Polarized ^{3}He as an effective neutron: Progress towards a relativistic 3N Faddeev calculation

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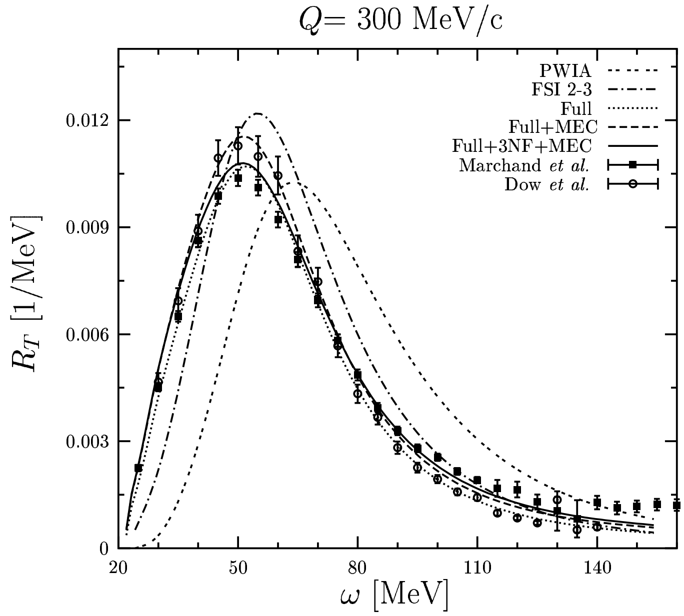
- Sketch of nonrelativistic Faddeev formalism
- Construction of a boosted potential
- Relativistic $2N$ and $3N$ bound states
- First applications: approximate treatment of reactions

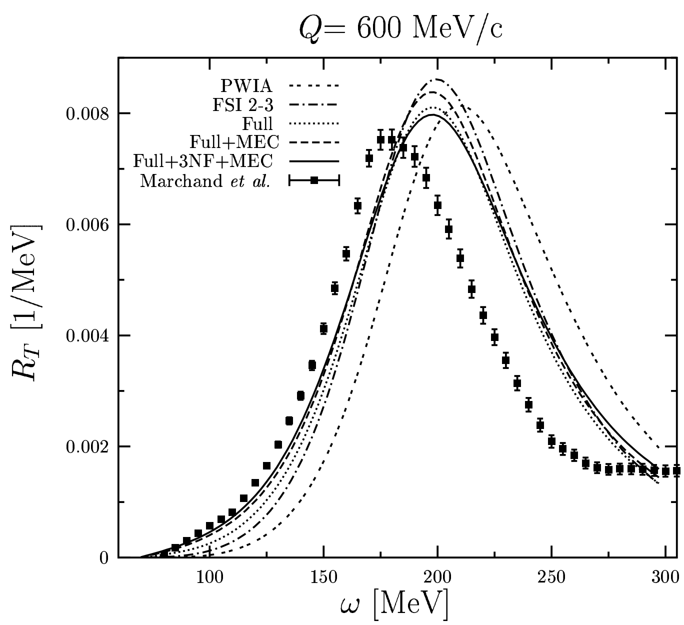
  $e +^3 \mathrm{He} \rightarrow e + p + d$

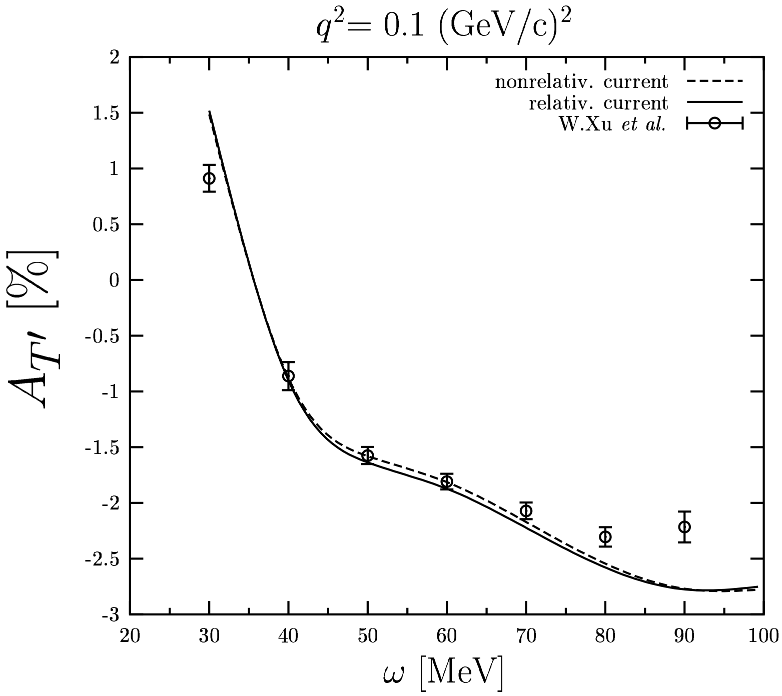
  $e +^3 \mathrm{He} \rightarrow e + p + p + n$
- Summary and outlook

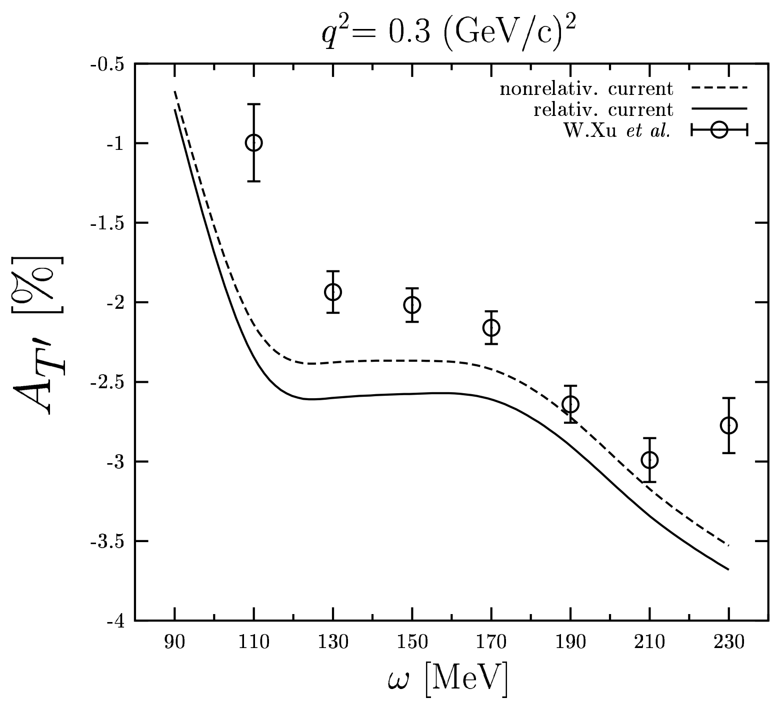
Why do we need relativity:



The shift between the nonrelativistic ($\omega_{QF} = \frac{Q^2}{2m}$) and relativistic ($\omega_{QF} = \sqrt{m^2 + Q^2} - m$) quasi-free conditions for Q= 600 MeV/c amounts to 16 MeV.

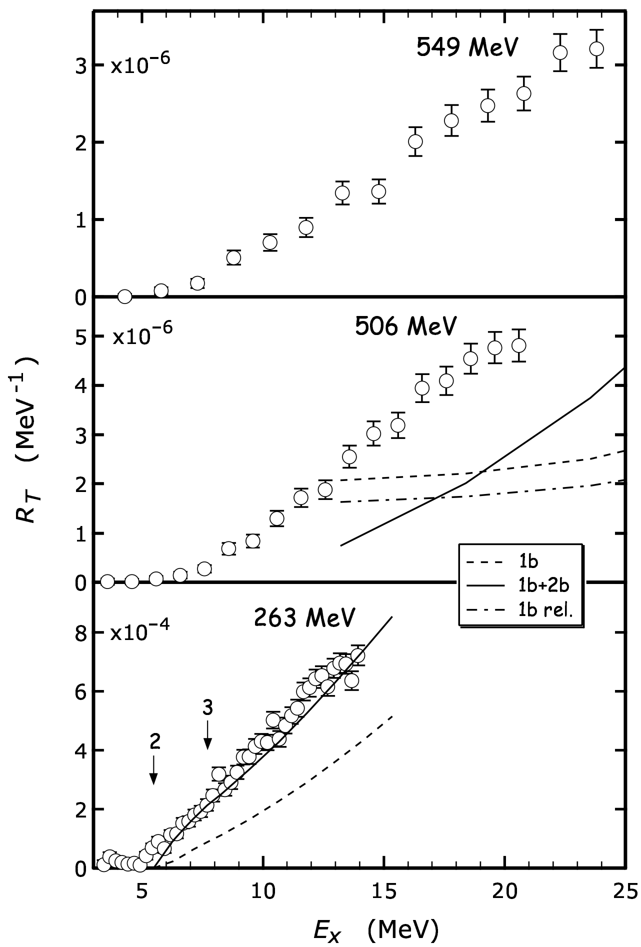






Adding *one* relativistic feature ist not sufficient !

Threshold Electrodisintegration of ^{3}He
[R.S.Hicks *et al.*, Phys. Rev. C**67**, 064004 (2003)]



Dashed curve: one-body current only; solid curve: one- and two-body currents; dot-dash: relativistic one-body currents.

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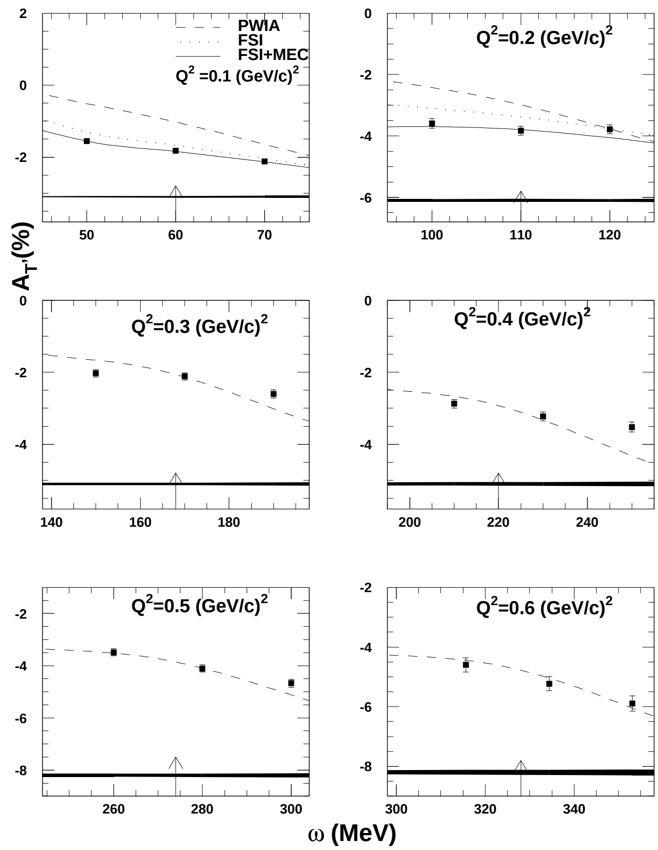
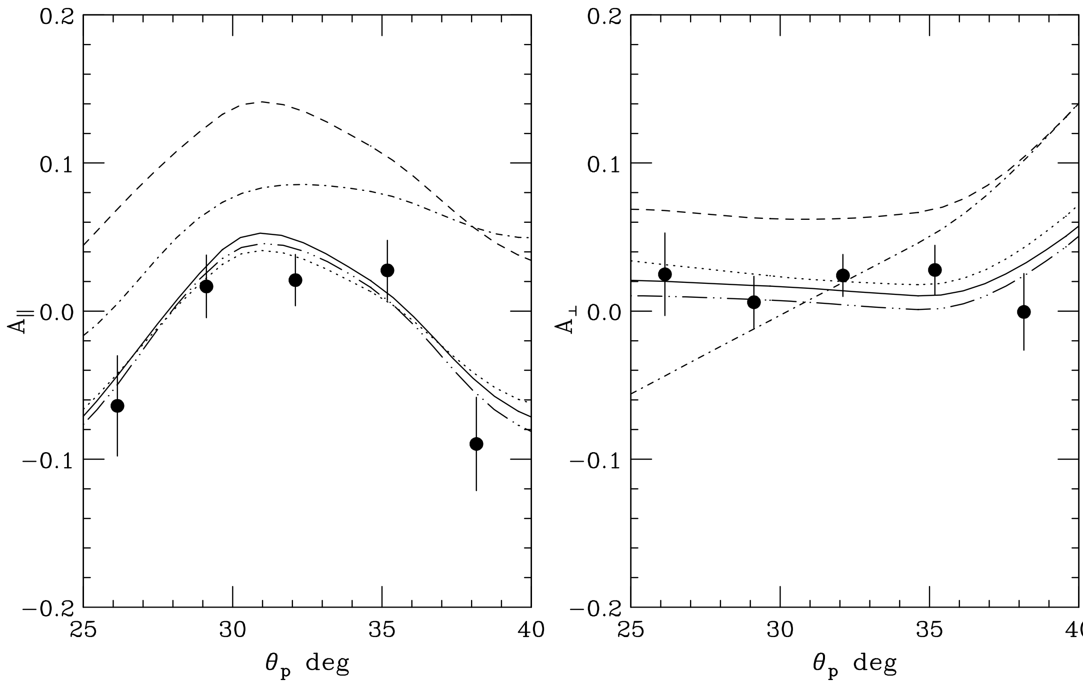


The transverse asymmetry $A_{T'}$ near the peak of quasielastic scattering at the six kinematics of the experiment [W. Xu *et al.*, Phys. Rev. Lett. 85, 2900 (2000)]. The solid and the dash-dotted curves are the Faddeev calculation [J. Golak *et al.*, Phys. Rev. C**63**, 034006 (2001)] which includes the FSI and MEC effects and FSI effect only, respectively. The dashed curve is the approximate calculation [A. Kievsky *et al.*, Phys. Rev. C**56**, 64 (1997)].

Asymmetries in $\overrightarrow{^3\mathrm{He}}(\vec{e}, e'p)$ [C.Carasco *et al.*, Phys. Lett. **B559**, 41 (2003)]



Experimental results of $A_\parallel$ (left) and $A_\perp$ (right) for the quasi–elastic peak region as a function of the scattering angle of the knocked out proton. The results of the full (PWIA) calculation are shown with solid (dashed) lines. The result of the full calculation with a non–relativistic current (dot), the effect of a $(v/c)^2$ correction (dot-dot-dash) and the same with non–relativistic kinematics (dot-dash) are also shown.

Electrodisintegration of ^{3}He

two-body (Nd) $\quad e + {}^3\text{He} \rightarrow \text{e} + \text{p} + \text{d}$

three-body (3N) $\quad e + {}^3\text{He} \rightarrow \text{e} + \text{p} + \text{p} + \text{n}$

All observables are given through $\langle \Psi_f \mid j_\mu \mid \Psi_i \rangle$

$$
\begin{aligned}
N_{\text{Nd}} \equiv \langle \Psi^{(-)}_{Nd} \mid j_\mu \mid \Psi_i \rangle &= \langle \Phi_{Nd} \mid (1+P) j_\mu \mid \Psi_i \rangle \\
&+ \langle \Phi_{Nd} \mid P \mid U_\mu \rangle,
\end{aligned}
$$

$$
\begin{aligned}
N_{\text{3N}} \equiv \langle \Psi^{(-)}_{3N} \mid j_\mu \mid \Psi_i \rangle &= \langle \Phi_{3N} \mid (1+P) j_\mu \mid \Psi_i \rangle \\
&+ \langle \Phi_{3N} \mid (1+P) \mid U_\mu \rangle,
\end{aligned}
$$

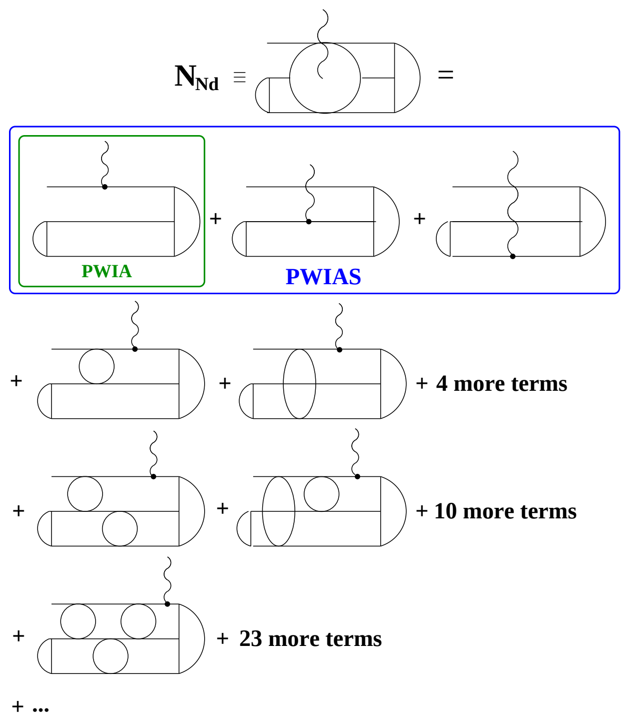
$$
\mid U_\mu \rangle = t_1 \, G_0 (1+P) j_\mu \mid \Psi_i \rangle + t_1 \, G_0 \, P \mid U_\mu \rangle
$$

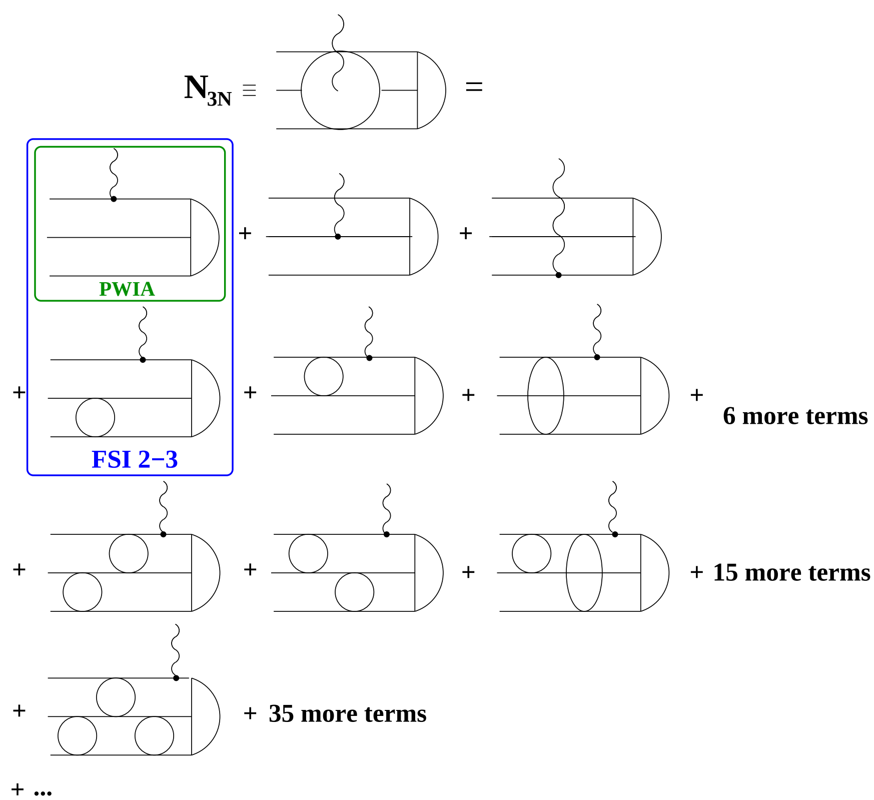
$$
\begin{aligned}
G_0 &\equiv (E - H_0 + i\epsilon)^{-1} \\
t_1 &= V_{23} + V_{23} \, G_0 \, V_{23} + \ldots \\
P &\equiv P_{12} P_{23} + P_{13} P_{23}
\end{aligned}
$$

$\mid \Phi_{Nd} \rangle$ and $\mid \Phi_{3N} \rangle$ are two- and three-body channel states

j_μ is the EM nuclear current operator

Diagrammatically:





Lorentz boosted potential

(1) Nonrelativistic NN potential $v^{(nr)}(\vec{p},\vec{p}\,')$
(for example AV18, CD Bonn, Nijmegen 93,I,II) used in the nonrelativistic Lippmann-Schwinger equation

$$t^{(nr)}(\vec{p},\vec{p}_0) = v^{(nr)}(\vec{p},\vec{p}_0) + \int d^3p' \, \frac{v^{(nr)}(\vec{p},\vec{p}\,')t^{(nr)}(\vec{p}\,',\vec{p}_0)}{\frac{\vec{p}_0^{\,2}}{m} - \frac{\vec{p}\,'^{\,2}}{m} + i\epsilon}$$

and in the nonrelativistic bound state equation

$$\psi_b^{(nr)}(\vec{p}) = \frac{1}{M_b - 2m - \frac{\vec{p}^{\,2}}{m}} \int d^3p' \, v^{(nr)}(\vec{p},\vec{p}\,') \, \psi_b^{(nr)}(\vec{p}\,')$$

(2) Relativistic NN potential $v(\vec{p},\vec{p}\,')$ in the NN c.m. system used in the relativistic Lippmann-Schwinger equation

$$t(\vec{p},\vec{p}_0) = v(\vec{p},\vec{p}_0) + \int d^3p' \, \frac{v(\vec{p},\vec{p}\,')t(\vec{p}\,',\vec{p}_0)}{\omega(\vec{p}_0) - \omega(\vec{p}\,') + i\epsilon}$$

and in the relativistic bound state equation

$$\psi_b(\vec{p}) = \frac{1}{M_b - \omega(\vec{p})} \int d^3p' \, v(\vec{p},\vec{p}\,') \, \psi_b(\vec{p}\,')$$

$$\omega(\vec{p}) \equiv 2\sqrt{m^2 + \vec{p}^{\,2}}.$$

(3) Boosted potential introduced as [W.Glöckle *et al.*, Phys. Rev. C**33**, 709 (1986)]

$$V \equiv \sqrt{[\omega(\vec{p}) + v]^2 + \vec{q}^{\,2}} - \sqrt{(\omega(\vec{p}))^2 + \vec{q}^{\,2}}$$

$\vec{q}$ is the total momentum of the two-nucleon system. $V(\vec{p}, \vec{p}\,'\,; \vec{q})$ is calculated in [H.Kamada *et al.*, Phys. Rev. C**66**, 044010 (2002)] and used to obtain a Lorentz boosted T-matrix

$$T(\vec{p}, \vec{p}\,'\,; \vec{q}) = V(\vec{p}, \vec{p}\,'\,; \vec{q})$$
$$+ \int d^3p\,'' \frac{V(\vec{p}, \vec{p}\,''\,; \vec{q})\, T(\vec{p}\,'', \vec{p}\,'\,; \vec{q})}{\sqrt{(\omega\,')^2 + \vec{q}^{\,2}} - \sqrt{(\omega\,'')^2 + \vec{q}^{\,2}} + i\epsilon}$$

$(\omega\,' \equiv 2\sqrt{m^2 + \vec{p}\,'^{\,2}},\ \omega\,' \equiv 2\sqrt{m^2 + \vec{p}\,''^{\,2}})$ and in the equation for the moving bound state

$$\psi_b(\vec{p}) =$$
$$\frac{1}{\sqrt{M_b^2 + \vec{q}^{\,2}} - \sqrt{\omega(\vec{p})^2 + \vec{q}^{\,2}}} \int d^3p'\, V(\vec{p}, \vec{p}\,'\,; \vec{q}) \psi_b(\vec{p}\,')$$

Note: $\psi_b(\vec{p})$ in (2) and $\psi_b(\vec{p})$ in (3) are *the same* !

Remarks:

- step (1) can be skipped and one can start directly with relativistic potential v
- in our case a scale transformation [H.Kamada, W.Glöckle, Phys. Rev. Lett. **80**, 2547 (1998)] generating a phase equivalent relativistic potential v from a nonrelativistic potential $v^{(nr)}$ for transition (1) $\longrightarrow$ (2) is used. See also discussion in [T.W.Allen *et al.*, Phys. Rev. C**62**, 054002 (2000)]
- expression for $V(\vec{p}, \vec{p}\,' ; \vec{q})$ given in [Phys. Rev. C**66**, 044010 (2002)] is general but quite complicated; for approximate boosted potential see [S.J.Wallace, Phys. Rev. Lett. **87**, 180401 (2001)] or use a simple prescription

$$V(\vec{p}, \vec{p}\,' ; \vec{q}) \approx v(\vec{p}, \vec{p}\,') - \frac{\vec{q}^{\,2}}{2} \frac{1}{\omega(\vec{p})} v(\vec{p}, \vec{p}\,') \frac{1}{\omega(\vec{p}\,')}$$

- for more information on relativistic calculations see for example references in [H.Kamada *et al.*, Phys. Rev. C**66**, 044010 (2002)]

Relativistic Faddeev equations

Since the formal structure of the $3N$ Hamiltonian

$$H = H_0 + \frac{1}{2}\sum_{i\neq j} V_{ij}, \quad V_{ij} - \text{boosted!}$$

is the same for relativistic and nonrelativistic dynamics, the formal derivation of the Faddeev equations is also the same in both cases. [W.Glöckle *et al.*, Phys. Rev. C**33**, 709 (1986)]

In a momentum space partial wave representation the relativistic Faddeev equation

$$\phi = G_0 T P \phi$$

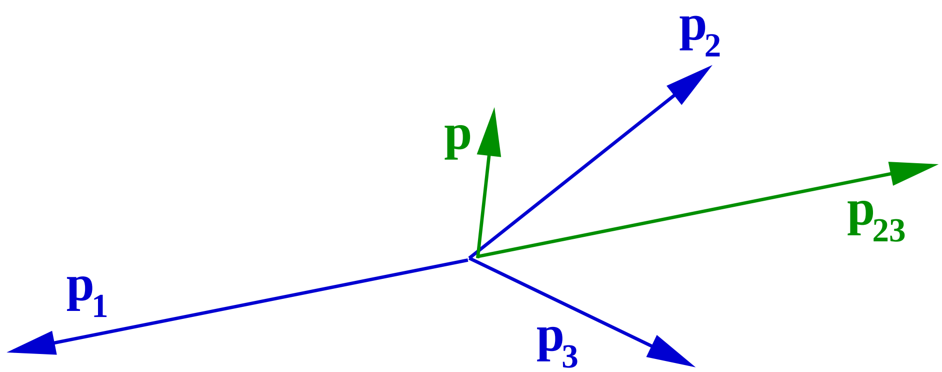
takes the form (ϕ is the Faddeev component)

$$\phi_\alpha(p,q) = \frac{1}{E_b - \mathcal{E}(p,q)} \sum_{\alpha'\alpha''} \int_0^\infty dq' {q'}^2 \int_{-1}^{1} dx \frac{T_{\alpha\alpha'}(p,\pi_1;q')}{{\pi_1}^{l'}} \frac{G_{\alpha'\alpha''}(q,q',x)}{\mathcal{N}_1(q,q',x)\mathcal{N}_2(q,q',x)} \frac{\phi_{\alpha''}(\pi_2,q')}{{\pi_2}^{l''}}$$

where $\mathcal{E}(p,q)$ is the $3N$ c.m. kinetic energy in terms of the subsystem (23) c.m. relative momentum p and the individual momentum of nucleon 1 in the $3N$ c.m. system q: $\mathcal{E}(p,q) \equiv \sqrt{(\omega(p))^2 + q^2} + \sqrt{m^2+q^2} - 3m$, the geometrical function $G_{\alpha\alpha'}(q,q',x)$ results from the permutation operator P.

$\mathcal{N}_1$, $\mathcal{N}_2$, π_1 and π_2 are analytically known functions of (q,q',x).

The wave function Ψ is later obtained as $(1+P)\phi$.



$\vec{p}$ represents the relative momentum *in the* (23) *two-body c.m. subsystem* and $\vec{q}$ stands for the momentum of the corresponding third particle.

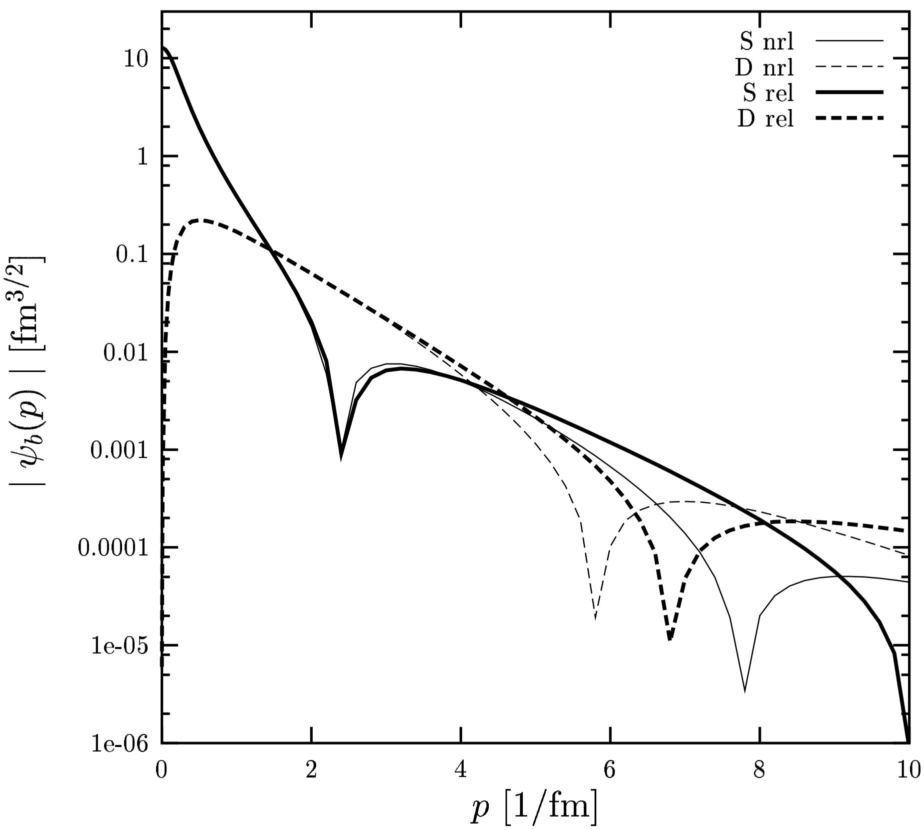
$$\vec{p} \equiv \vec{p}\,(\vec{p}_2, \vec{p}_3) = \frac{1}{2}\,(\vec{p}_2 - \vec{p}_3)$$

$$-\frac{1}{2}\,\vec{p}_{23}\,\left[\frac{\Omega_2 - \Omega_3}{(\Omega_2 + \Omega_3) + \sqrt{(\Omega_2 + \Omega_3)^2 - \vec{p}_{23}^{\,2}}}\right],$$

where $\Omega_2 \equiv \sqrt{m^2 + p_2^2}$, $\Omega_3 \equiv \sqrt{m^2 + p_3^2}$ and $\vec{q} = \vec{p}_1 = -(\vec{p}_2 + \vec{p}_3) \equiv -\vec{p}_{23}$ in the $3N$ c.m. system.

$$\begin{aligned} \mathcal{N}(\vec{p}_2, \vec{p}_3) &= \left[\left|\frac{\partial(\vec{p}_2, \vec{p}_3)}{\partial(\vec{p}, \vec{p}_{23})}\right|\right]^{1/2} \\ &= \left[\frac{4\Omega_2\Omega_3}{\sqrt{(\Omega_2 + \Omega_3)^2 - \vec{p}_{23}^{\,2}}(\Omega_2 + \Omega_3)}\right]^{1/2}. \end{aligned}$$

The deuteron wave function



p is the relative momentum in the two-nucleon c.m. system

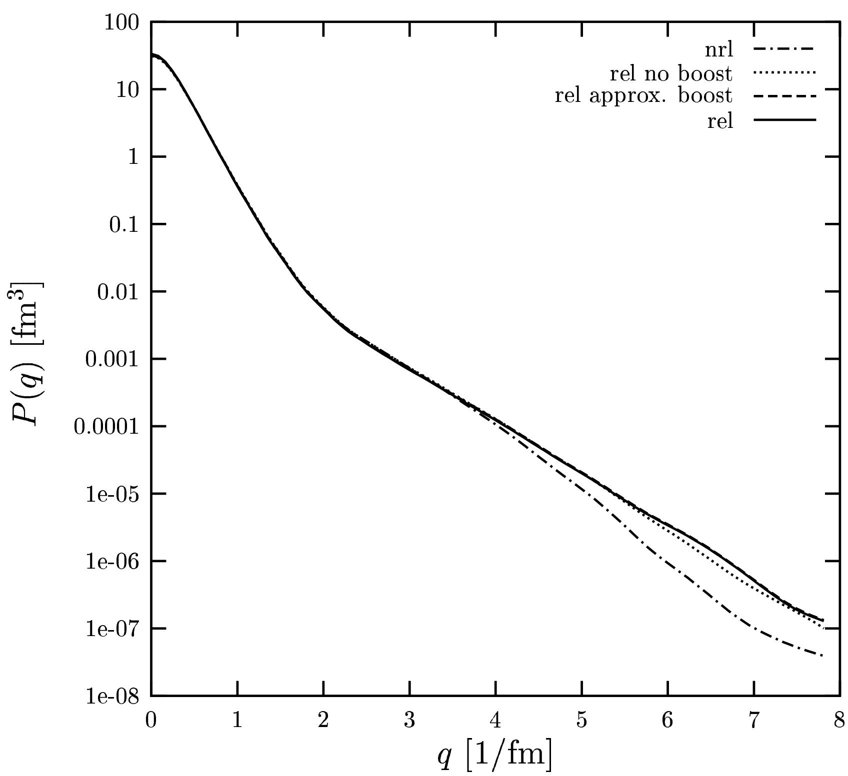
Relativistic and nonrelativistic binding energies are (by construction) the same !

3N bound state from five-channel calculation

interaction	E_b	$E_b^{(nr)}$	$E_b - E_b^{(nr)}$
CD-Bonn	-7.97	-8.33	0.36
Nijmegen II	-7.20	-7.65	0.45
Nijmegen I	-7.70	-8.00	0.30
Nijmegen 93	-7.44	-7.76	0.32
AV18	-7.23	-7.66	0.43
exp.	-8.48		

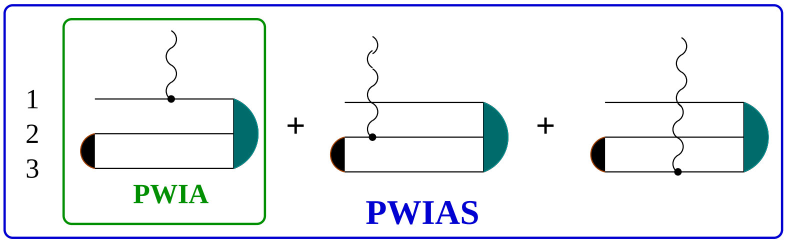
Relativistic binding energies are smaller !

The single-nucleon momentum distribution for CD Bonn and 34-channel calculation

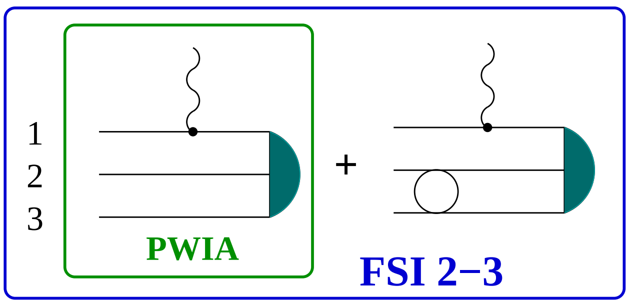


Application to electron scattering

$e + {}^3\text{He} \rightarrow \text{e} + \text{p} + \text{d}$



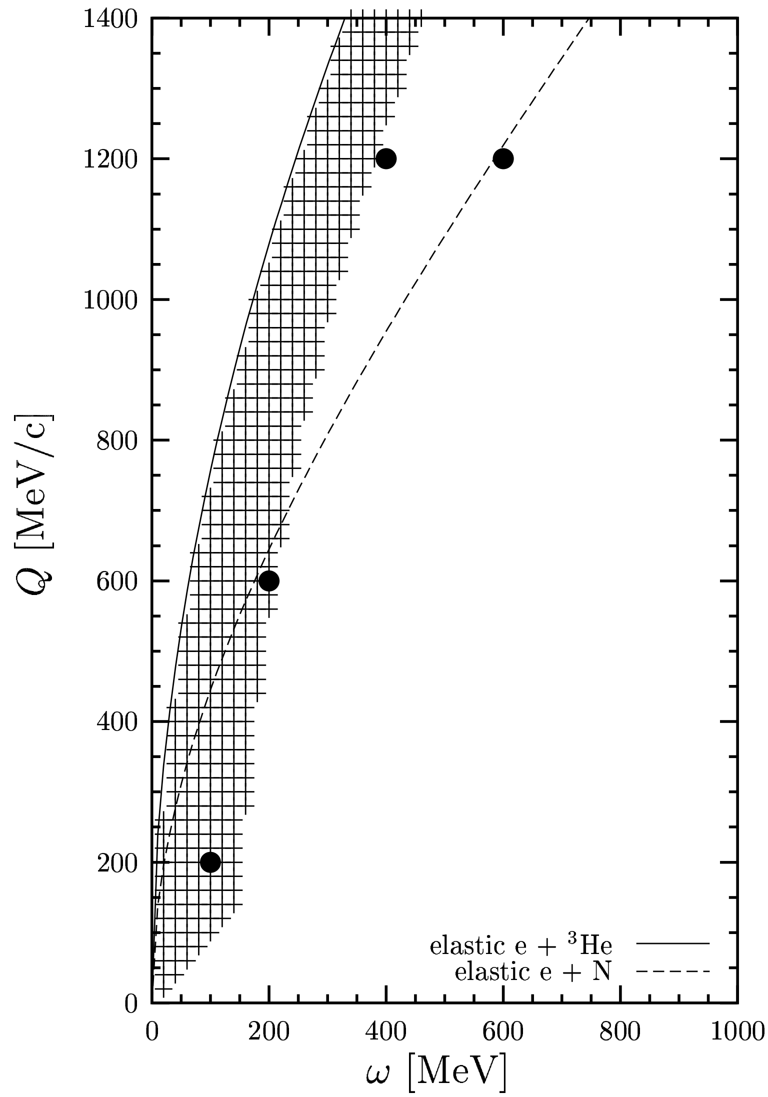
$e + {}^3\text{He} \rightarrow \text{e} + \text{p} + \text{p} + \text{n}$



Consistent

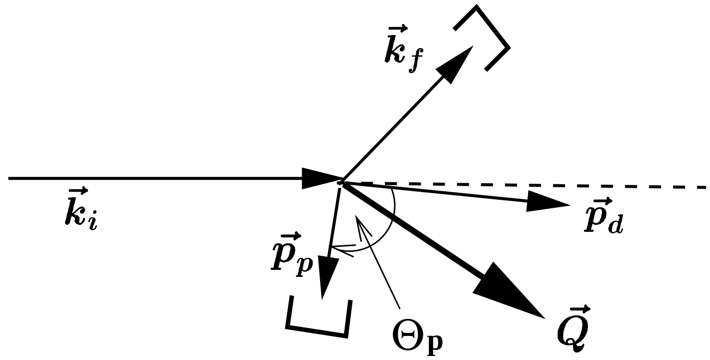
- relativistic deuteron and $3N$ bound states
- relativistic boosted T-matrix
- relativistic single nucleon current operator
- relativistic kinematics in the intermediate and final states

(ω, Q) points considered in this talk



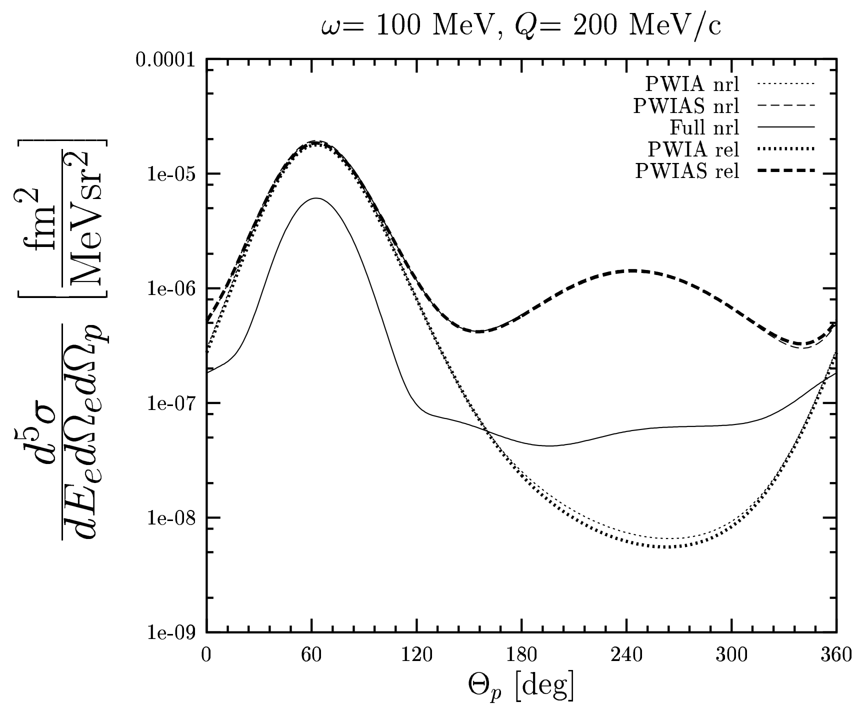
The shaded area shows the (ω, Q) points for which the $3N$ c.m. kinetic energy is smaller than the pion mass.

Examples for two-body electrodisintegration of ^{3}He

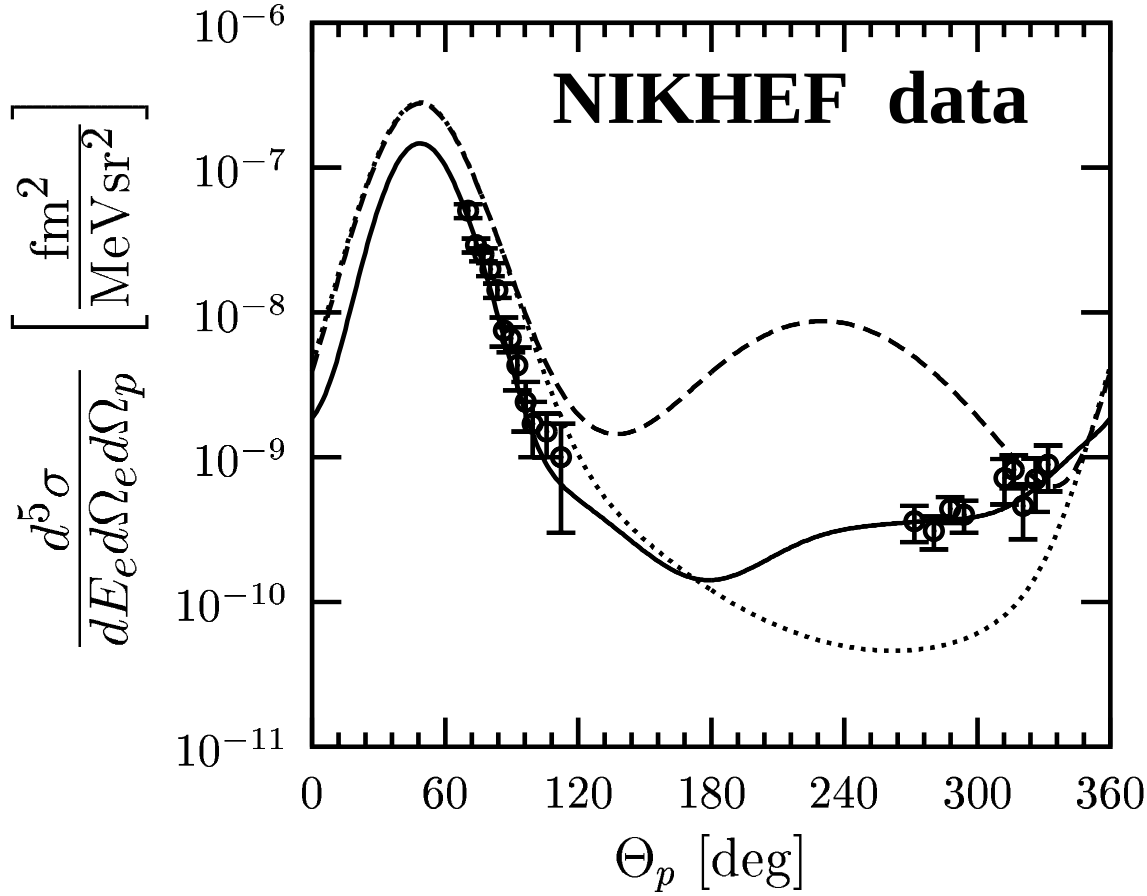


In all following examples $|\vec{k}_i| = 2$ GeV

Unpolarized cross sections

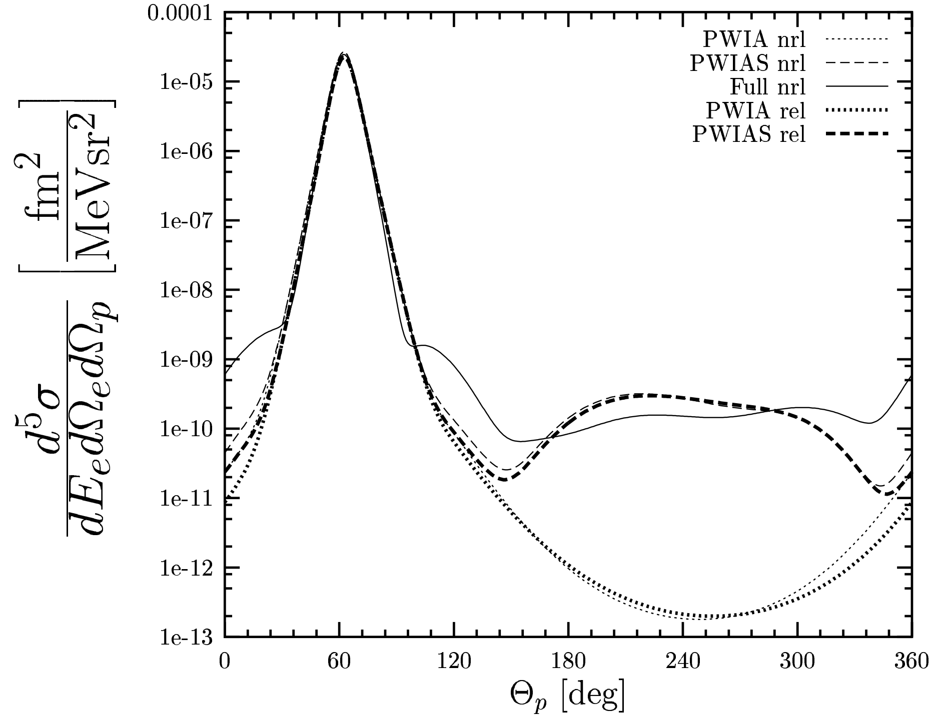


ω= 113 MeV, Q= 250 MeV/c

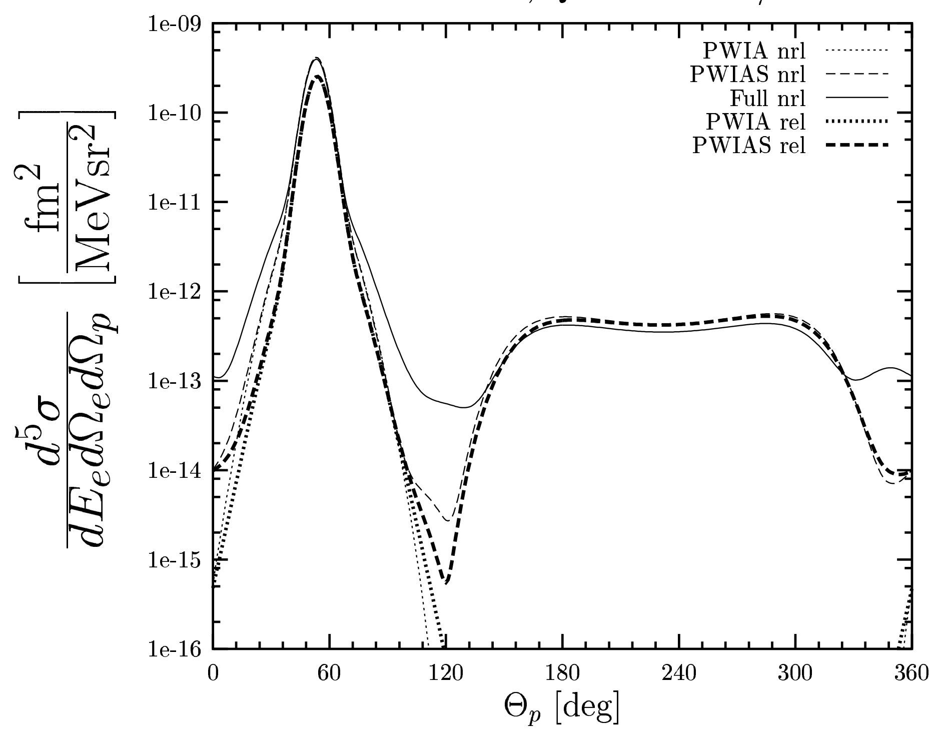


Only *Full* calculations describe the data at similar electron kinematics

ω= 200 MeV, Q= 600 MeV/c



ω= 400 MeV, Q= 1200 MeV/c



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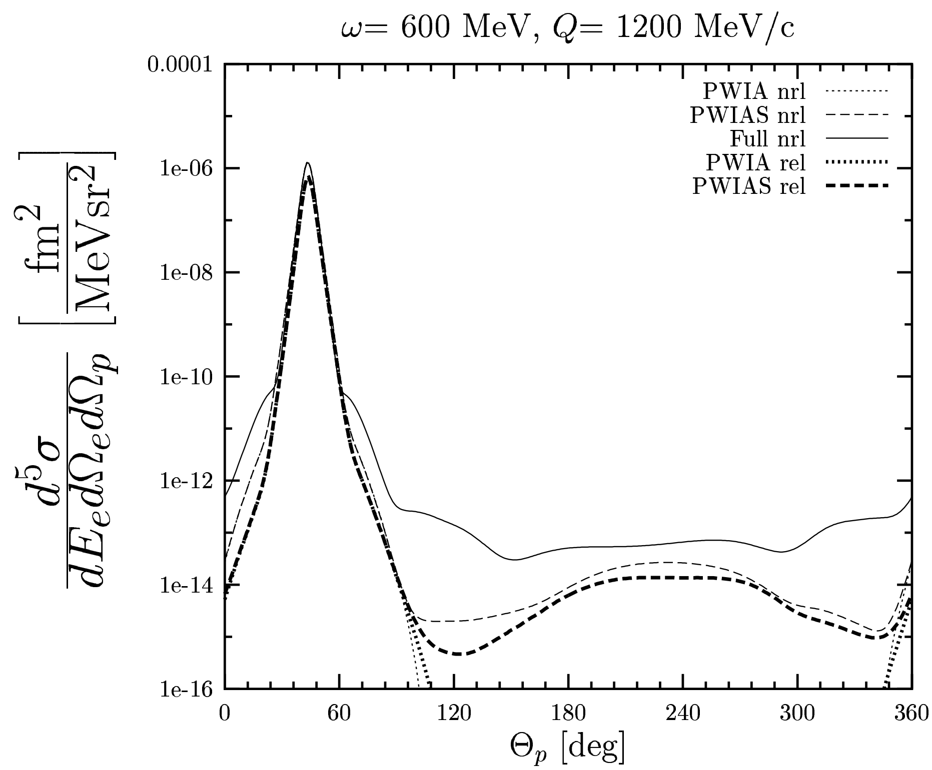
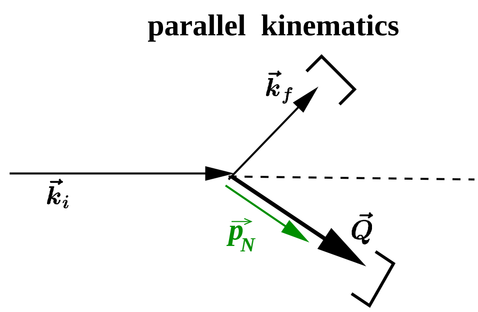


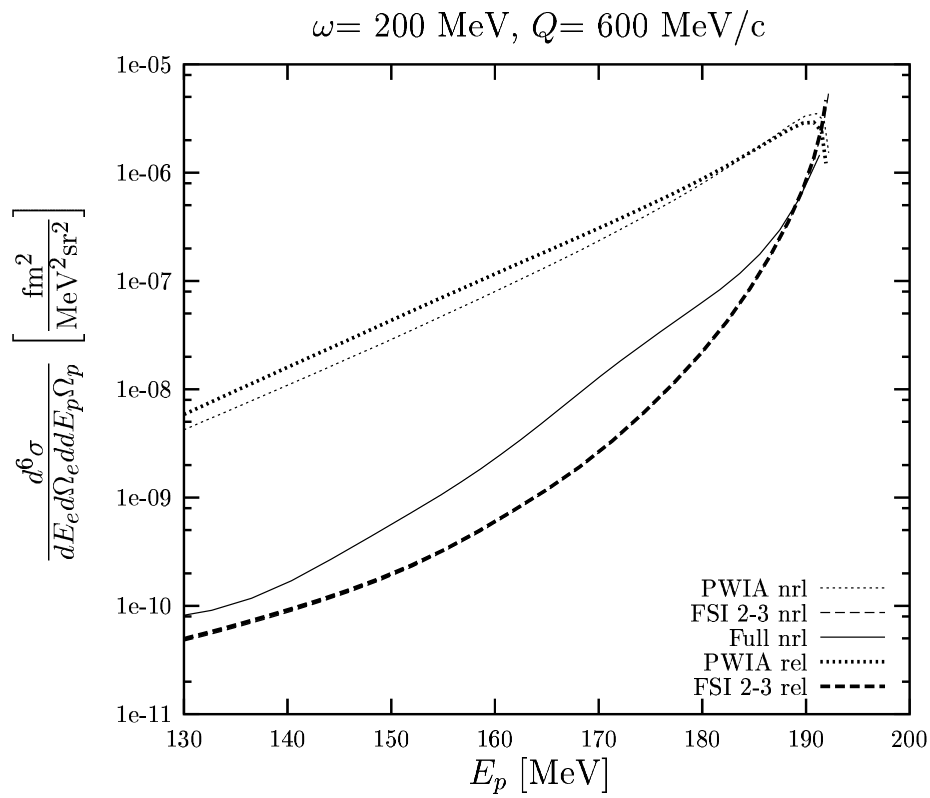
Full calculations are illegal !!!

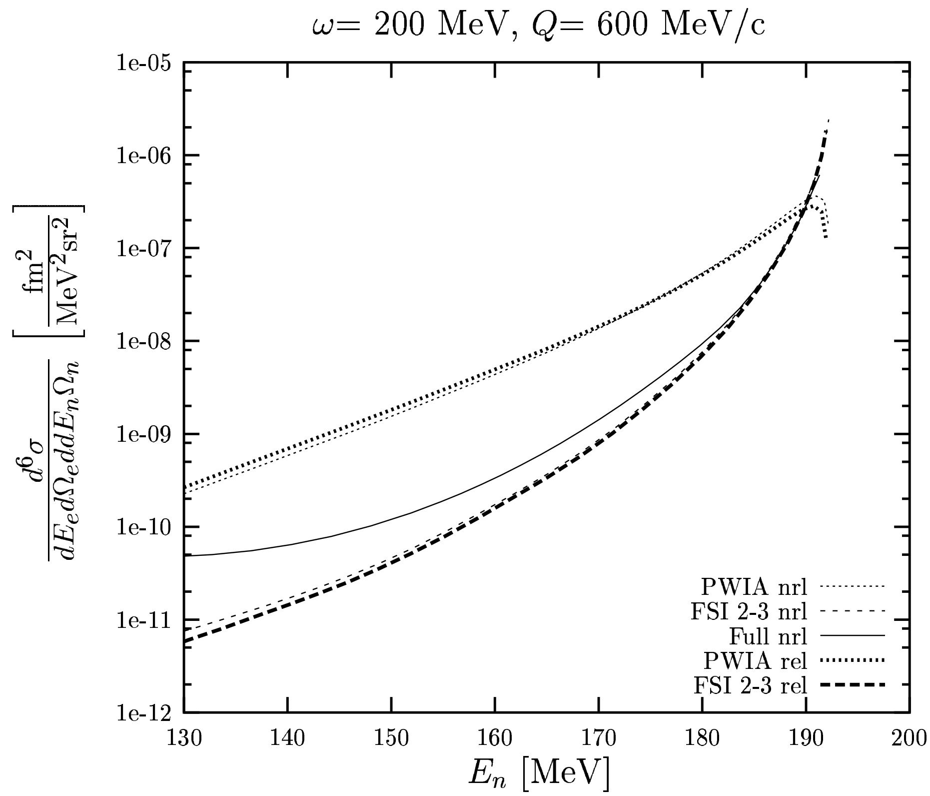
Note: in all examples for two-body electrodisintegration of ^{3}He relativistic kinematics was used for the pd final state.

Examples for three-body electrodisintegration of ^{3}He



Unpolarized cross sections





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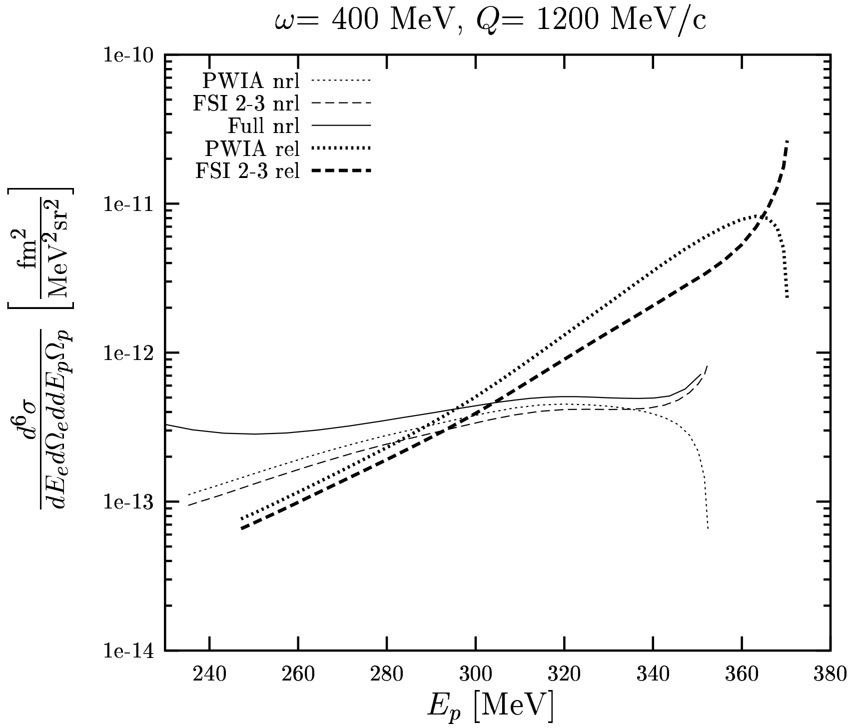

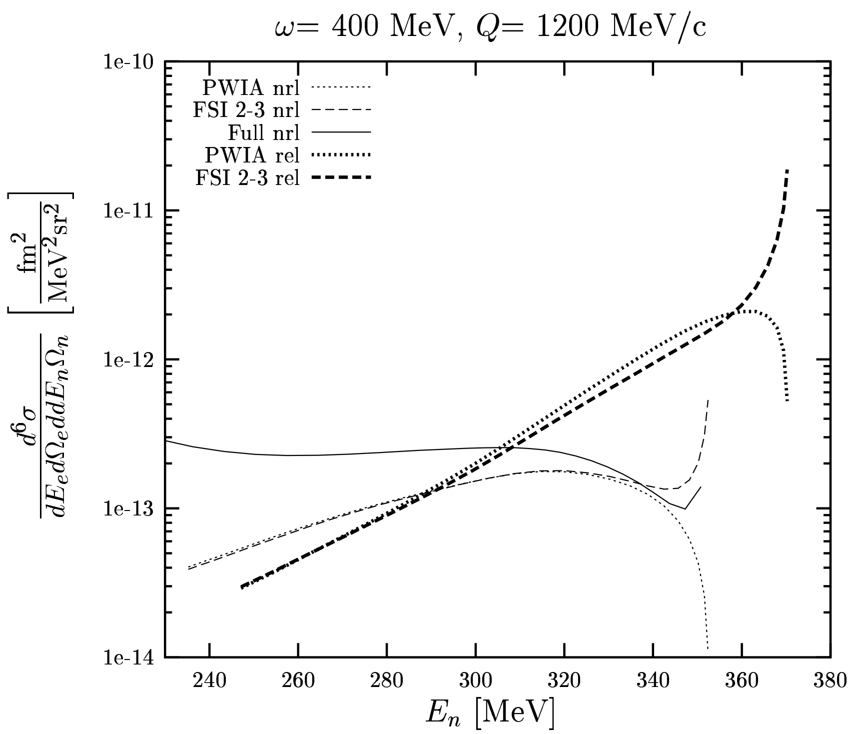

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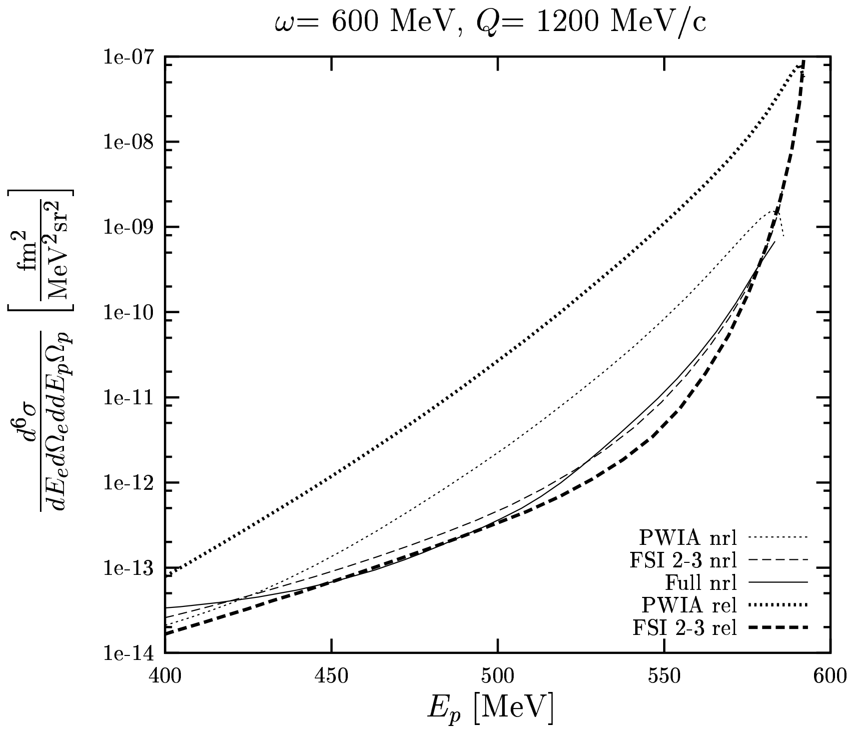
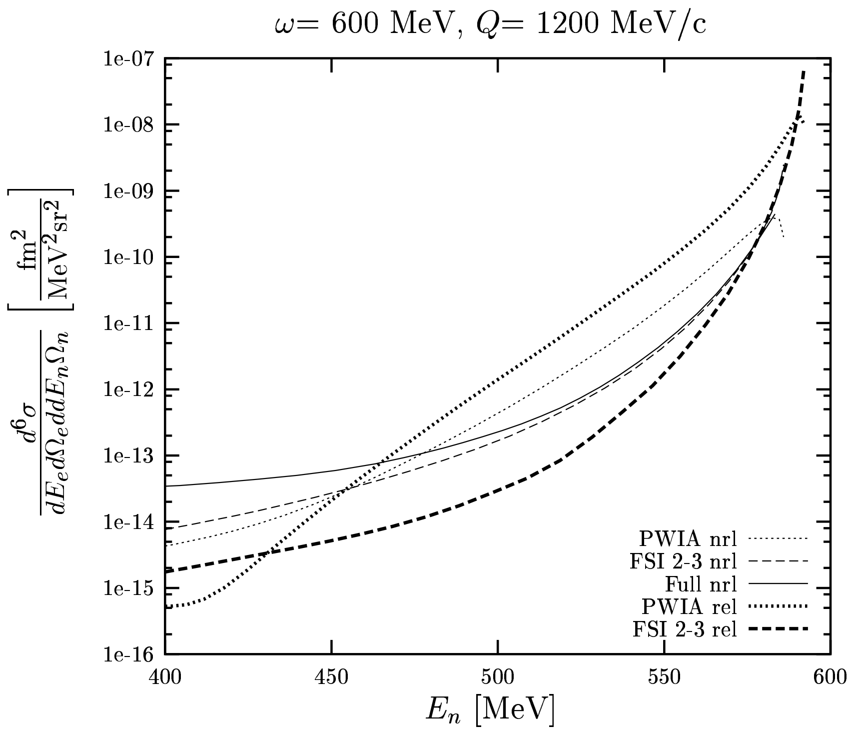
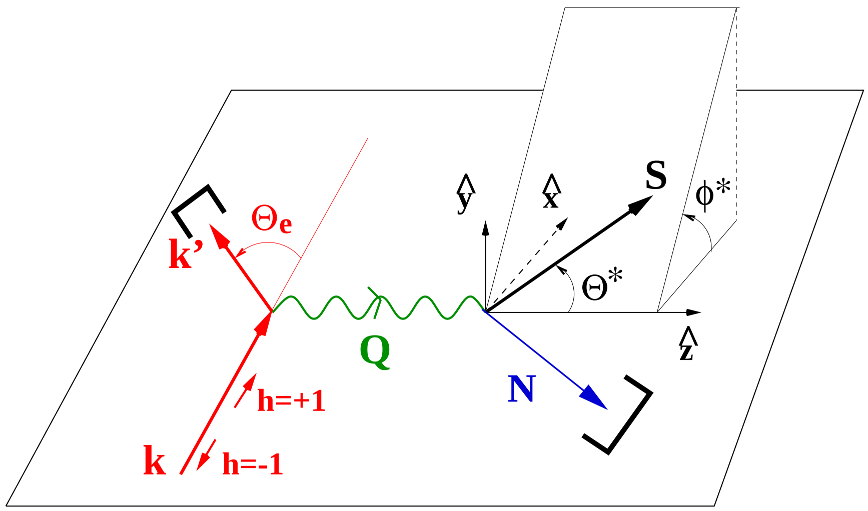




Full calculations are illegal !!!

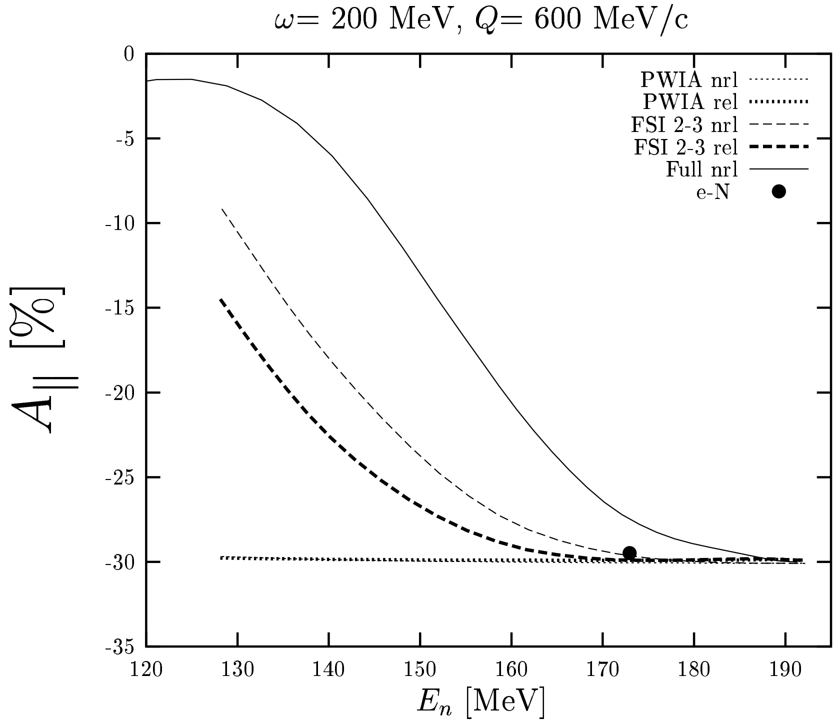
Spin dependent helicity asymmetries for $\overrightarrow{^3\mathrm{He}}(\vec{e}, e'N)$

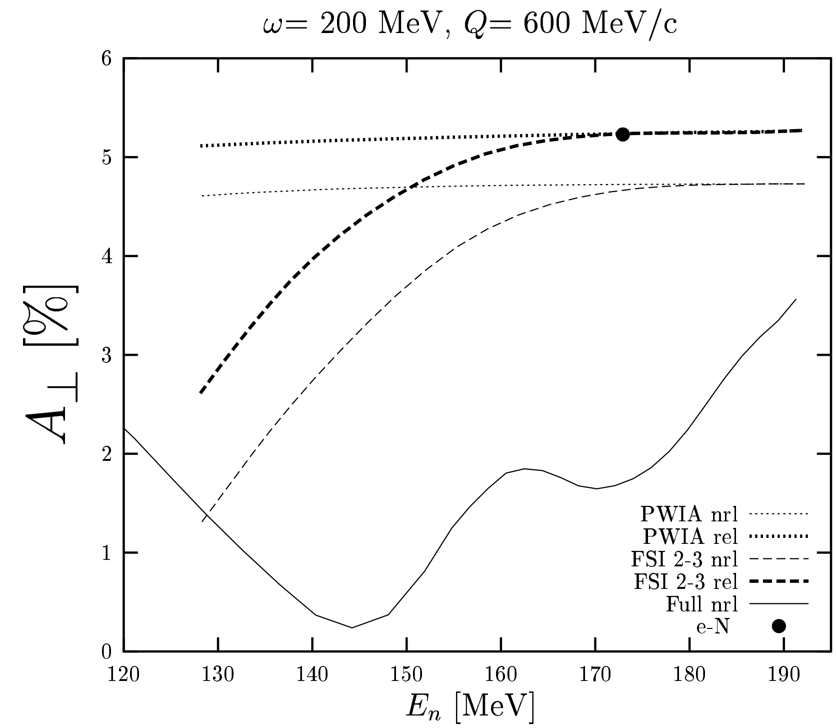


$$A(\hat{S}) = \frac{\sigma(h=+1,\hat{S}) - \sigma(h=+1,\hat{S})}{\sigma(h=+1,\hat{S}) + \sigma(h=+1,\hat{S})}$$

$$A_{\parallel} \equiv A(\theta^{\star} = 0^{\circ}, \phi^{\star} = 0^{\circ})$$

$$A_{\perp} \equiv A(\theta^{\star} = 90^{\circ}, \phi^{\star} = 0^{\circ})$$





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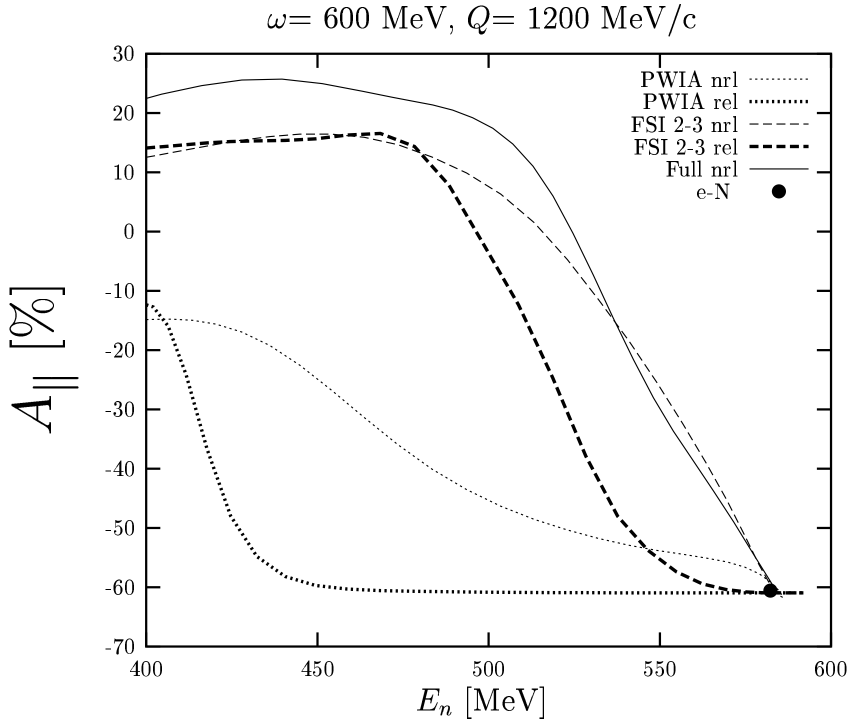

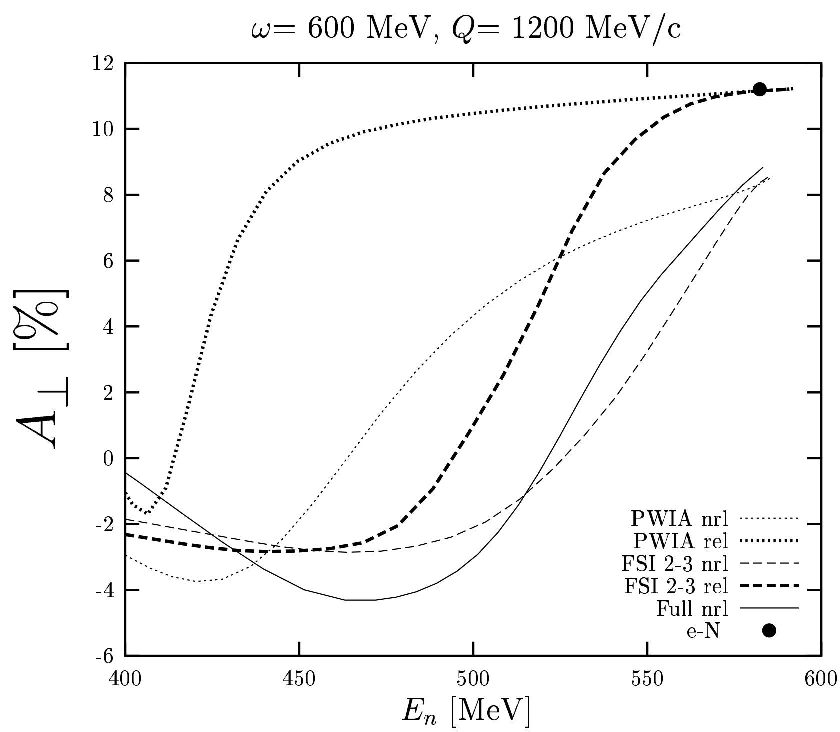

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Summary

- Relativistic Faddeev formalism for electron induced breakup of ^{3}He below the pion production threshold is under construction
- Main features:
  - natural generalization of the nonrelativistic framework
  - applicable to exclusive, inclusive and semi-exclusive processes with and without polarization degrees of freedom
  - dynamical input based on realistic NN interactions
- First approximate calculations for two- and three-body breakup processes give better control over *relativistic effects*
- Are approximations good enough in the *forbidden* area where full calculations are impossible ? (see for example [C.Carasco *et al.*, Phys. Lett. **B559**, 41 (2003)])
- Need for experiments at many different (ω, Q) locations

Outlook

- Full solution of the Faddeev equations in continuum (see [H.Kamada, Few-Body Syst., Suppl. **12**, 433 (2000)] for some hints)
- Verification in the pure $3N$ system
  - $N + d \to N + d$
  - $N + d \to N + p + n$
- Application of the full formalism in electron scattering
- Relativistic rotations of the spin states
- Relativistic MEC
- $\cdots$