Three Nucleons at Very Low Energies

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Bedaque/hg/Hammer/Rupak: *Nucl. Phys.* **A714** (2003), 589 [nucl-th/0207034]

Bedaque/hg: *Nucl. Phys.* **A671** (2000), 357 [nucl-th/9907077]

Bedaque/Gabbiani/hg: *Nucl. Phys.* **A675** (2000), 601 [nucl-th/9911034]

Reviews: Beane/Bedaque/Haxton/Phillips/Savage: nucl-th/0008064;

Bedaque/van Kolck: [nucl-th/0203055]

1. Why Few Nucleon Physics at Very Low Energies?

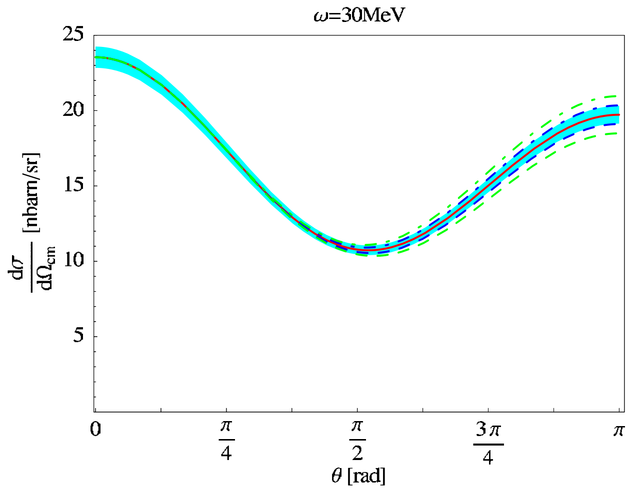
Need for model-independent, systematic predictions and extractions.

– Determination of fundamental neutron properties from light nuclei?

No free neutron targets $\Longrightarrow$ How strong are nuclear binding effects?

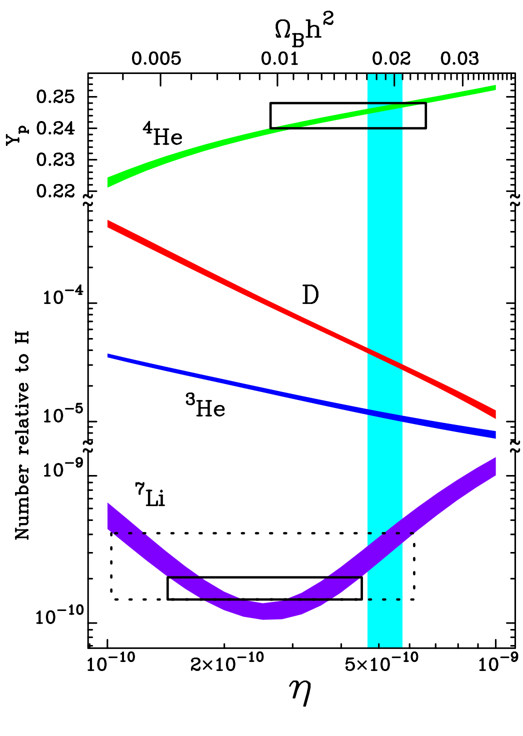
e.g. neutron-polarisabilities from Compton scattering on deuteron $\gamma d \to \gamma d$ at $30\ \mathrm{MeV}$

(hg/Rupak 1999 for TUNL-proposal)



– Plethora of pivotal physical processes at $E < 1\ \ \mathrm{MeV}$ which are hard to access experimentally (rates, targets, . . .):

- Big Bang Nuclear Synthesis, e.g. dependence of light element abundances on baryon density.
- neutrino-nucleus interactions, e.g. calibrating SNO by νd elastic & inelastic scattering to test Standard Model
- Stellar Evolution, e.g. proto-star lifetime, supernova explosion



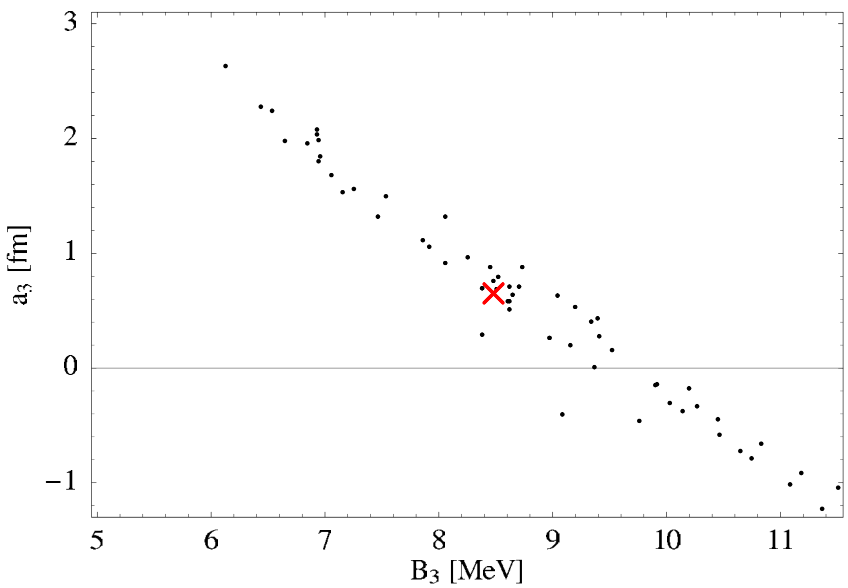
(b) The Three-Body Force Puzzle

Given NN scattering to very high precision, predict 3-body system?

Then, one could determine properties of all nuclei by getting NN right.

Answer: No. NN potentials match all two-nucleon data to $\chi^2/\text{d.o.f.} \approx 1$, but vast differences in predicted triton binding energy and scattering length:

Phillips line (1969)



nd scatt. length vs. triton binding energy

×: exp.

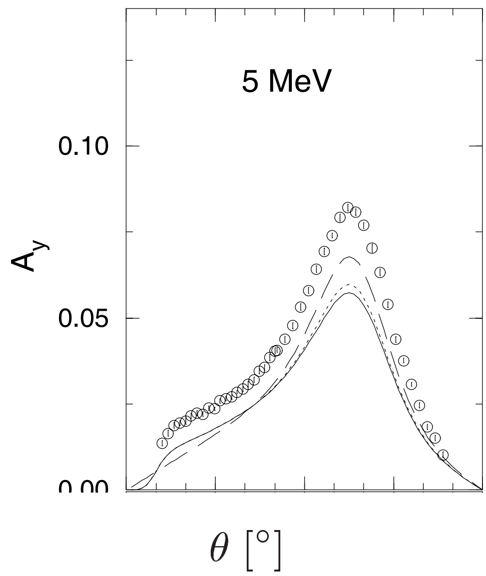
$\Longrightarrow$ At least one Three-body force exists: New physics special to three-body system. Origin?

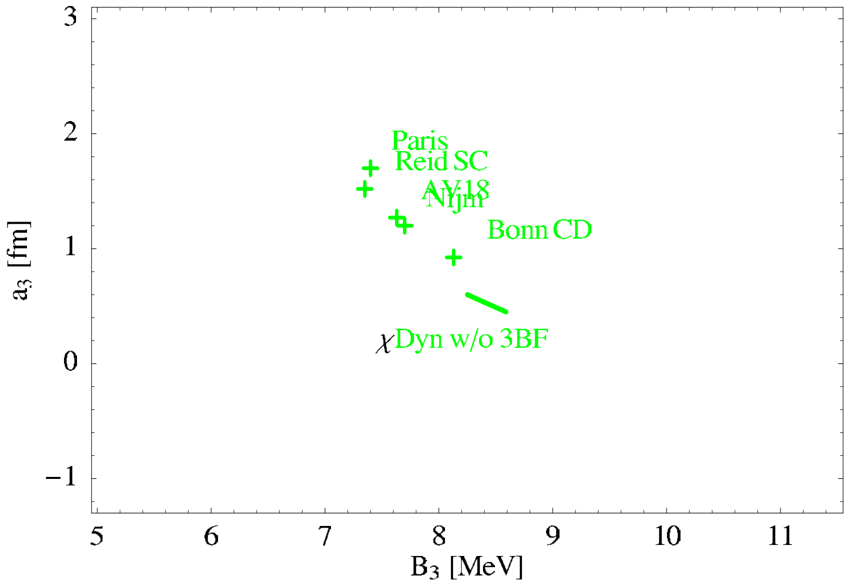
Traditionally, some ad-hoc three-body forces added to make up for difference to experiment.

e.g. A_y-puzzle: spin-asymmetry in $\vec{N}d$ scattering at $1-30\ \text{MeV}$: no good 3-body force found yet.

Predict relevance/size of 3-body contributions?

$\Longrightarrow$ Systematic, model-independent approach to low-energy physics needed





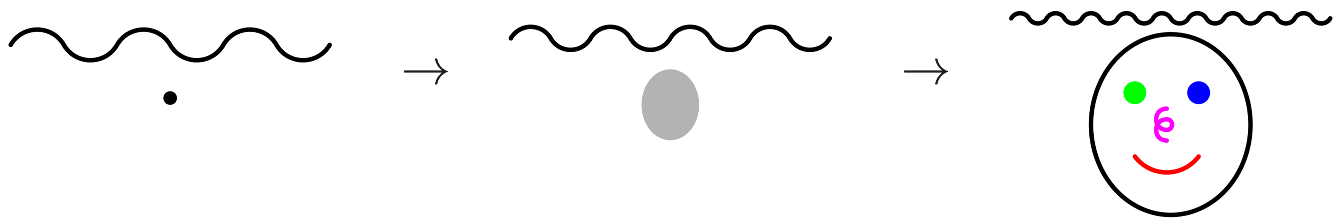
3
2
1
0
−1
a_3 [fm]
Paris
Reid SC
AV18
Nijm
Bonn CD
χDyn w/o 3BF
5
6
7
8
9
10
11
B_3 [MeV]

2. A Cartoon of Effective Field Theory (Weinberg 1991, ...)

(a) EFTs use Separation of Scales

To probes with typical momentum q, an object of size R appears

point-like when $q \ll \frac{1}{R}$, blurry for $q \lesssim \frac{1}{R}$, composed for $q \gtrsim \frac{1}{R}$.



QCD: Different *effective* degrees of freedom at different energies

$q \gtrsim 1\mathrm{GeV}$: quarks and gluons

$q \lesssim 400$ MeV: Chiral Perturbation Theory with nucleon, pion

$q \lesssim m_\pi$: nucleon as effective low-energy degree of freedom;

pion not seen $\Longrightarrow$ breakdown scale $\Lambda \sim m_\pi \sim \frac{1}{R_N}$.

$\Longrightarrow$ Systematic expansion in small parameter:

$$Q = \frac{\text{typ. momentum}}{\text{breakdown scale}} = \frac{q}{\Lambda} \sim q\, R_N \quad : \quad \Lambda \sim \frac{1}{R_N}$$

$\Longrightarrow$ ***Effective Field Theory***: Separation of scales factorises physics into

– correct IR behaviour: local interactions between "***Effective Fields***";

– complex UV physics, integrated out into local coefficients/operators, ordered by small parameter: simpler short-distance behaviour.

Physics cannot depend on details at short distance, cut-off.

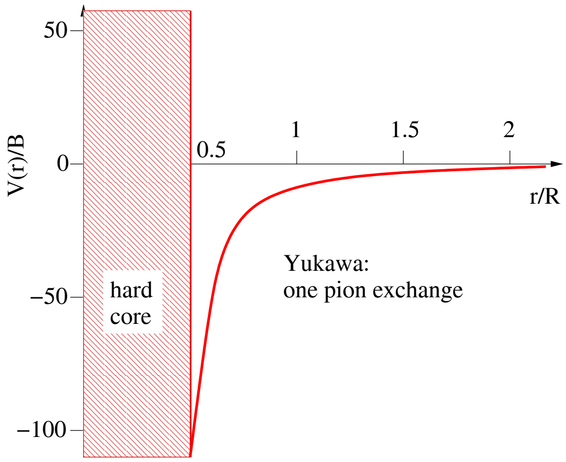
Knowing underlying theory not necessary: Model-independent, ***Theory***.

Comparing Deuteron Wave Functions: Unnatural scales in NN

$$-1/a(^1\mathrm{S}_0) = 8\ \mathrm{MeV},$$
$$\gamma_{\mathrm{deut.}} = 45\ \mathrm{MeV} \ll m_\pi,\ \Lambda_{\mathrm{QCD}}$$

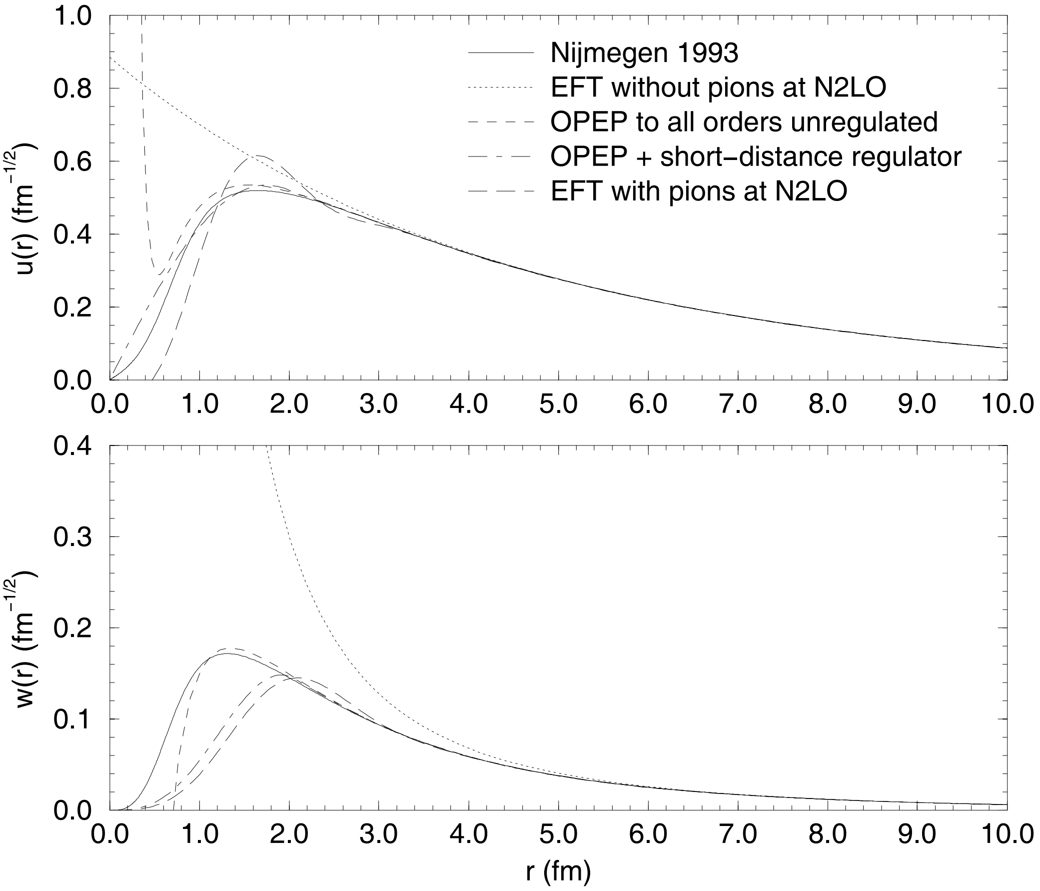
Traditional potential models use (unphysical) hard core at short distances to regulate One Pion Exchange:

Fine-tuning.



$^3\mathrm{S}_1$

$^3\mathrm{D}_1$



Indeed: Short distance very different but unimportant,

long distance asymptotics identical: $\propto \frac{\mathrm{e}^{-\gamma r}}{r}$,

mid distance feels pion effects.

Systematic expansion in $Q = \frac{\text{typ. momentum}}{\text{breakdown scale}} = \frac{\text{target size}}{\text{resolution}} \ll 1$

(b) Interactions and Q Power Counting in EFT($\not\pi$)

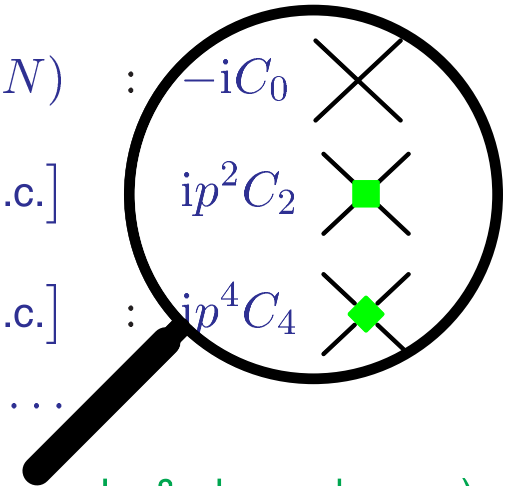
(Bethe 1949, Kaplan/Savage/Wise & van Kolck 1997)

Symmetries independent of scale $\Longrightarrow$ preserved at low energies

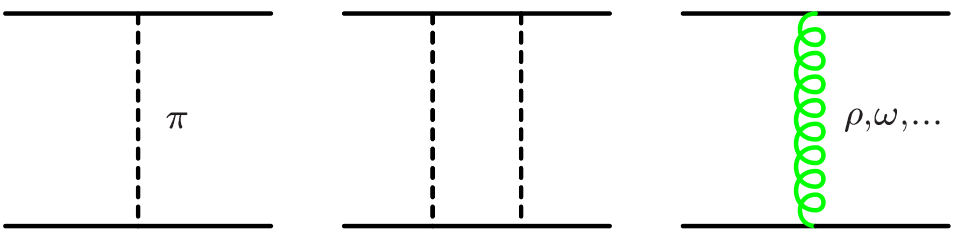
Most general Lagrangean out of local interactions between low-energy degrees of freedom respecting all symmetries of underlying theory (Galilei/Lorentz order by order, particle conservation, flavour, gauge, ...):

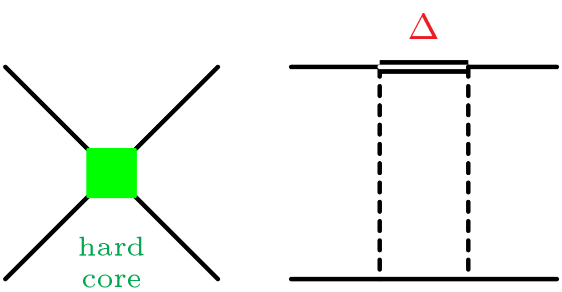
$N = \binom{p}{n}$, P^i: Projector on $^3S_1$ channel

$$- C_0 (N^T P^i N)^\dagger (N^T P^i N) \quad :$$
$$+ \frac{C_2}{8} \left[(N^T P^i N)^\dagger (N^T P^i \overleftrightarrow{\partial}^2 N) + \text{H.c.} \right] \quad :$$
$$+ \frac{C_4}{8^2} \left[(N^T P^i N)^\dagger (N^T P^i \overleftrightarrow{\partial}^4 N) + \text{H.c.} \right] \quad :$$
$$+ \ \ldots$$



Coefficients C_{2n} encode UV physics (π, Δ, ω, quarks & gluons, branes) as strengths of point-like interactions.





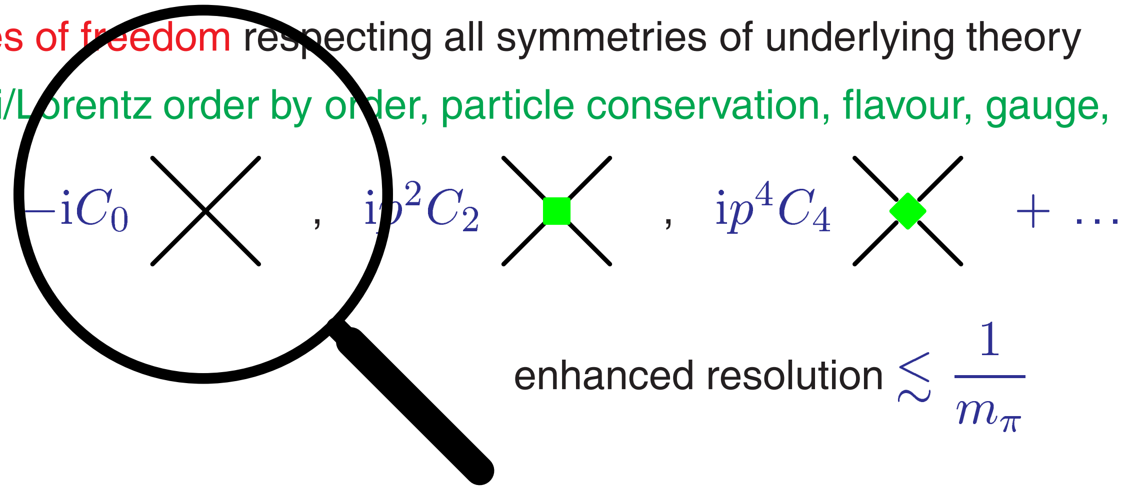
etc.

+ quarks, gluons
+ electro-weak
+ strings, branes
+ whacks

(b) Interactions, Q Power Counting and the Deuteron in EFT($\not{\pi}$)

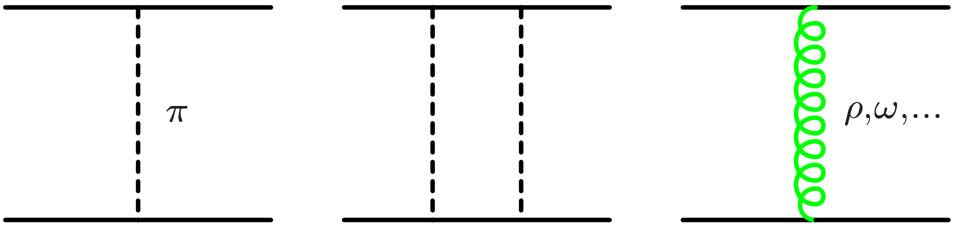
(Bethe 1949, Kaplan/Savage/Wise & van Kolck 1997)

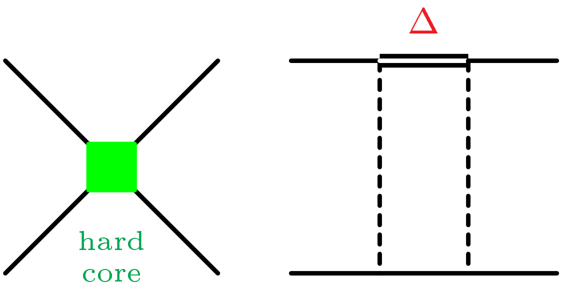
Most general Lagrangean out of local interactions between low-energy degrees of freedom respecting all symmetries of underlying theory (Galilei/Lorentz order by order, particle conservation, flavour, gauge, ...):



enhanced resolution $\lesssim \dfrac{1}{m_\pi}$

Coefficients C_{2n} encode UV physics (π, Δ, ω, quarks & gluons, branes) as strengths of point-like interactions.





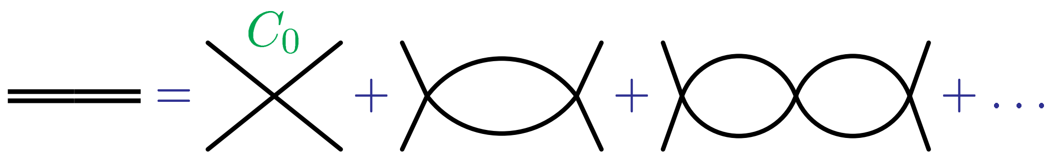
etc.

+ quarks, gluons
+ electro-weak
+ strings, branes
+ whacks

Typical p-resolution $\gamma = \sqrt{M B_{\text{deut}}} \approx 45\ \text{MeV}$: $Q = \dfrac{\gamma}{\Lambda \approx m_\pi} \simeq \dfrac{1}{3}$

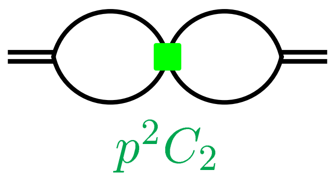
At very large distances, only momentum-independent interactions survive:

LO (30%) :



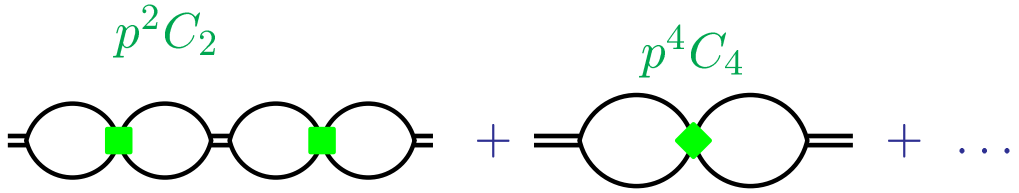
$C_0(\mu) = \frac{4\pi}{M} \frac{1}{\gamma - \mu}$: correct asymptotics $\rightarrow \Psi_d(r) \propto \dfrac{e^{-\gamma r}}{r}$

NLO (10%) :



C_2: correct asymptotic normalisation Z_d

N2LO (3%) :



$\longrightarrow$ D wave contribution etc.

Extensions simple: Relativistic effects, retardation, exchange currents, inelastic reactions, . . . , gauge inv.: $\partial_\mu \rightarrow \partial_\mu + \text{i}e\, Q\, A_\mu$

Parameters C_{2n} can not (yet) be calculated from QCD

(d) Why Theorists Like the Three-Nucleon System

- calculable ("precision")
- clean (simple constituents)
- non-trivial (bound states, many observables)
- rich (electro-magnetic, strong, weak)
- good sale (big bang nucleo-synthesis, stellar evolution, triton β-decay, neutrino masses, Bose-Einstein condensates, atomic trimers,. . .)

Understanding three nucleons is a

minimal criterion

for success in the microscopic description of heavier systems.

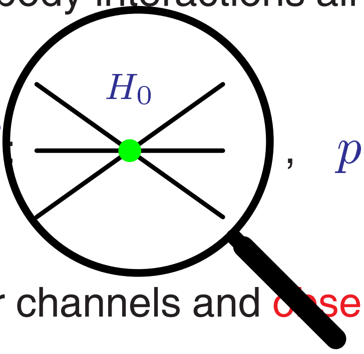
3. The Three-Body System: Neutron-Deuteron Scattering

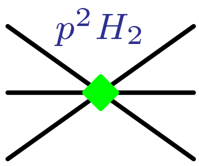
(a) Three-Body Forces in EFT($\not\pi$)

In this simple theory, is 3-body system ruled by 2-body interactions?

EFT: Write down all interactions permitted by the symmetries of QCD

$\Longrightarrow$ 3-body interactions already present:

$H_0 \left(N^\dagger N\right)^3$: 

, $p^2\ H_2 \left(N^\dagger N\right)^3$: 

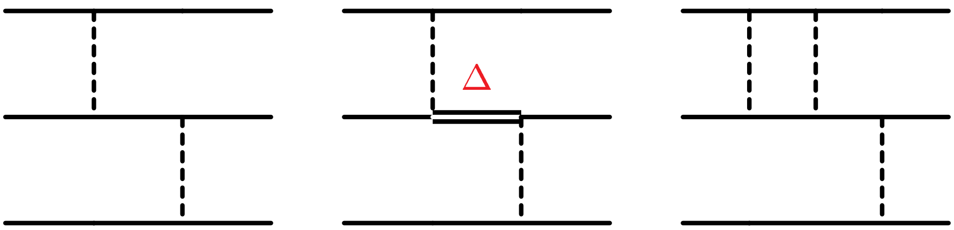
, etc.

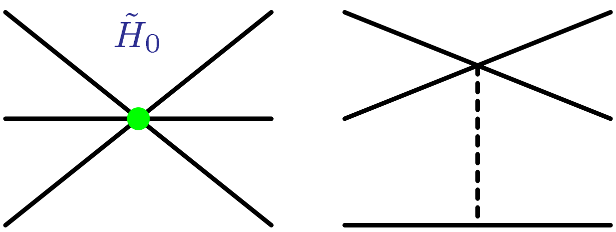
$\Longrightarrow$ Look for channels and observables most sensitive to these new forces.

How important are they?

At which order in Q do they start to contribute?

What are they?

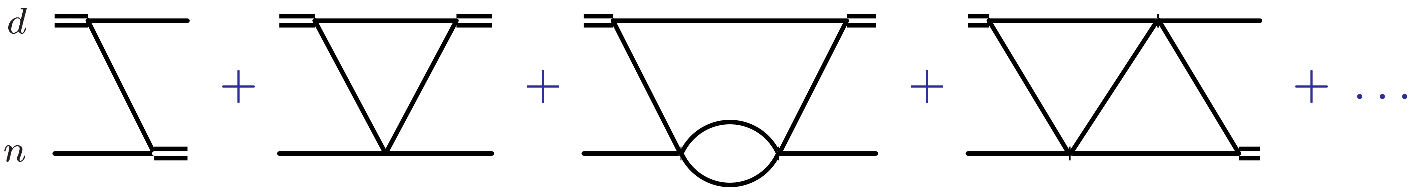




etc.

(b) 3-Body System in EFT: Nd scattering, Quartet Channels

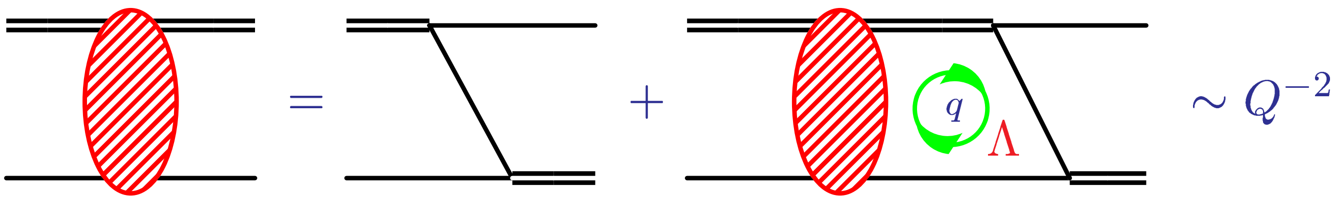
"Pinball diagrams": "exchange" of a nucleon, $\mathcal{O}(Q^{-2})$



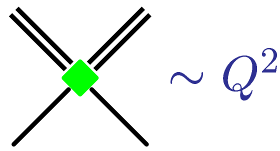
Sum deuteron and use iteration to Faddeev integral equation

for half off-shell amplitude $\mathcal{A}(k,q)$: (Skorniakov/Ter-Martirosian 1957)

LO:



All spins up $\Longrightarrow$ Pauli exclusion, 3-body force p-dependent:

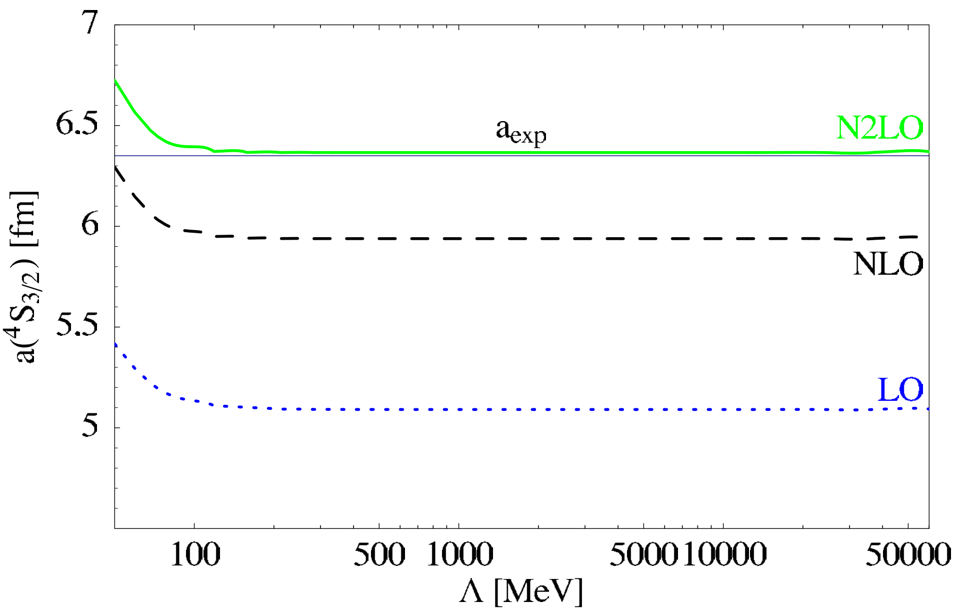


$\Longrightarrow$ negligible for low momenta, N4LO naïvely.

nd scattering length, Quartet-S wave

Observable cut-off independent, good convergence.

parameter-free



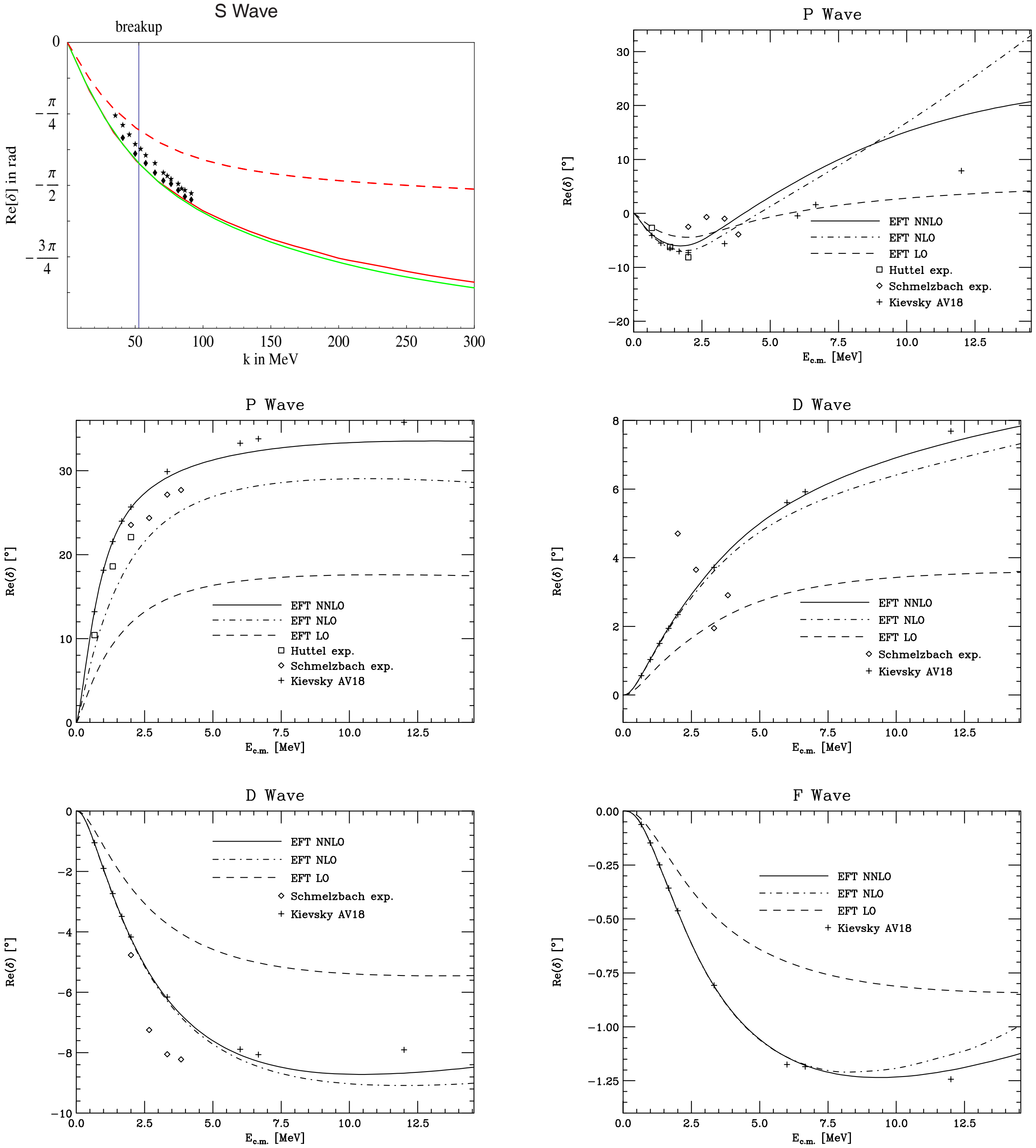
Similar picture in all Quartet partial waves, and in Doublet higher waves:

Pauli exclusion or centrifugal barrier forbids momentum-independent 3BF.

Phase Shifts of Neutron-Deuteron Scattering to 3% Accuracy

Bedaque/hg: NP**A671**(2000), 357; Bedaque/Gabbiani/hg: NP**A675**(2000), 601.

Quartet Channel $(s = \frac{3}{2})$ Doublet Channel $(s = \frac{1}{2})$

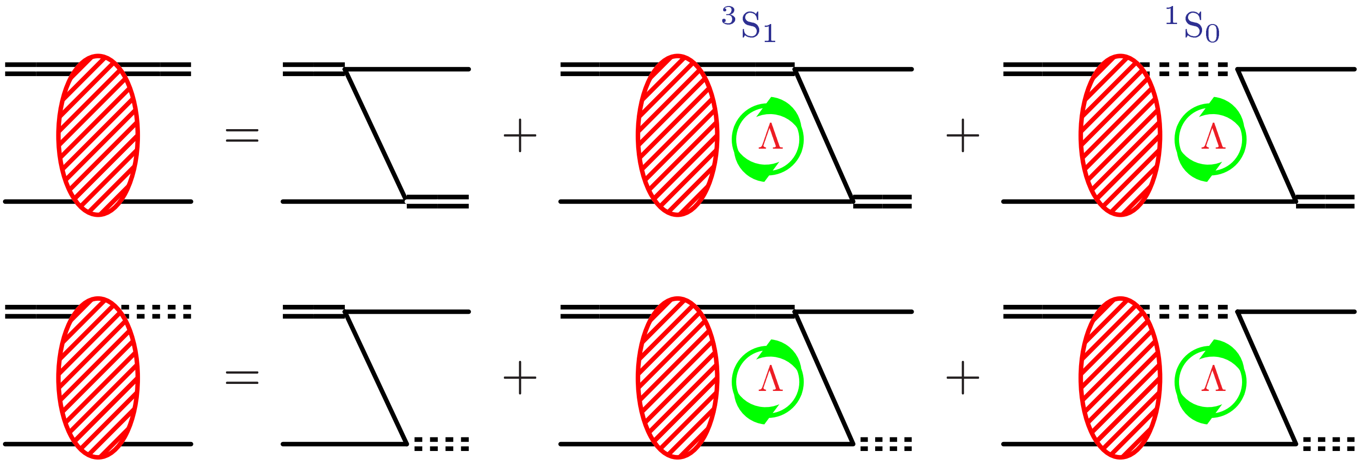


Numerically simple: N2LO code runs within a minute on PC

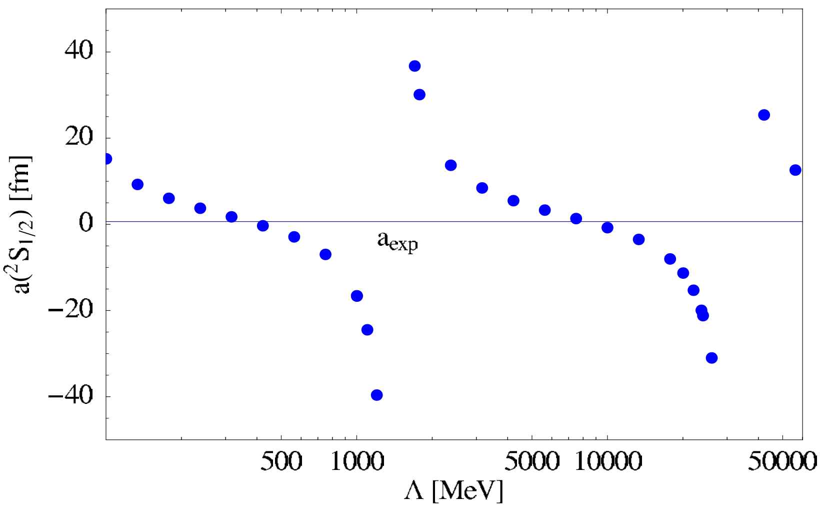
Agrees well with sophisticated, modern potential model calculations.

N3LO (3-body force!): Splitting/mixing of partial waves $\Longrightarrow A_y$

(d) The Problem: nd-Scattering, ${}^2\mathrm{S}_{\frac{1}{2}}$ Wave ("triton channel")



No Pauli principle, no centrifugal barrier $\Longrightarrow$ 3-body forces at N2LO?



Danilov, Minlos/Faddeev 1961:

Slight cut-off variation has dramatic effect on scattering length $a({}^2\mathrm{S}_{\frac{1}{2}})$.

$\Longrightarrow$ Can 3-body scattering length not be extracted independently of UV?

Naïve interpretation: As cut-off is removed, more and more 3-body bound states appear, each being first shallow and then getting deeper and deeper.

Thomas Effect (1935): Fourier: $\left[\frac{1}{R}\frac{\partial}{\partial R}R\frac{\partial}{\partial R} - \frac{{s_0}^2}{R^2} + ME\right]F_0(R) = 0.$

$s_0^2 < 0$: attractive $\frac{1}{r^2}$-pot. has infinitely many, deeply bound states.

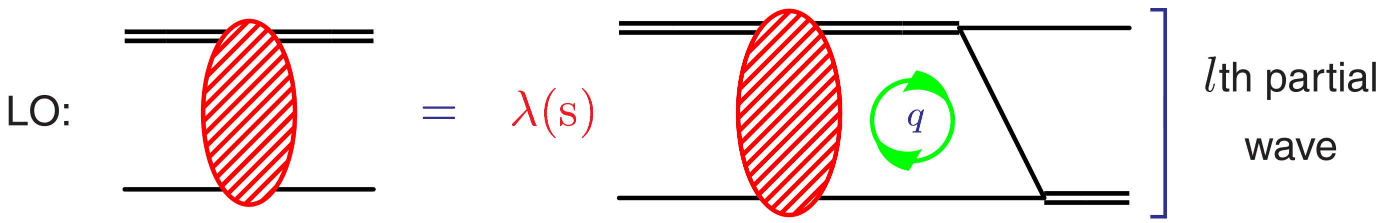
Efimov Effect (1971): As 2-body scatt. length $a(np) \to \infty$, infinitely many shallow 3-body bound states for $\Lambda \to \infty$.

Cut-off Dependence: Numerical Glitch or Real Problem?

(Danilov 1961; Bedaque/Hammer/van Kolck 1998; hg)

UV physics $\Longrightarrow$ drop all external scales: $\Lambda \gg q \gg k,\ \gamma,\ \dots$

Faddeev equations decouple: Wigner's $\mathrm{SU}(4)$-(spin-flavour)-symmetry



$$\mathcal{A}_{(l,s)}(0,p) \quad = \quad \frac{4\ \lambda(\mathrm{s})}{\sqrt{3}\,\pi} \int\limits_0^\infty \frac{\mathrm{d}q}{p}\ \mathcal{A}_{(l,s)}(0,q)\ Q_l\left[\frac{p}{q}+\frac{q}{p}\right]$$

Naïvely: UV $\sim \frac{1}{p^2}$ $\Longrightarrow$ no cut-off dependence.

UV-behaviour of half off-shell amplitude $\mathcal{A}_{(l,s)}(0,p)$ depends crucially on parameters λ (spin) and l (partial wave) considered.

Ansatz: $\mathcal{A}_{(l,s)}(0,p) \propto \dfrac{p^{-s_0}}{p}$ Naïve: $\dfrac{1}{p^2}$: $s_0 = 1$

$s_0(l,s)$	$l=0$	$l=1$	$l=2$	$l=3$
Quartet ($s=\frac{3}{2}$): $\lambda=-\frac{1}{2}$	2.16	2.27	3.10	4.04
Doublet ($s=\frac{1}{2}$): $\lambda=1$	$\pm 1.0062\ \mathrm{i}$	1.56	2.82	3.92

UV-arbitrariness at LO only in triton (${}^2\mathrm{S}_{\frac{1}{2}}$) channel:

$$\mathcal{A}_{(l=0,s=\frac{1}{2})}(k\to 0,q) \propto \frac{\cos[1.0062\ln q + \delta]}{q}$$

arbitrary phase δ, $\mathcal{A}$ not square integrable $\Longrightarrow$ undetermined on shell.

(e) The Solution

(Bedaque/Hammer/van Kolck 1999, +hg+Rupak 2002)

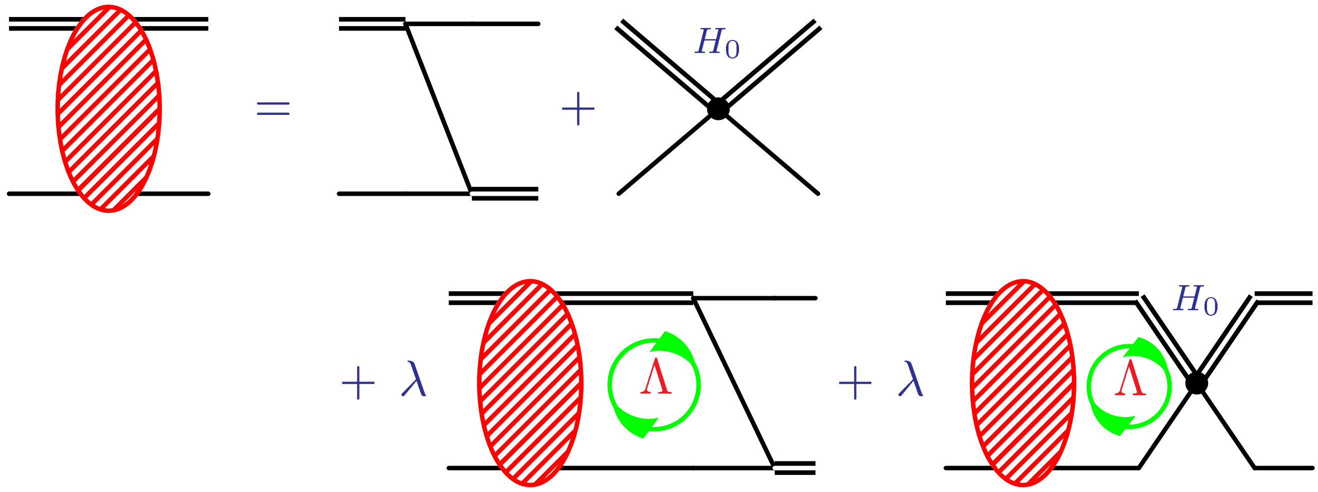
Power counting in one and two-body sector fixed.

No self-consistent Effective Range Expansion in 3-body system!

IR/long-range physics insensitive to UV/short-distance.

Tenet: Include specific 3-body force if and only if counter term needed to cancel cut-off/off-shell dependence of on-shell $\mathcal{A}$.

Spin-flavour $\mathrm{SU}(4)$-symmetric three-body contact interaction with strength $H_0(\Lambda) \sim Q^{-2}$ promoted to LO to absorb cut-off dependence.



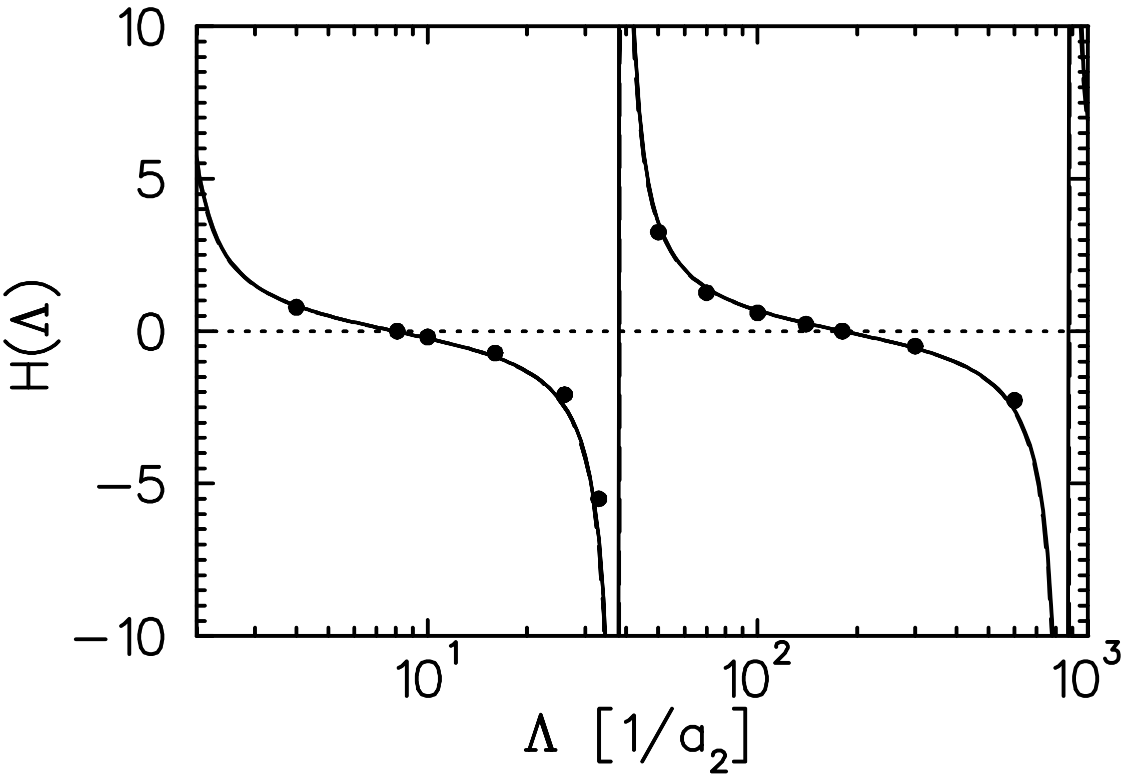
Tune running coupling $H_0(\Lambda)$ such that

Numerically: one observable fixed, e.g. scatt. length.

Analytically: $\mathcal{A}$ cut-off independent; $\mathcal{A}$ known analytically in UV

$$\Longrightarrow \quad H_0(\Lambda) = \frac{\sin[s_0 \ln \frac{\Lambda}{\Lambda_0} + \arctan s_0]}{\sin[s_0 \ln \frac{\Lambda}{\Lambda_0} - \arctan s_0]} \quad \text{at LO for } \Lambda \to \infty.$$

(f) Comparing Numerical and Analytical Cut-off Dependence



numerically: H_0 such that scattering length $a_3 = 0.64 \text{ fm}$.

analytically: H_0 such that scattering amplitude $\mathcal{A}$ cut-off independent in UV.

$H_0(\Lambda)$ has no fixed-point as $\Lambda \to \infty$, but periodic in cut-off.

Limit Cycle: new Renormalisation Group phenomenon.

Different 2-body off-shell behaviour leads to different 3-body force.

Strength of three-body force itself has no physical meaning.

Not three-body force is large, but its effect!

One physical scale Λ_0 must be determined experimentally

such that e.g. $a_3 = 0.64 \text{ fm}$. cf. $\alpha_s \leftrightarrow \Lambda_{\text{QCD}}$ in QCD

Are the other observables cut-off independent, too?

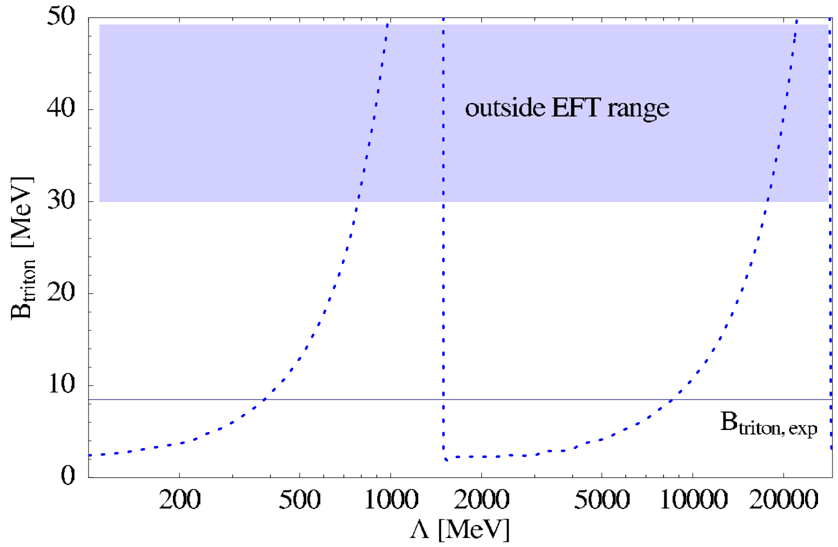
numerically: H_0 such that scattering length $a_3 = 0.64\ \mathrm{fm}$,

i.e. on-shell amplitude cut-off independent at $\mathcal{A}(k = 0, p = 0)$.

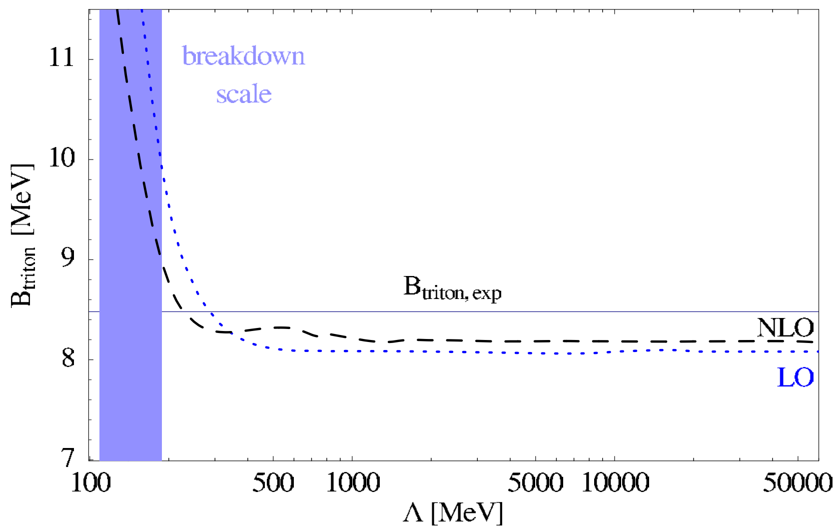
Is on-shell amplitude $\mathcal{A}(k \neq 0, p = k)$ cut-off independent everywhere?

$\Longrightarrow$ Calculate position of pole in $\mathcal{A}$: Triton binding energy B_3.

Cut-off sensitive without three-body force

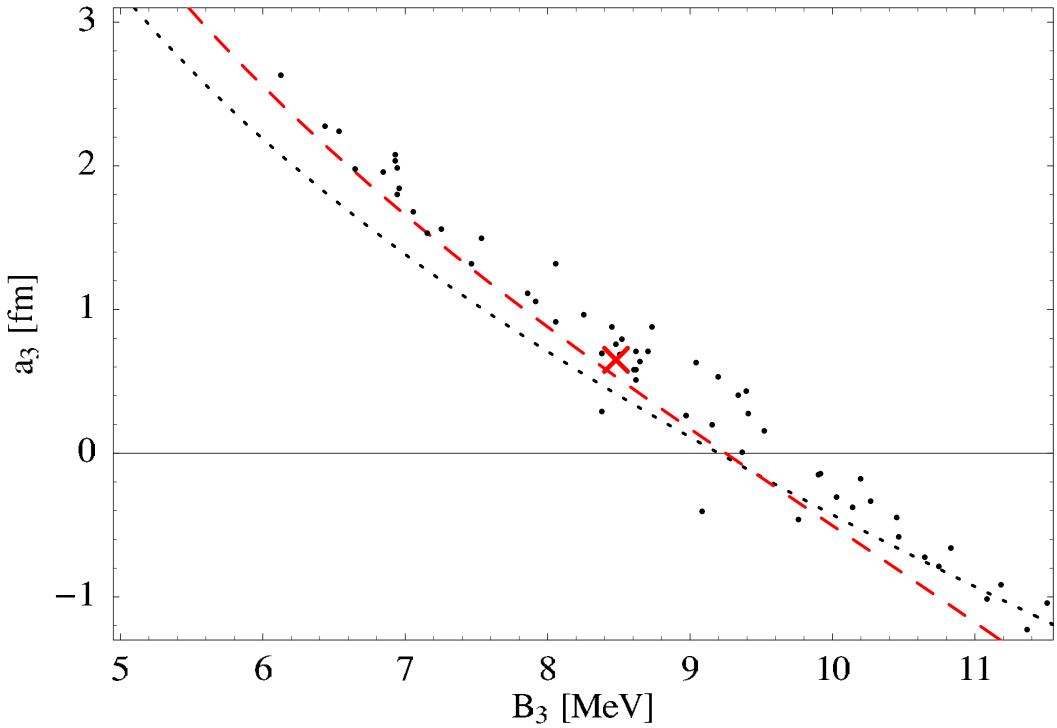


Cut-off in-sensitive with three-body force (here $a_3 = 0.64\ \mathrm{fm}$)



Once a_3 given, B_3 fixed. $\Longrightarrow$ Explains Phillips line.

The one new, free parameter Λ_0 explains Phillips line of Nuclear Physics.



×: exp.

LO EFT

NLO EFT

$a_3 = 0.64$ fm

$\Longrightarrow B_3$ at

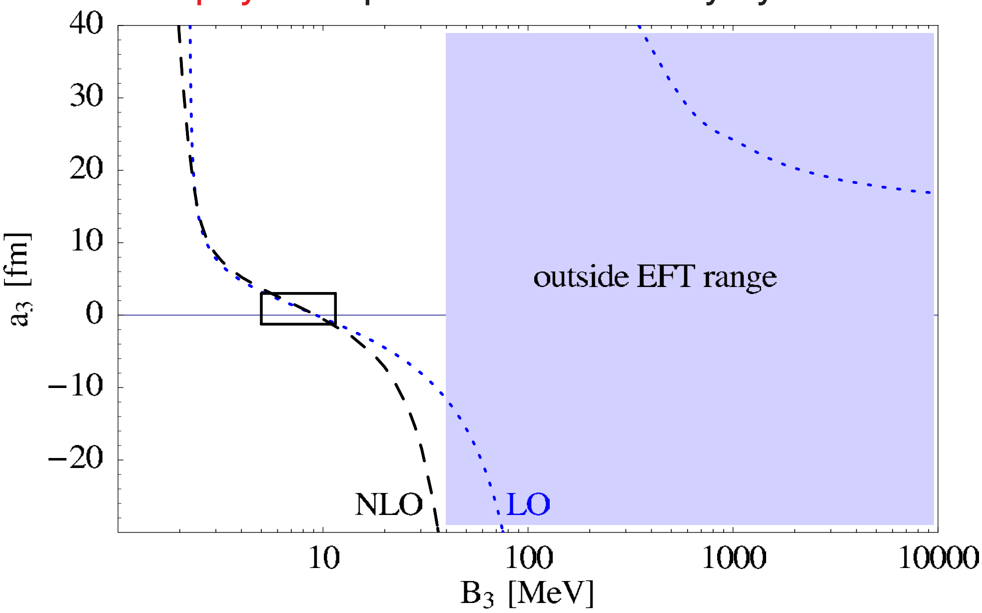
LO: 8.0 MeV

NLO: 8.35 MeV

exp.: 8.48 MeV

$\Longrightarrow$ At least one Three-body force exists:

New physics special to three-body system.

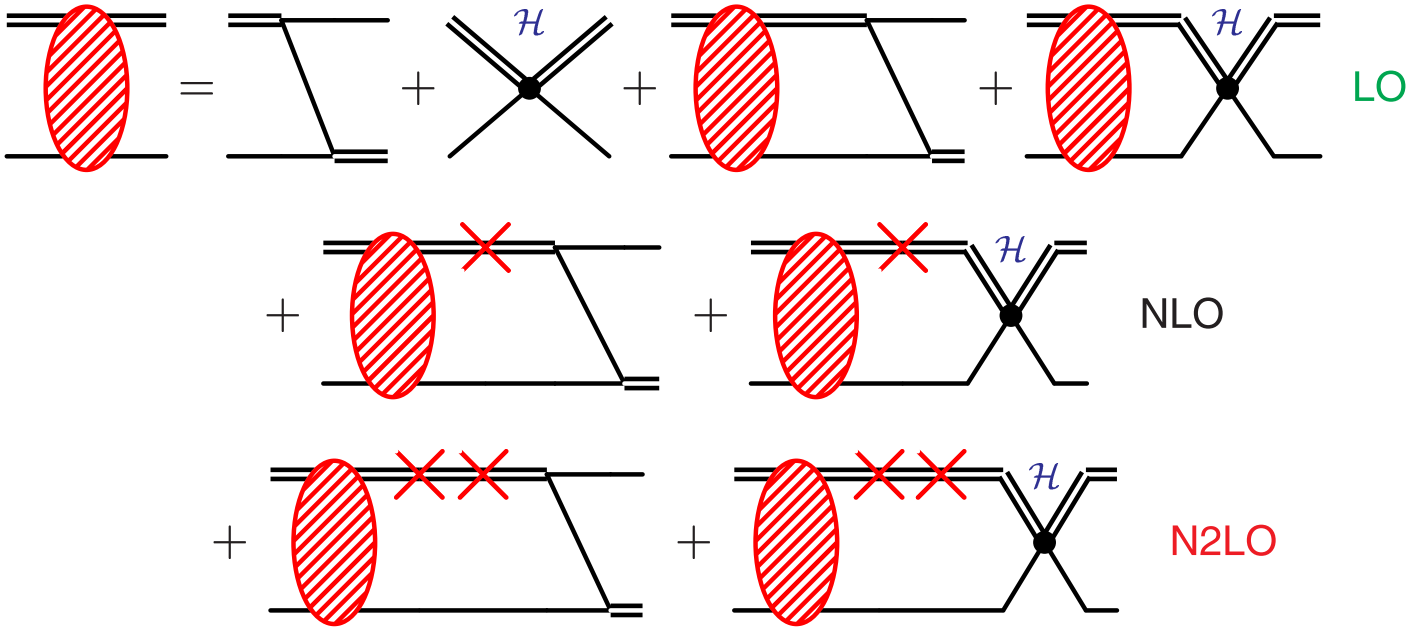


“big picture”: Efimov and Thomas effects

but $B_3 \gtrsim 20$ MeV outside range of EFT applicability (pions).

(g) Higher Orders

(Bedaque/hg/Hammer/Rupak 2002)



Expand *potential* in powers of Q and iterate.

Expand generic 3-body force in on-shell momentum:

$$\mathcal{H}(\Lambda) = H_0(\Lambda) + \frac{k^2}{\Lambda^2} H_2(\Lambda) + \dots$$

EFT Tenet: Include specific 3-body force if and only if counter term needed to cancel cut-off/off-shell dependence of on-shell $\mathcal{A}$.

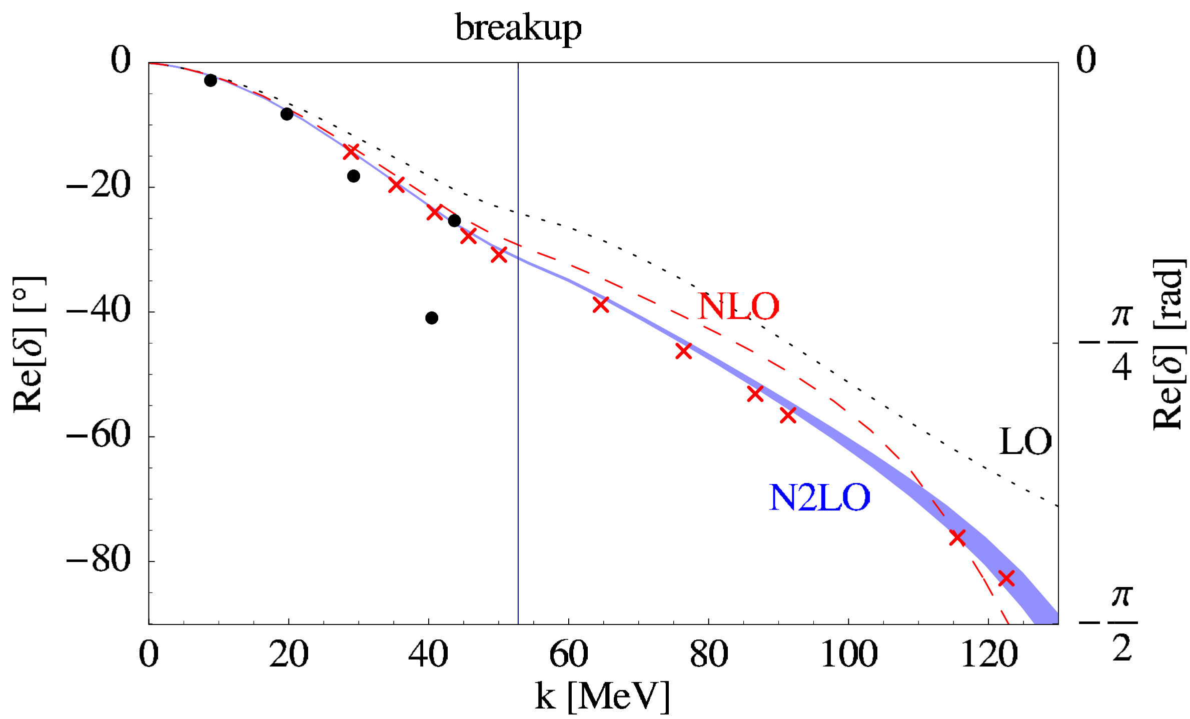
$\Longrightarrow$ 3-body forces Wigner-$\mathrm{SU}(4)$-symmetric in UV limit

LO and NLO ($< 10\%$ accuracy): One free parameter H_0,
fixed e.g. by triton binding energy (very accurately known).
H_0 as important as two-body physics!

N2LO and N3LO ($< 1\%$ accuracy): One more free parameter H_2,
fixed best by scattering length (poorly known).

(h) Doublet-S Wave nd Phase Shift

(Bedaque/hg/Hammer/Rupak 2002)



blue corridor: N2LO with $\Lambda \in [200; \infty]$ MeV:

estimates higher order effects ↔ variation of resolution

×: AV18+Urbana IX (Kievsky 2002)

●: partial wave analysis 1967 (Seagrave/van Oers)

Agrees well with sophisticated, modern potential model calculations, no free parameters after a_3 fixed, plus B_{triton} at N2LO.

Mathematica code: `http://www-nsdth.lbl.gov` or

`http://www.ph.tum.de/~hgrie`

(i) A Problem with Experiment

nd-scattering and $\Psi_{\rm triton}$ need knowledge of three-body forces

Theory error by neglecting "higher order" interactions at N2LO: $1-2\%$

Exp. induced error by uncertainty in nd scattering length: $6-10\%$

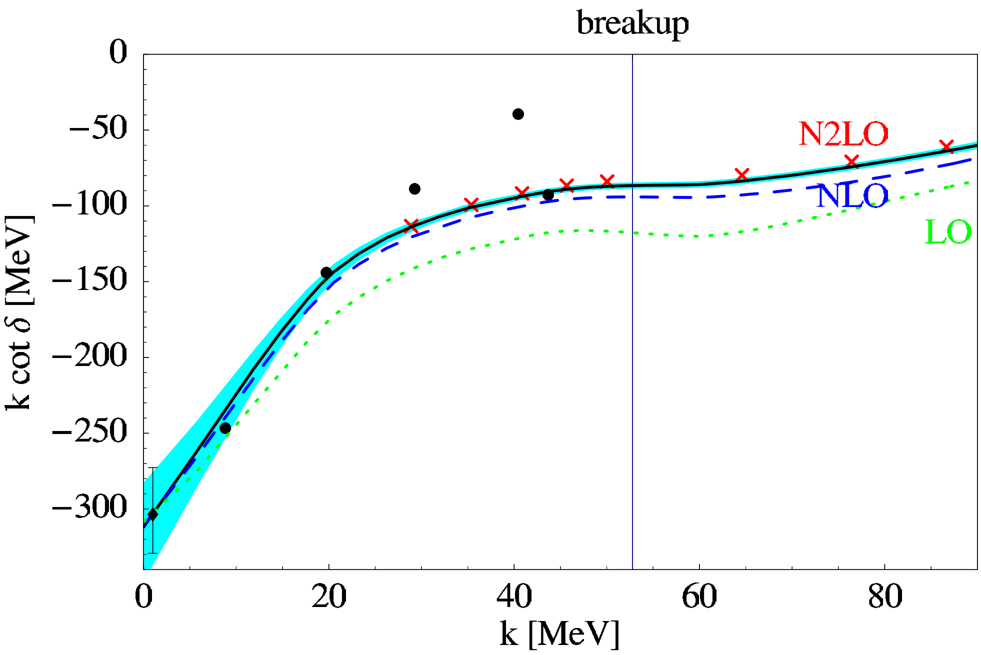
$$a_3(\exp) = [0.64 \pm 0.04] \;\; \text{fm}$$ (Dilg/Koester/Nistler 1971)

Measure for triton wave function at low momenta: "inverse K-matrix":

$$\mathcal{A} \propto \frac{1}{k\cot\delta - \mathrm{i}k} \quad \text{with} \quad k\cot\delta\,|_{k=0} = -\frac{1}{a_3}$$

blue corridor:
EFT prediction
when a_3 varied
within exp. error.

● PWA
× AV18+Urbana IX



Most sensitive to change in scattering length at low momenta.

Re-measure Doublet-S Wave scattering length!

TUM-PSI-Saclay Collaboration Zimmer/Glattli/hg et al., approved by PSI.

(k) Universality of EFT: Lessons Learned Applied Elsewhere

When anomalously large 2-body scatt. length/shallow 2-body bound state:

Hypertriton Λnp (Hammer 2001)

Shallow bound state of Λ-hyperon and deuteron:

$B_3 = [0.13 \pm 0.05]$ MeV (exp.)

$\Longrightarrow a_{\Lambda d} = 16.8^{+4.4}_{-2.4}$ fm, $\quad r_{\Lambda d} = [2.3 \pm 0.3]$ fm

Atomic Physics (Bedaque/Hammer/van Kolck 1999)

Bound state of two He-atoms ("He-dimer") very shallow: $[62 \pm 10]$ Å!

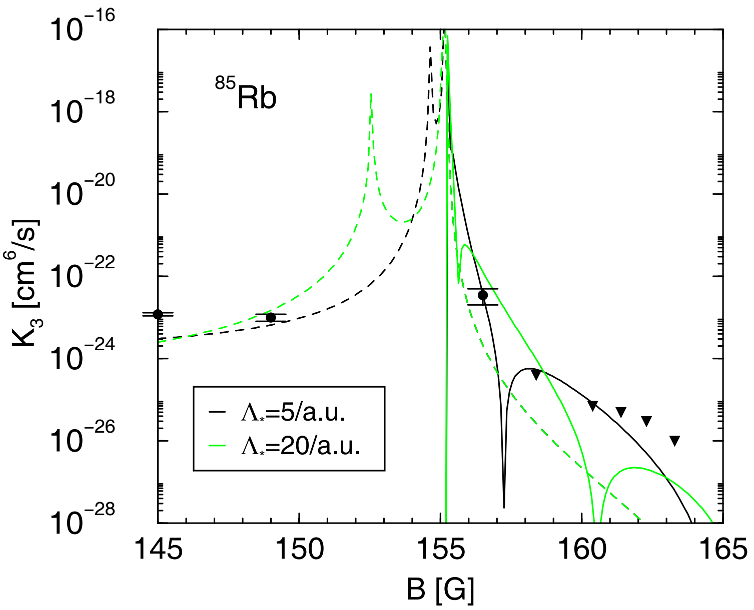
Prediction for ^{4}He trimer: Bound state energy not calculable from dimer properties only. If $a_3 = 195$ Å, predict $B_3 = [1.2 \pm 0.1]$ mK.

Bose-Einstein Condensates (Braaten/Hammer 2001)

Feshbach resonances:
two-body scattering length
tunable at will by magnetic field.

Strong contribution to loss rate
from
three-particle recombination into
shallow three-particle bound state.

Compares well to experiment.



4. Summary and Rewards

(a) Summary

- Few-nucleon system at very low energies:

  3-body force puzzle: Phillips line, A_y-problem etc.

  $\Longrightarrow$ Systematic, model-independent approach needed:

- **Effective Field Theory** with local interactions between nucleons only.

- In most channels, 3-body force only for very high precision.

- Doublet-S wave (triton as bound state):

  3-body observables independent of high-energy/off-shell behaviour

  only if spin-iso-spin symmetric 3-body forces included.

  Naïve power counting is naïve.

- Limit Cycle:

  Only combination of 2-body and 3-body force physically meaningful.

- Need 2 empirical three-body data for 1% accuracy:

  triton binding energy,

  nd scattering length in triton channel (to be determined better!).

- Wide range of applications: atomic trimers, BEC, etc.

- Successful test of EFT methods.

Bedaque/hg/Hammer/Rupak: *Nucl. Phys.* **A714** (2003), 589 [nucl-th/0207034]

Bedaque/hg: *Nucl. Phys.* **A671** (2000), 357 [nucl-th/9907077]

Bedaque/Gabbiani/hg: *Nucl. Phys.* **A675** (2000), 601 [nucl-th/9911034]

(b) Challenges and Rewards Once H and H_2 are Fixed

- model-independent understanding of 3BF at low p to $\lesssim 1\%$ accuracy;
- accurate triton wave function;
- Can H_0, H_2 be saturated by one-pion exchange?
- fundamental three-body processes like $nd \to t\gamma$ at thermal energies:

  AV 18 $+$ Urbana IX potential model : $\sigma = 0.6$ mb

  experiment : $\sigma = [0.508 \pm 0.015]$ mb
- triton form factors, Compton scattering (nucleon polarisabilities);
- comparing $^3\mathrm{H}$ and $^3\mathrm{He}$: iso-spin breaking, charge dependence;
- better description of triton β-decay, parity violations;

  $\longrightarrow$ reduce number of free parameters in search for neutrino masses
- stellar & primordial nucleosynthesis,
  e.g. deuteron abundance sensitive to primordial baryon density.
  theory side uncertainty $\approx$ observational error

  $dd \to pt@100\ \mathrm{keV}$:
  Need for $2 - 3\%$ accurate cross-sections at $30 - 300\ \mathrm{keV}$.

  $^3\mathrm{H}$ and $^3\mathrm{He}$ wave functions biggest source of uncertainty.
  30% of error bar in d abundance
  also $dt \to n\alpha$, $n^3\mathrm{He} \to pt$