_2 = \frac{G_M - G_E}{1+\tau}$$

$G_E(Q^2)$ and $G_M(Q^2)$ describe charge and magnetic moment, respectively.

Static limits: Total change and magnetic moment of the nucleon

$$G_{Ep}(0) = 1 \qquad G_{Mp}(0) = 2.79$$

$$G_{En}(0) = 0 \qquad G_{Mn}(0) = -1.91$$

Rosenbluth cross section:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \cdot \left[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\frac{\theta}{2}\right]$$

Breit frame and charge distribution

In the Breit frame (or "brick wall" frame) incoming and outgoing electron have opposite momenta:

$$\begin{aligned}\vec{p} &= +\vec{q}/2\\ \vec{p}\,' &= -\vec{q}/2\end{aligned}$$

- No energy transfer by the virtual photon, $\omega = 0$

$$q^\mu = (0, \vec{q}), \qquad Q^2 = \vec{q}^{\,2}$$

- Electric and magnetic form factor G_E and G_M can be interpreted as three-dimensional Fourier transform of charge and magnetisation density.
- For each Q^2, there is a different Breit frame in which $G_E(\vec{q}^{\,2})$ and $G_M(\vec{q}^{\,2})$ are defined.

Mean squared radii

The mean squared radius is determined by the slope of the form factor at $Q^2 = 0$:

$$\langle r^2 \rangle = -\frac{6}{G(0)} \cdot \left.\frac{dG}{dQ^2}\right|_{Q^2=0}$$

(The neutron “charge radius” is defined without the normalisation factor, since $G_{En}(0) = 0$.)

For the charge distribution of a spherical heavy object, the mean squared radius is also given by:

$$\langle r^2 \rangle = \frac{1}{Q} \int_0^\infty r^2 \rho(r) dV$$

Foldy term

By separation of G_{En} in Dirac and Pauli formfactors F_1^n and F_2^n, one obtains for the neutron charge radius:

$$\begin{aligned}\langle r_{En}^2 \rangle &= -6 \frac{d}{dQ^2} \left(F_1(Q^2) - \underbrace{\frac{Q^2}{4M^2}}_{\tau} F_2(Q^2) \right)_{Q^2=0} \\ &= -6 \left. \frac{dF_1}{dQ^2} \right|_{Q^2=0} \underbrace{+ \frac{3}{2M^2} \overbrace{F_2(Q^2=0)}^{-1.91}}_{\text{Foldy term}}\end{aligned}$$

The so-called Foldy term gives a contribution to the neutron charge radius of

$$-0.127\,\text{fm}^2$$

very close to the experimental value for $\langle r_{En}^2 \rangle$.

[Foldy, Rev. Mod. Phys. **30** (1958) 471]

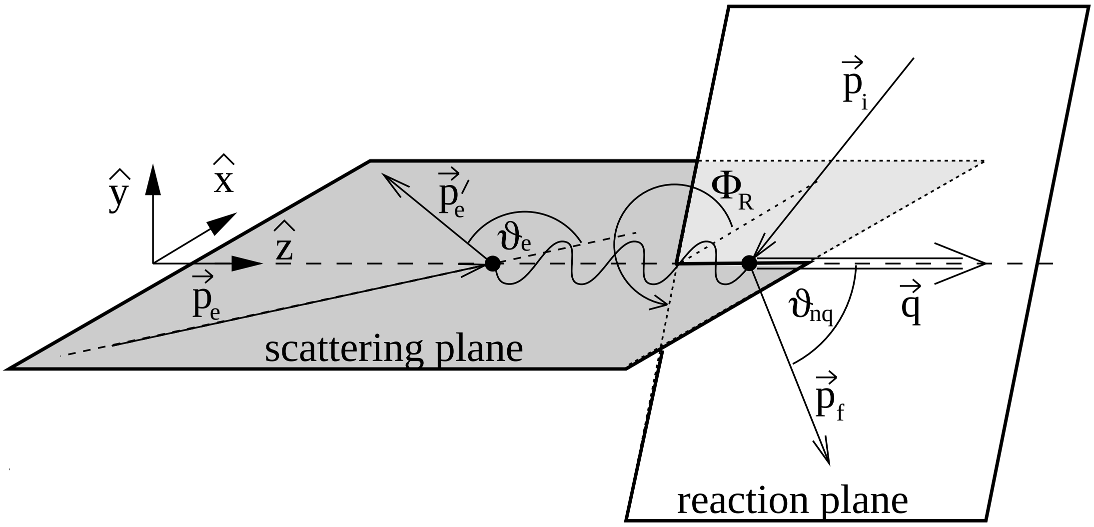
- Relativistic zitterbewegung of (anomalous) magnetic moment
- Some caution required for interpretation of G_{En} in terms of a static charge distribution
- Foldy term may be cancelled by other relativistic effects in constituent quark models [Isgur, PRL **83** (1999) 272]

Double polarisation experiments

Polarisation degrees of freedom → interference terms

Example: Measurement of G_{En}

Kinematics of the $n(\vec{e}, e'\vec{n})$ reaction:



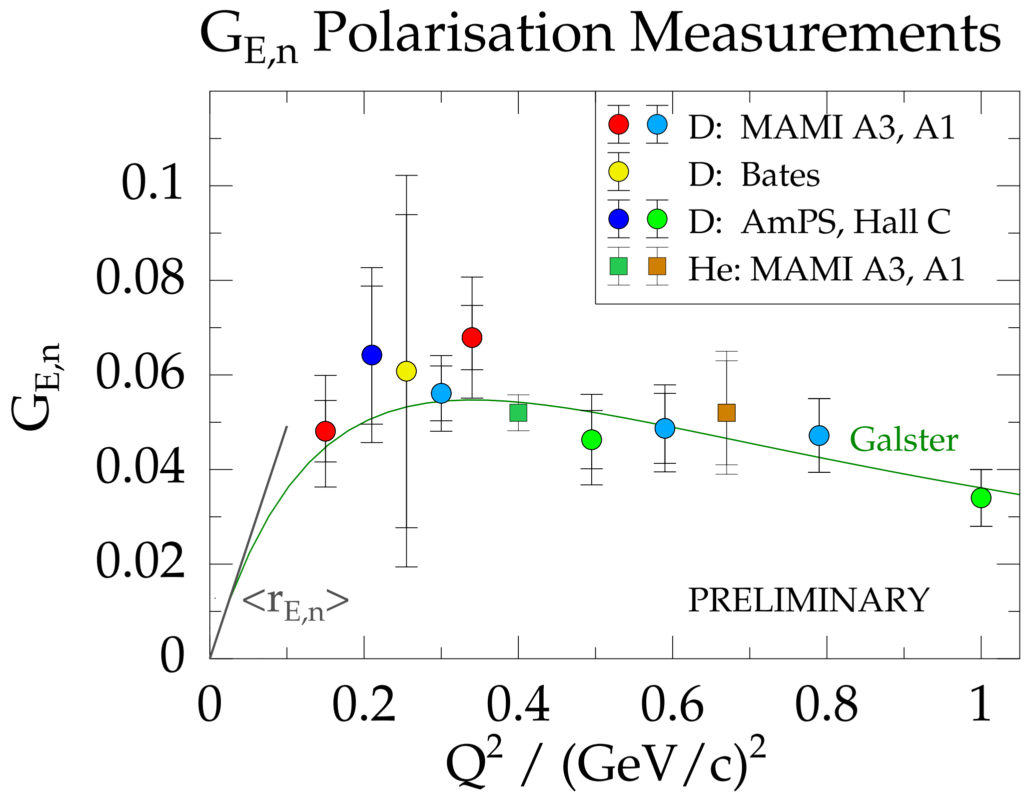
Polarisation transfer to the recoil neutron:

[Arnold, Carlson, Gross, PRC **23** (1981) 363]

$$
\begin{aligned}
P_x &= -P_e \frac{2\sqrt{\tau(1+\tau)}\tan(\theta/2) G_E G_M}{G_E^2 + \tau G_M^2(1+2(1+\tau)\tan^2(\theta/2))} \\
P_y &= 0 \\
P_z &= P_e \frac{2\tau\sqrt{1+\tau+(1+\tau)^2\tan^2(\theta/2)}\tan(\theta/2) G_M^2}{G_E^2 + \tau G_M^2(1+2(1+\tau)\tan^2(\theta/2))}
\end{aligned}
$$

P_x contains interference term $G_{En}G_{Mn}$.

World data on G_{En}



All data up to $Q^2 = 0.5\,(\mathrm{GeV}/c)^2$ have been corrected for final state interaction.

$D(\vec{e}, e'\vec{n})p$
Bates: [Eden et al., PRC **50** (1994) R1749]
MAMI A3: [Herberg et al., EPJA **5** (1999) 131]
MAMI A1: preliminary

$\vec{D}(\vec{e}, e'n)p$
AmPS: [Passchier et al., PRL **82** (1999) 4988]
Hall C: [Zhu et al., PRL **87** (2001) 081801]

$^3\vec{He}(\vec{e}, e'n)pp$
MAMI A3: [Becker et al., EPJA **6** (1999) 329]
MAMI A1: [Rohe et al., PRL **83** (1999) 4257]

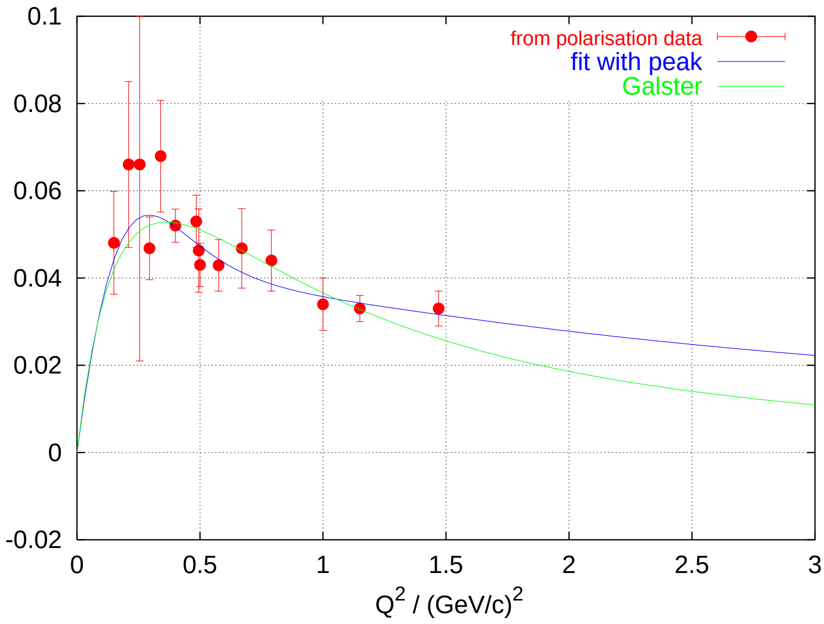
Form factors of the nucleons

[Friedrich und Walcher, hep-ph/0303054, → EPJA]

Parametrisation of G_{En}:

- Usually quoted: Galster parametrisation for G_{En}

$$G_{\text{Galster}}(Q^2) = \frac{a\tau}{1+b\tau} \cdot \frac{1}{(Q^2/0.71\,(\text{GeV}/c)^2)^2}$$



- Phenomenological description of G_{En} world data
- Results in “aperiodic” charge distribution in Breit system: positive inner and negative outer region
- But: Does not follow strictly from the data

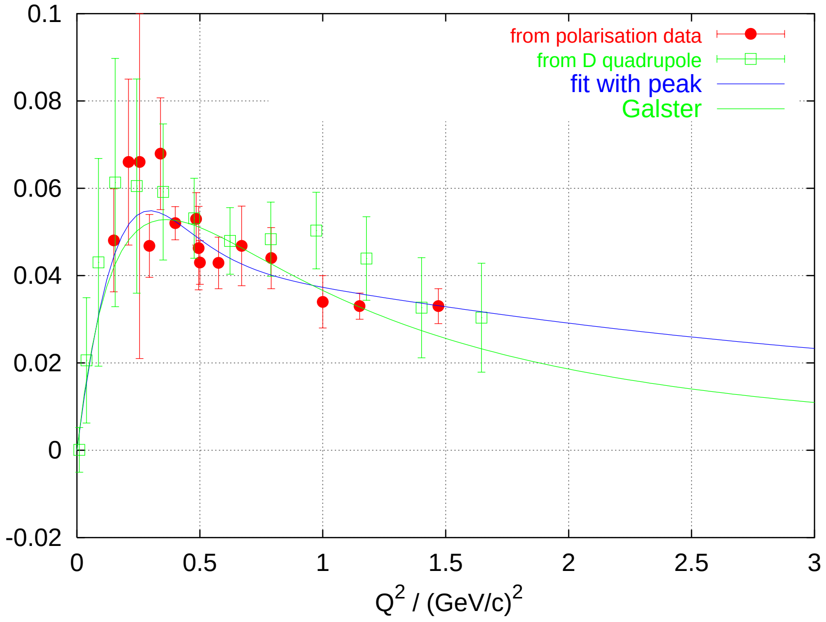
Alternative interpretation:

Data are also described by broad distribution and a “peak” at $Q^2 = 0.3\,(\text{GeV}/c)^2$.

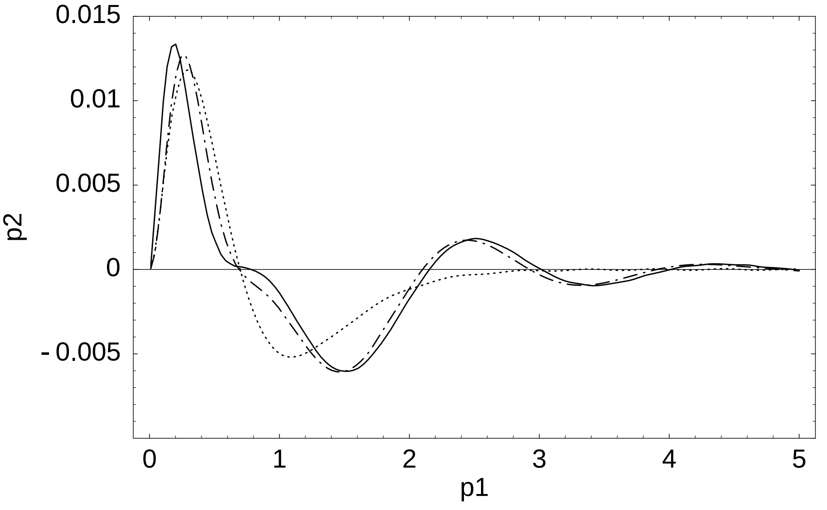
Fourier transformation → oscillations in charge distribution

Form factors of the nucleons

G_{En} from polarisation experiments and from deuteron F_{C2}



Charge distribution of the neutron in the Breit system



A phenomenological ansatz

Idea: describe all elastic nucleon form factors consistently

- Simple dipole: only limited description of the data, e. g. G_{Ep} at small and large Q^2 values.

New ansatz as sum of a broad distribution and a “bump”:

$$G_N(Q^2) = G_s(Q^2) + a_b Q^2 G_b(Q^2)$$

- G_s: two dipoles superimposed

$$G_s(Q^2) = \frac{a_{10}}{(1 + Q^2/a_{11})^2} + \frac{a_{20}}{(1 + Q^2/a_{21})^2}$$

- Bump G_b: “Gaussian” distribution

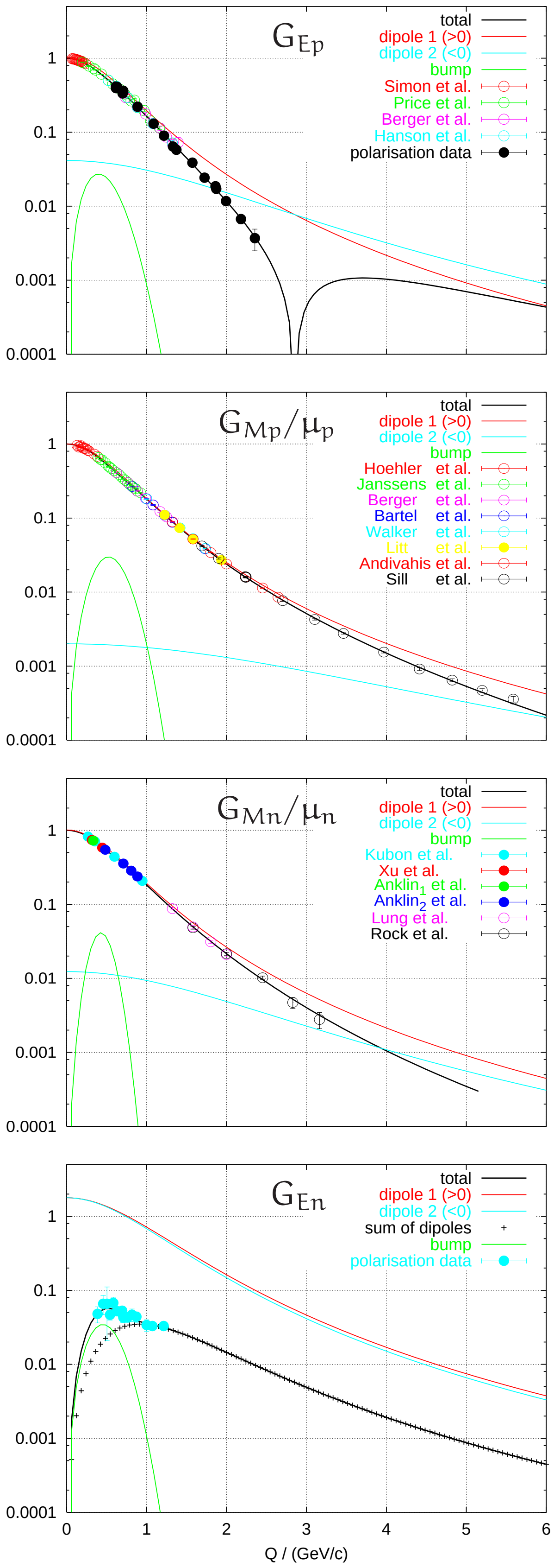
  [Sick, NPA 218 (1974) 509]

  (only even powers of Q allowed for form factors:)

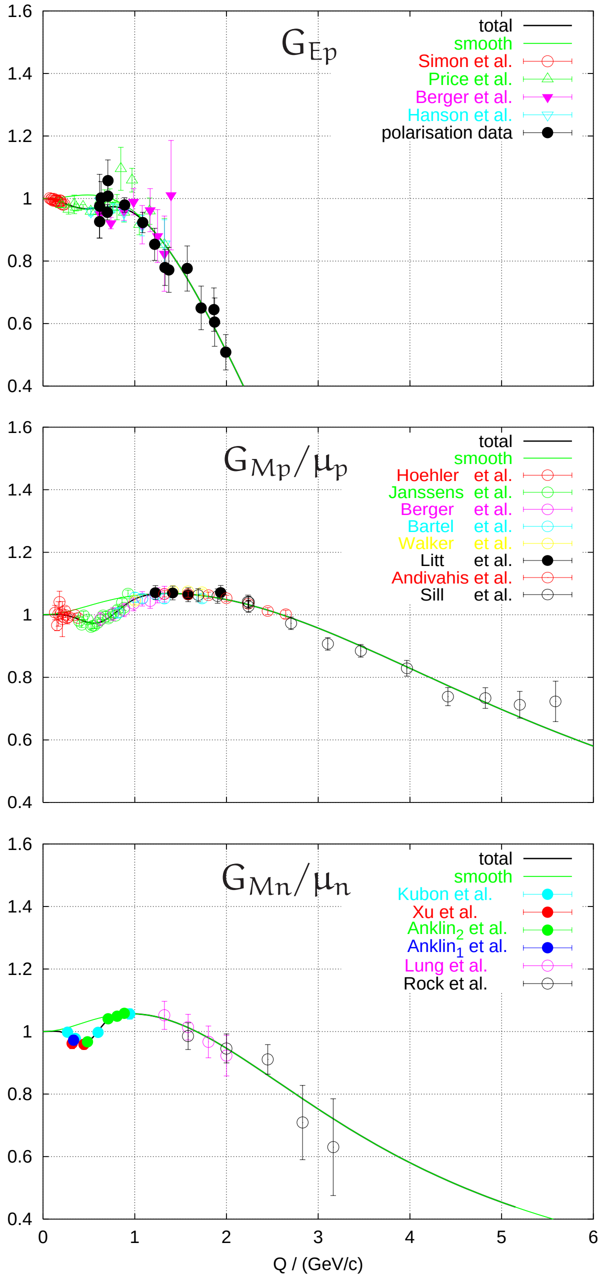
$$G_b(Q^2) = e^{-\frac{1}{2}\left(\frac{Q-Q_b}{\sigma_b}\right)^2} + e^{-\frac{1}{2}\left(\frac{Q+Q_b}{\sigma_b}\right)^2}$$

- Normalisation: $G_N(Q^2 = 0) = G_s(Q^2 = 0)$
  i. e. independent of G_b

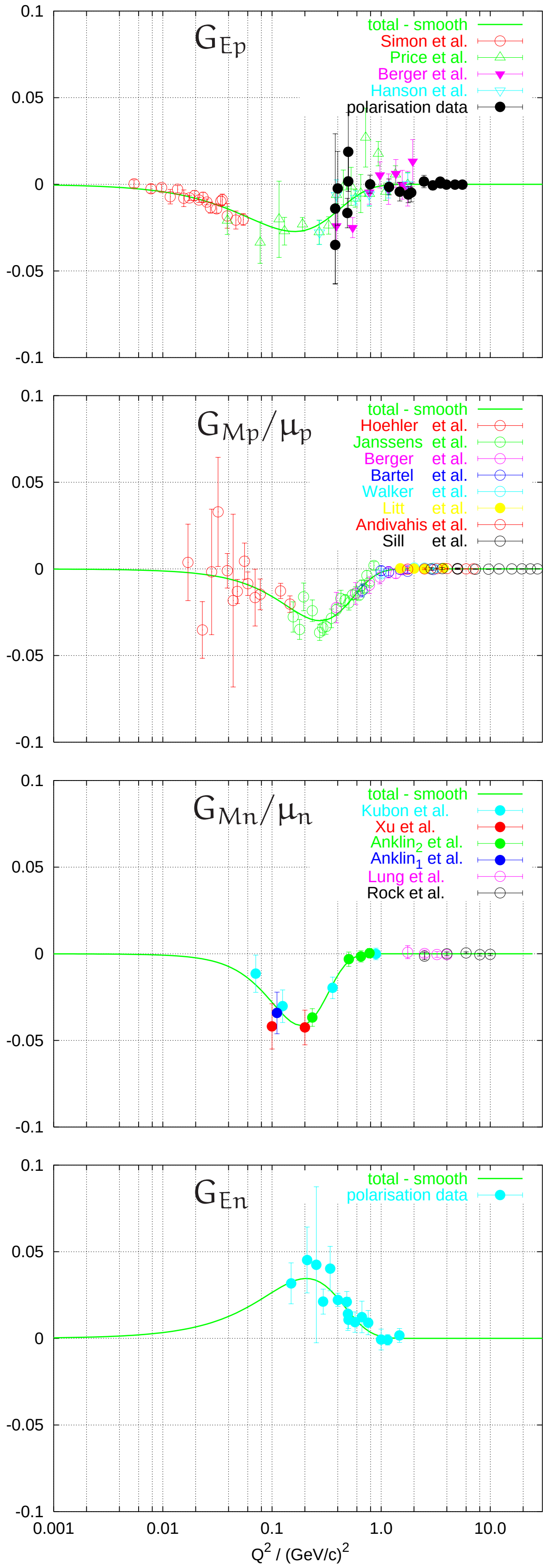
Form factors



FF / standard dipole

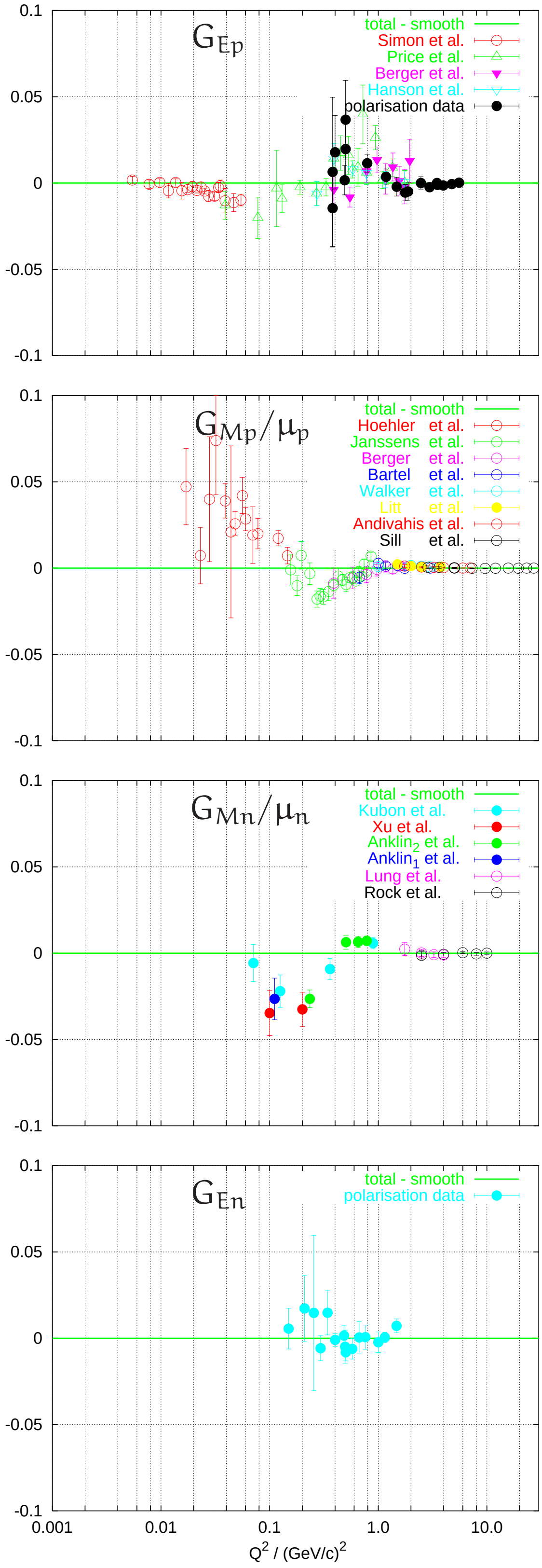


Form factors — “smooth distribution”

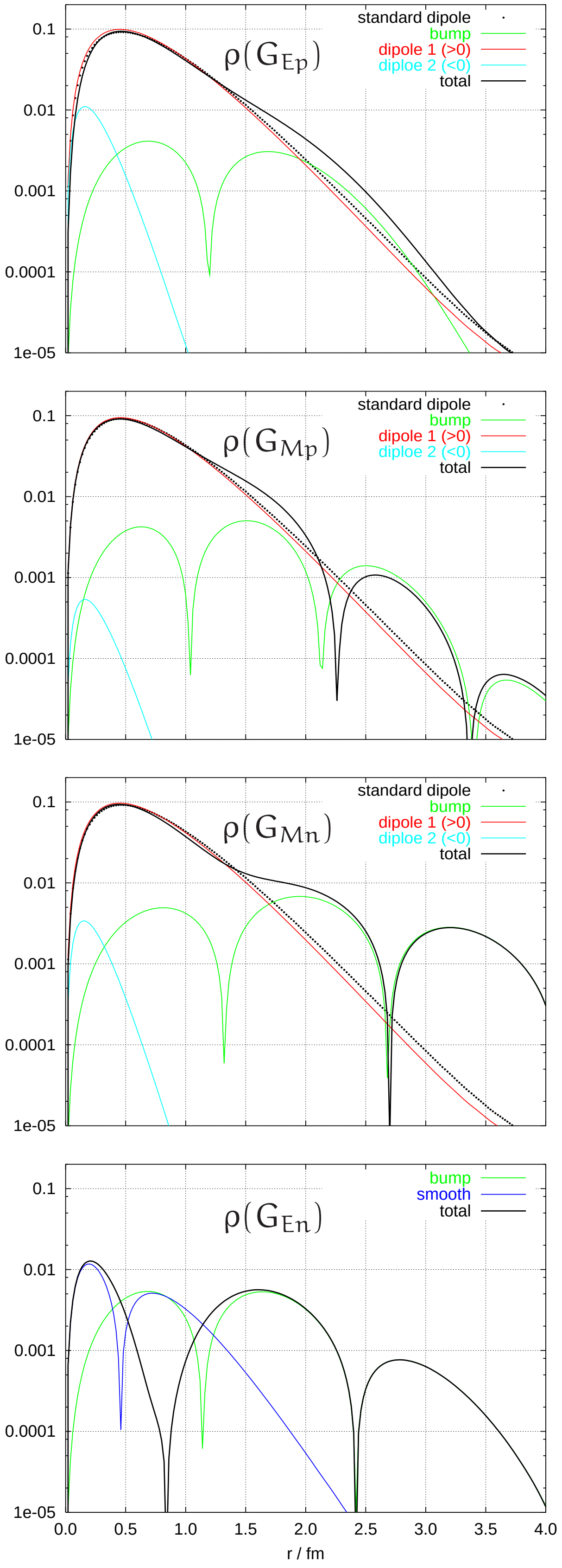


Form factors — “smooth distribution”

(only two dipoles fitted)



$r^2\rho(r)$ in Breit system (phenomenological ansatz)



The pion-cloud model

Describe the structure of a broad distribution + bump,
$\rightarrow$ physically motivated model for the nucleons:
Sum of “bare” nucleon and polarisation part ($a_N + b_N = 1$)

$$\begin{array}{rcrcl} p & = & a_p \cdot p^0 & + & b_p \cdot (n^0 + \pi^+) \\ & = & p^0 & + & b_p \cdot (-p_0 + n^0 + \pi^+) \\ n & = & a_n \cdot n^0 & + & b_n \cdot (p^0 + \pi^-) \\ & = & n^0 & + & b_n \cdot (+p_0 - n^0 + \pi^-) \end{array}$$

The form factors can then be written in the following form:

$$G_{p,n} = G^0_{p,n} + G^{pol}_{p,n}$$

with polarisation components

$$G^{pol}_p = b_p \cdot (G^0_n - G^0_p + G^{\pi^+})$$

$$G^{pol}_n = b_n \cdot (G^0_p - G^0_n + G^{\pi^-})$$

The pion-cloud model

- Bare nucleons:
  Constituent quarks $p = (uud)$ und $n = (udd)$
  Ansatz of dipole type for quark q in nucleon N:

$$G^{qN}(Q^2) = \frac{a_0^{qN}}{\left(1 + Q^2/a_1^{qN}\right)^2}$$

  The proton is described by the dipoles G^{up} and G^{dp}, the neutron correspondingly.

- Pions:
  p wave ($l = 1$) due to negative parity
  Ansatz as oscillator wave function

$$G^{\pi}(Q^2) = a_0^{\pi} \cdot (1 - \frac{1}{6}(Q/a_1^{\pi})^2)\, e^{-\frac{1}{4}(Q/a_1^{\pi})^2}$$

Model ansatz

... for electric form factors

The proton:

$$\begin{aligned} G_{ep} = {} & (G^{u,p}_{ep} + G^{d,p}_{ep}) \\ & + b_p \cdot (- (G^{u,p}_{ep} + G^{d,p}_{ep}) + (G^{u,n}_{ep} + G^{d,n}_{ep}) + G^{\pi^+}_{e}) \end{aligned}$$

The neutron:

$$\begin{aligned} G_{en} = {} & (G^{u,n}_{en} + G^{d,n}_{en}) \\ & + b_n \cdot ((G^{u,p}_{en} + G^{d,p}_{en}) - (G^{u,n}_{en} + G^{d,n}_{en}) + G^{\pi^-}_{e}) \end{aligned}$$

Normalisation:

$$\begin{aligned} p:\ G_{ep}(0) &= G_{ep}(0) + b_p[-G^0_{ep}(0) + G_{\pi^+}(0)] = 1 + 0 \\ n:\ G_{en}(0) &= \quad 0 \quad + b_n\underbrace{[\ G^0_{ep}(0) + G_{\pi^-}(0)]}_{=0} = 0 + 0 \end{aligned}$$

Polarisation part: charge = 0

"Bump" = pion part: charge = $+b_p$, $-b_n \neq 0$!!

Amplitudes of dipole formfactors in smooth distribution ...

... in proton: $a^{u,p}_{0,e} = \frac{4}{3}$, $a^{d,p}_{0,e} = -\frac{1}{3}$

... in neutron: $a^{u,n}_{0,e} = \frac{2}{3}$, $a^{d,n}_{0,e} = -\frac{2}{3}$

6 free parameters per nucleon:

Proton: $a^{u,p}_{1,p}$, $a^{d,p}_{1,p}$, $a^{u,n}_{1,p}$, $a^{d,n}_{1,p}$, b_p, $a^{\pi^+}_1$

Neutron: $a^{u,n}_{1,n}$, $a^{d,n}_{1,n}$, $a^{u,p}_{1,n}$, $a^{d,p}_{1,n}$, b_n, $a^{\pi^-}_1$

Model ansatz

… for magnetic form factors

The proton:

$$\begin{aligned} G_{mp} &= \big(G^{u,p}_{mp} + G^{d,p}_{mp}\big) \\ &\quad + b_p \cdot \big(- \big(G^{u,p}_{mp} + G^{d,p}_{mp}\big) + \big(G^{u,n}_{mp} + G^{d,n}_{mp}\big) + G^{\pi^+}_{m} \big) \end{aligned}$$

The neutron:

$$\begin{aligned} G_{mn} &= \big(G^{u,n}_{mn} + G^{d,n}_{mn}\big) \\ &\quad + b_n \cdot \big(\big(G^{u,p}_{mn} + G^{d,p}_{mn}\big) - \big(G^{u,n}_{mn} + G^{d,n}_{mn}\big) + G^{\pi^-}_{m} \big) \end{aligned}$$

Normalisation of all terms unknown

→ 11 parameters per nucleon: non determinable

Ansatz: an “inner” quark distribution
+ an “outer” quark distribution
+ pion distribution

$$G_m = a_0^{out} \cdot G^{out} + a_0^{in} \cdot G^{in} + a_0^{\pi} \cdot G^{\pi}$$

G^{out}, G^{in}: dipole form factors
G^{π}: 1p wave in harmonic oscillator

Pion-cloud model: Interpretation

- In the phenomenological model:

  Deviation from smooth distribution,
  “bump” at $Q^2 = 0.2\,(\mathrm{GeV}/c)^2$

- Physical interpretation?

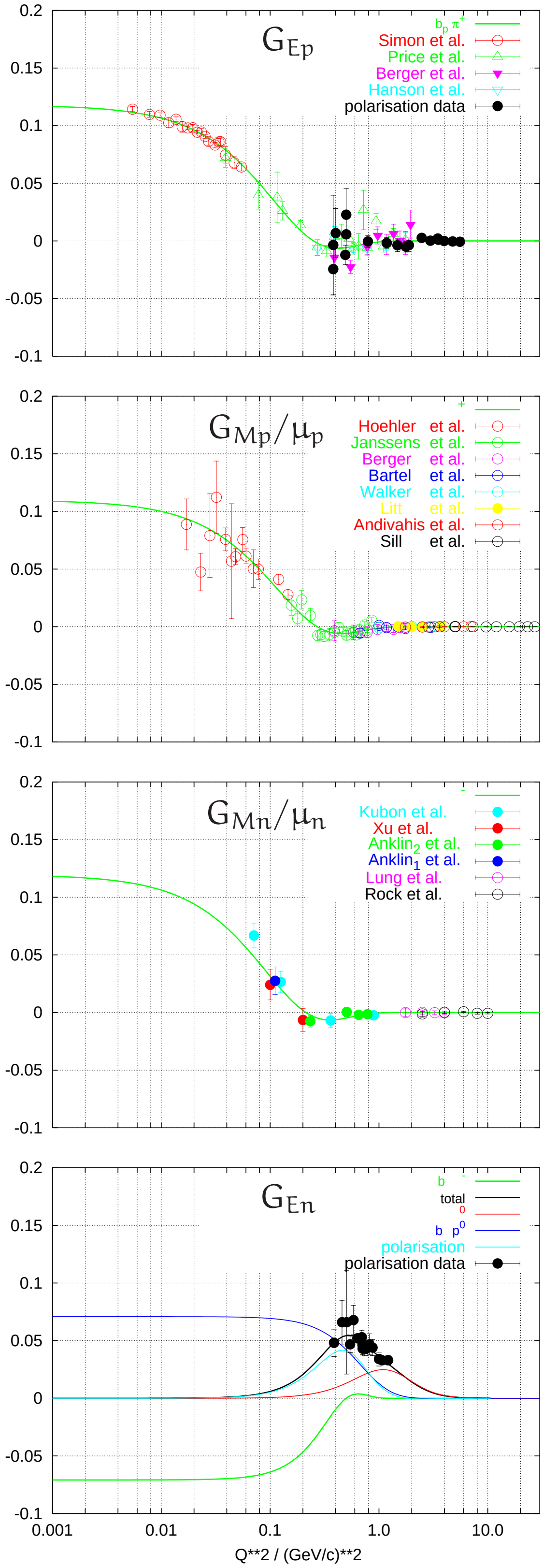
  Bare nucleon $+$ polarisation component (pion cloud)

- Pion cloud
  - Bump at $Q^2 = 0$
  - Polarisation component $\approx 10\,\%$
  - Leads to shoulder in distributions,
    extending beyond 2 fm.

- Interpretation of G_{En}
  - Main part from polarisation (“Galster” like)
  - Bare neutron at considerably smaller radius r

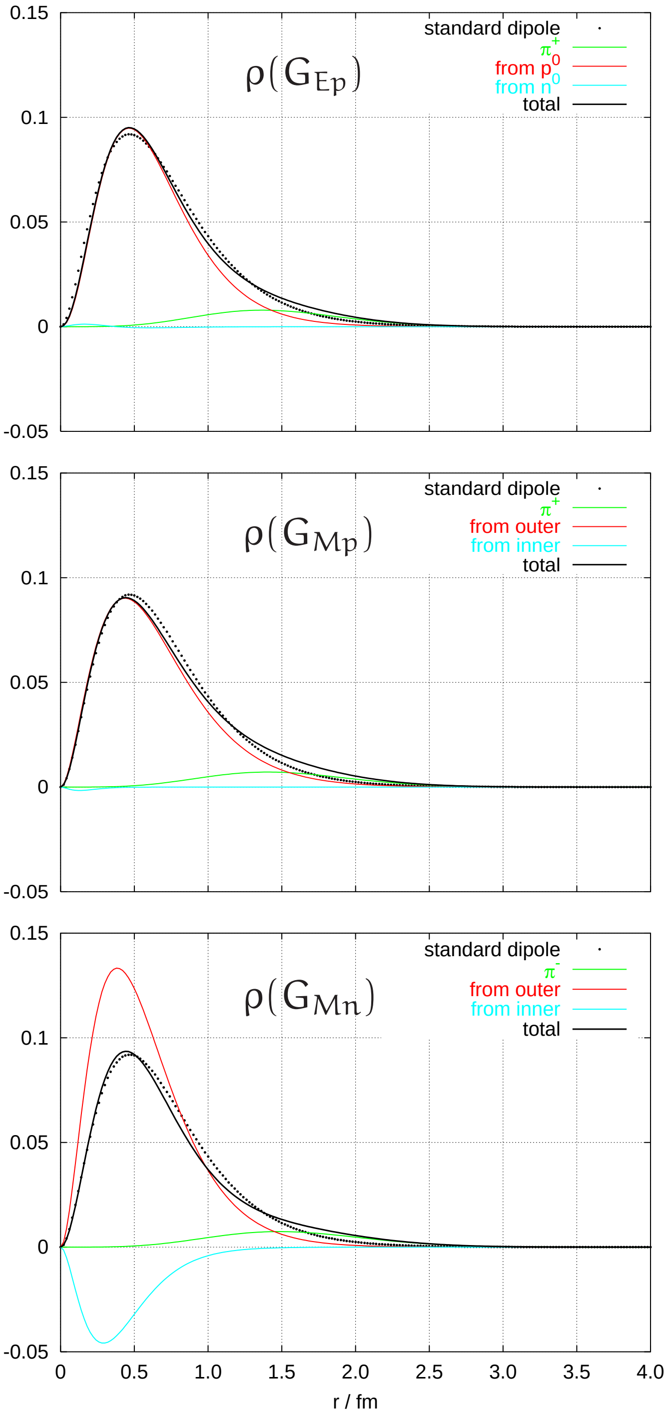
Pion cloud contribution to the form factors

Data of standard form factors: Measurement — two dipoles



Distributions in the Breit system

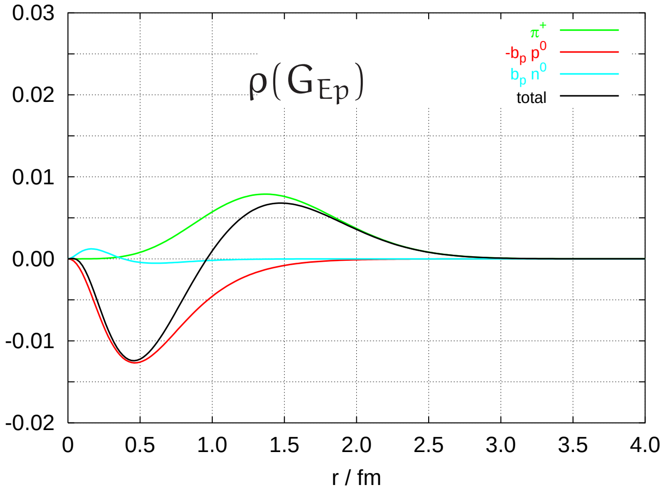
Model: pion cloud (1p wave) and quarks



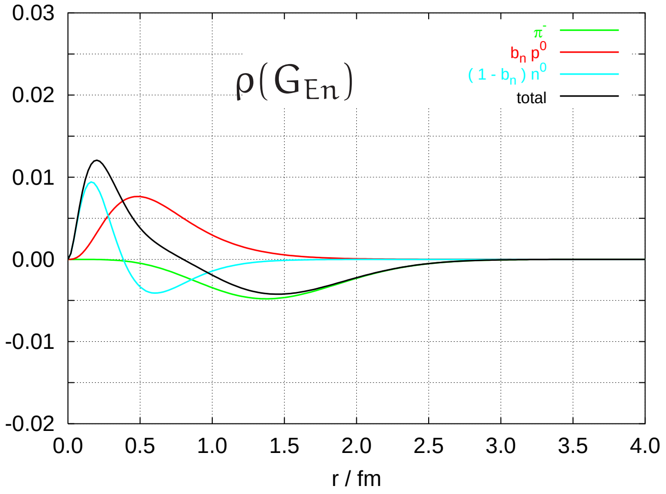
Charge distributions in the Breit system

Model: pion cloud (1p wave) and quarks

Polarisation in the proton:



Charge in the neutron:



Summary

New model-independent measurements of nucleon elastic form factors (particularly G_{En}) in double polarisation experiments.

All elastic form factors of the nucleon exhibit a similar structure at momentum transfers near $Q^2 = 0.2\,(\mathrm{GeV}/c)^2$.

Physically motivated ansatz als bare nucleon with a pion cloud:

- Pions: p wave, polarisation part $\approx 10\,\%$
- Nucleon: Superposition of dipoles interpreted as distribution of constituent quarks

Further experiments, in particular at very small (and here very precise!) and large momentum transfers are much needed for the understanding of the nucleon.