Application of chiral 2N and 3N forces to few–nucleon systems

- Effective Field Theory and nuclear forces
- Two, three and four nucleons at NNLO
- Improving the convergence of the chiral expansion
- First results for two nucleons at NNNLO
- Summary and outlook

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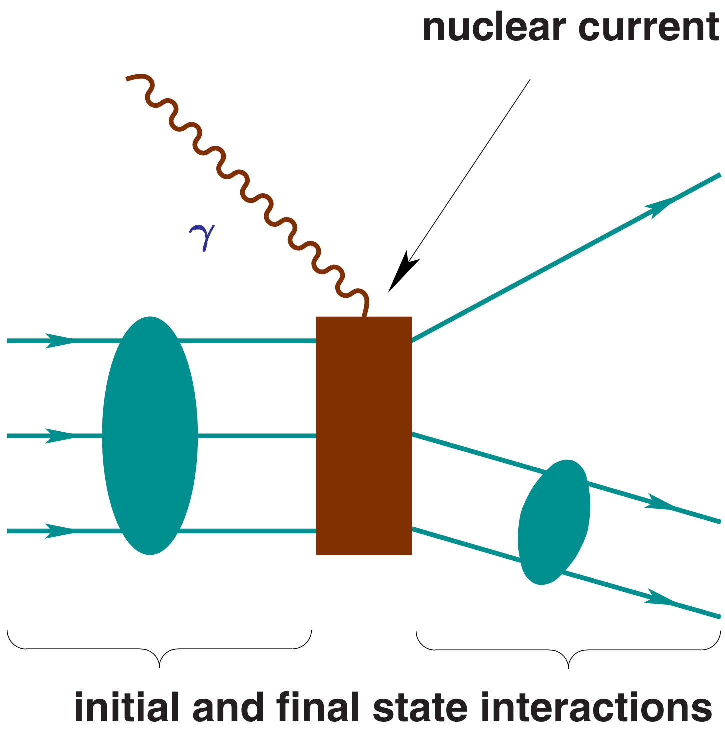


Chiral EFT is an **ideal tool** to study such problems at low Q :

- Nuclear forces and currents are derived in a **consistent** and **model independent** way.
- **Link to QCD** due to chiral symmetry.
- Once the unknown coupling constants are fixed from some observables, **all other low–energy observables can be predicted**. Theoretical uncertainty can be estimated.
- **Can be improved** in a straightforward way.

Chiral Effective Field Theory

Goal: describe interaction between pions and nucleons at $Q \sim M_\pi$.

Degrees of freedom: pions and nucleons (for $Q < M_\pi$ pions can be integrated out. Talk by Harald Grießhammer.)

Symmetries: Lorenz and isospin invariance, **chiral symmetry**.

QCD Lagrangian (2 flavors):

$$\mathcal{L}_{\rm QCD} = \bar q \, i \not{D} \, q - \frac{1}{4} G^\alpha_{\mu\nu} G^{\alpha, \mu\nu} - \bar q \mathcal{M} q$$

$SU(2)_L \times SU(2)_R$ -invariant $m_{u,d}$ – small

$SU(2)_A$ – **spontaneously broken**

$\Rightarrow$ Goldstone bosons (pions)

Effective Lagrangian for pions and nucleons:

$$\begin{aligned}
\mathcal{L}_{\pi\pi} &= \frac{F^2}{4} \langle \nabla^\mu U \, \nabla_\mu U^\dagger + 2\, B\, \mathcal{M}(U + U^\dagger) \rangle + L_1 \langle \partial_\mu U \, \partial^\mu U^\dagger \rangle^2 + \ldots \\
\mathcal{L}_{\pi N} &= \bar N_v \left[i (v \cdot D) + g_A (S \cdot u) + 2\, c_1 \, B \langle \mathcal{M}(U + U^\dagger) \rangle \right] N_v + \ldots \\
\mathcal{L}_{NN} &= C_S \, (\bar N_v N_v)^2 + C_T \, (\bar N_v S_\mu N_v)^2 + \frac{C_1}{4} \left[(\bar N_v \, \vec\partial_\mu \, N_v)^2 + h.c. \right] + \ldots
\end{aligned}$$

where

$U = \frac{1}{F}[\sqrt{F^2 - \boldsymbol{\pi}^2} + i \boldsymbol{\pi} \cdot \boldsymbol{\tau}]$, $\quad u = \sqrt{U}$, $\quad u_\mu = i(u^\dagger \nabla_\mu u - u \nabla_\mu u^\dagger)$,

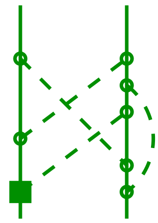
$D_\mu = \partial_\mu + \Gamma_\mu$, $\quad \Gamma_\mu = \frac{1}{2}[u^\dagger, \, \partial_\mu u]$, $\quad S_\mu = \frac{1}{2} i \gamma_5 \sigma_{\mu\nu} v^\nu$.

How to calculate observables?

- $\pi\pi$, πN – **perturbative** expansion of the amplitude in powers of Q/Λ .
  - – $Q \sim M_\pi$: typical momenta involved (soft scale);
  - – Λ : momenta at which new physics appears (hard scale) ($\Lambda_\chi , \; M_\rho ,$ ultraviolet cutoff).
- NN – **non–perturbative** problem.

Weinberg's idea:

- Use time–ordered perturbation theory to calculate $V_{\rm eff} = \sum \left(\begin{array}{c}\text{all irreducible} \\ \text{diagrams}\end{array} \right)$. Perturbative expansion in powers of Q/Λ using the rules of the **chiral power counting**: any connected irreducible diagram is $\sim (Q/\Lambda)^\nu$, where



$\sim \left(\frac{Q}{\Lambda}\right)^\nu$

$\mathcal{N}$ of nucleons | $\mathcal{N}$ of loops | $\mathcal{N}$ of vertices of type i

$$\boldsymbol{\nu = -4 + 2\,N + 2\,L + \sum_i V_i\,\Delta_i}$$

$$\Delta_i = d_i + \tfrac{1}{2} n_i - 2$$

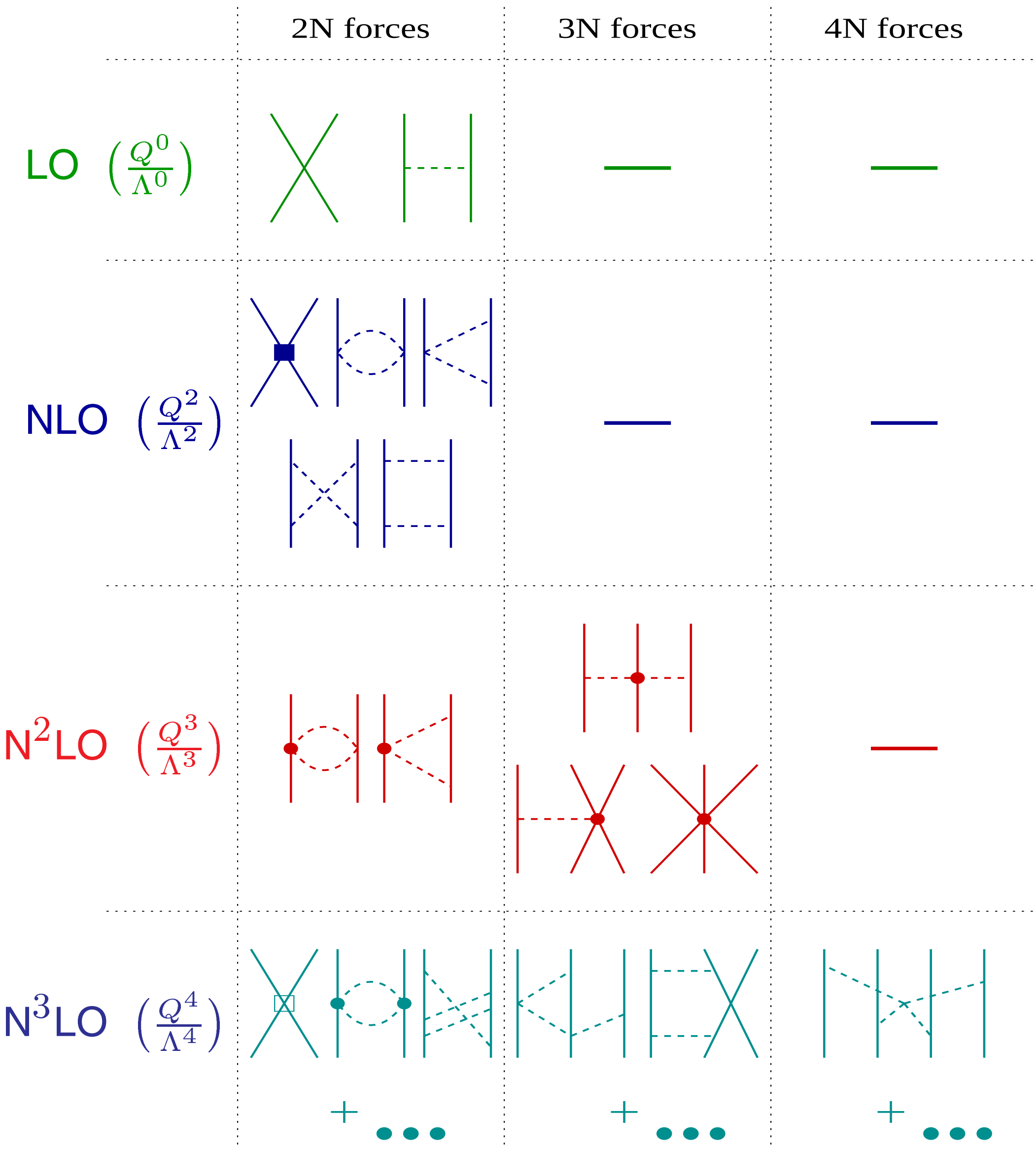
$\mathcal{N}$ of derivatives/M_π–insertions | $\mathcal{N}$ of nucleon fields

($\Delta_i \geq 0$ due to spontaneously broken chiral symmetry)

- Solve the Lippmann–Schwinger equation $T = V_{\rm eff} + V_{\rm eff} G_0 T$

Hierarchy of nuclear forces

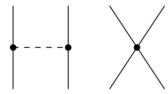
$$\left(\nu = -4 + 2\,N + 2\,L + \sum_i V_i\,\Delta_i\,, \qquad \Delta_i = d_i + \tfrac{1}{2}n_i - 2\right)$$



Chiral effective field theory offers a natural explanation why 3N forces are less important than the 2N ones, 4N forces are less important than the 3N ones, ...

Two nucleons at NNLO: $V_{\rm eff} = V^{(0)} + V^{(2)} + V^{(3)}$

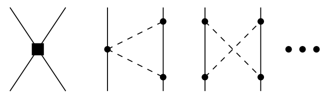
- **Leading order** $\sim Q^0/\Lambda^0$



$$V^{(0)}_{\rm OPE} = - \left(\frac{g_A}{2F_\pi} \right)^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \, \frac{\vec{\sigma}_1 \cdot \vec{q} \, \vec{\sigma}_2 \cdot \vec{q}}{q^2 + M_\pi^2}, \qquad V^{(0)}_{\rm cont} = C_S + C_T \, \vec{\sigma}_1 \cdot \vec{\sigma}_2 \, ,$$

where $\vec{q} \equiv \vec{p}\,' - \vec{p}$; $\vec{p}$ ($\vec{p}\,'$) are initial (final) nucleon momenta in cms.

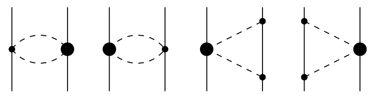
- **Next–to–Leading order** $\sim Q^2/\Lambda^2$



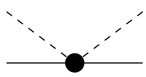
$$\begin{aligned} V^{(2)}_{\rm TPE} &= - \frac{\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{384 \pi^2 F_\pi^4} \, L(q) \left\{ 4M_\pi^2 (5g_A^4 - 4g_A^2 - 1) + q^2 (23 g_A^4 - 10 g_A^2 - 1) \right. \\ &\quad \left. + \frac{48 g_A^4 M_\pi^4}{4M_\pi^2 + q^2} \right\} - \frac{3 g_A^4}{64 \pi^2 F_\pi^4} \, L(q) \left\{ \vec{\sigma}_1 \cdot \vec{q} \, \vec{\sigma}_2 \cdot \vec{q} - q^2 \, \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right\} \\ V^{(2)}_{\rm cont} &= C_1 \, \vec{q}^{\,2} + C_2 \, \vec{k}^2 + \ldots + C_7 \, (\vec{\sigma}_1 \cdot \vec{k}) \, (\vec{\sigma}_2 \cdot \vec{k}) \, , \end{aligned}$$

where $k \equiv \frac{1}{2} (\vec{p}\,' + \vec{p})$, The 9 LECs C_i contribute to S– and P–waves.

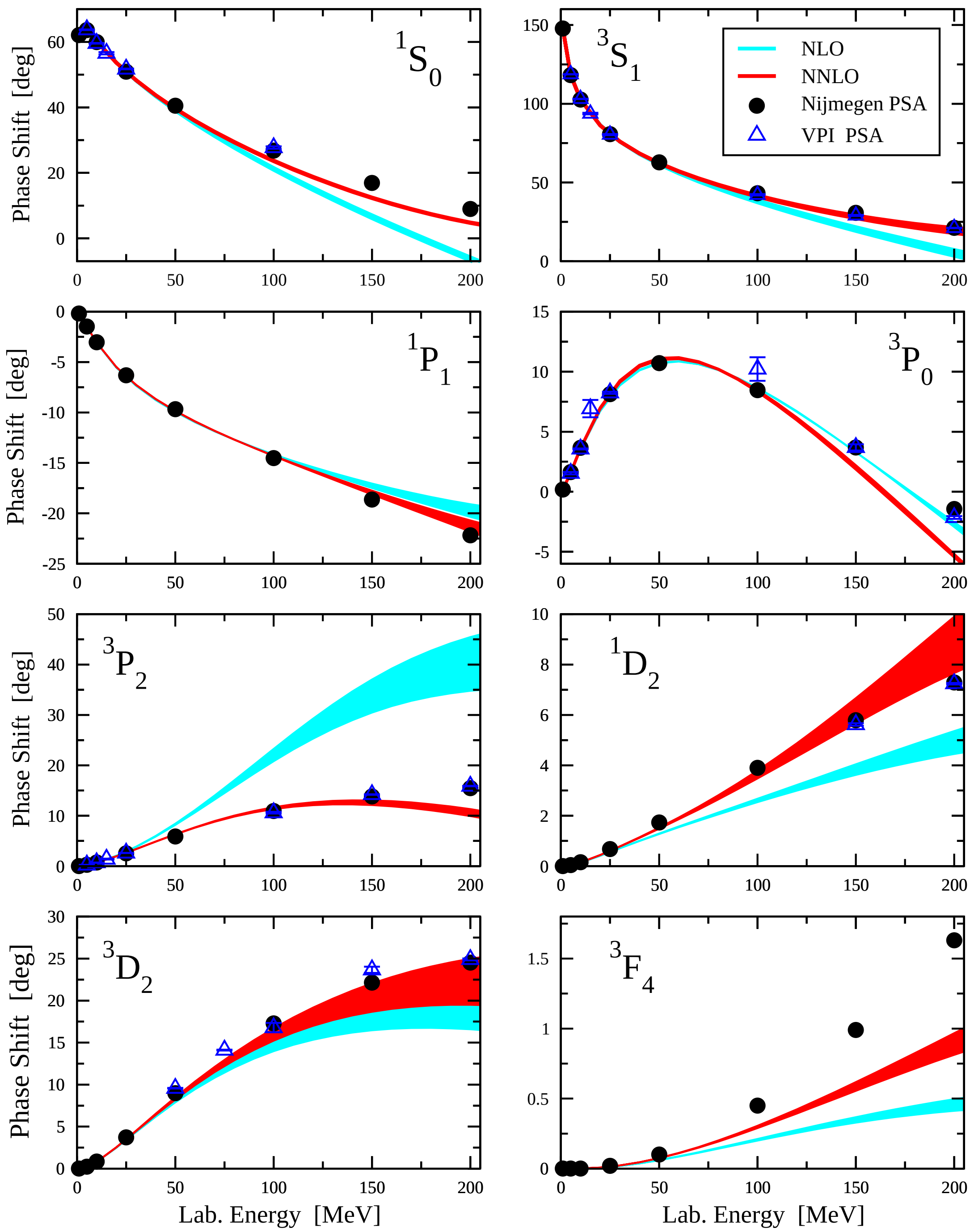
- **Next–to–Next–to–Leading order** $\sim Q^3/\Lambda^3$



$$\begin{aligned} V^{(3)}_{\rm TPE} &= - \frac{3 g_A^2}{16 \pi F_\pi^4} \left\{ 2M_\pi^2 (2c_1 - c_3) - c_3 q^2 \right\} (2M_\pi^2 + q^2) A(q) \\ &\quad - \frac{g_A^2}{32 \pi F_\pi^4} \, c_4 (4M_\pi^2 + q^2) A(q) \, (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \left[(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q}) - q^2 (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \right] \end{aligned}$$

New LECs $c_{1,3,4}$ are related to  and start to contribute at NNLO. We use $c_3 = -1.29, \; c_4 = 1.75$ GeV^{-1} which are significantly smaller than the values $c_3 = -4.69 \pm 1.34, \; c_4 = 3.40 \pm 0.04$ GeV^{-1} from the Q^3–analysis of πN scattering.

NN phase shifts



The cutoff Λ in the LS–equation is varied between 500 and 600 MeV.

Data from: http://nn-online.sci.kun.nl (Nijmegen PSA),

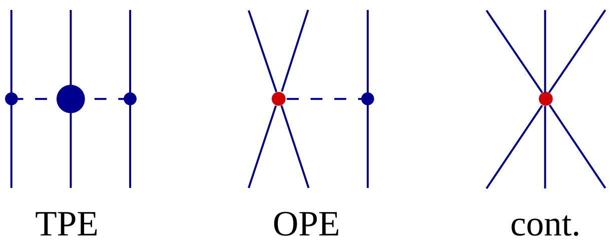
http://gwdac.phys.gwu.edu/ (VPI PSA).

3 and 4 nucleons at NNLO

Since the 2N forces are fixed from the NN scattering study of > 2 nucleon systems opens an excellent way to test the chiral forces.

- **LO, NLO:** only 2N forces $\Rightarrow$ parameter–free results for > 2 N systems (see E.E. et al., PRL 86 (2001) 4787)
- **NNLO:** the first nonvanishing chiral 3NF appears (see E.E. et al., PRC 66 (2002) 064001)

Chiral 3NF at NNLO is given by:

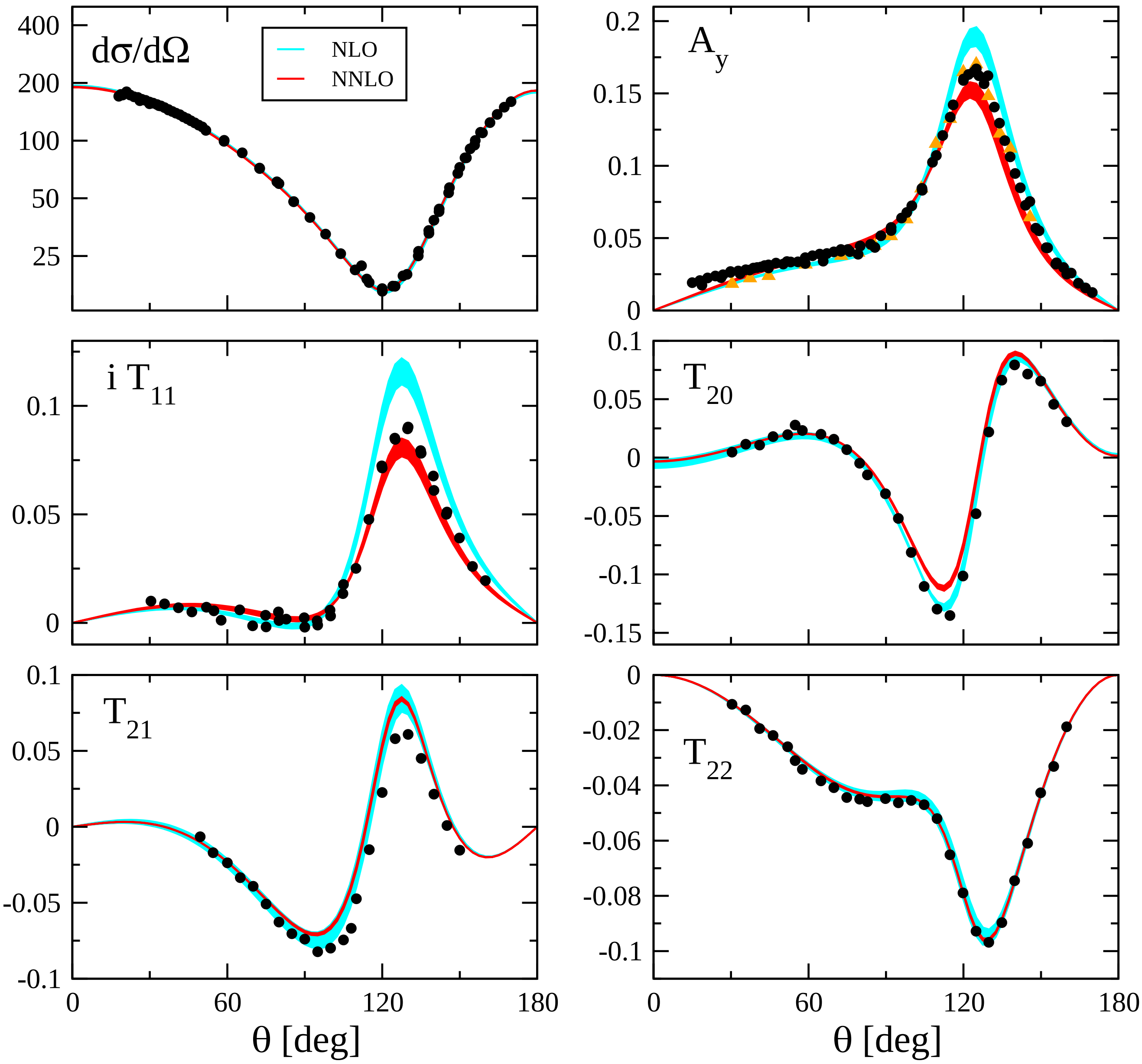


We have fixed two new low–energy constants D and E from the triton binding energy and *nd* scattering length.

3N and 4N binding energies

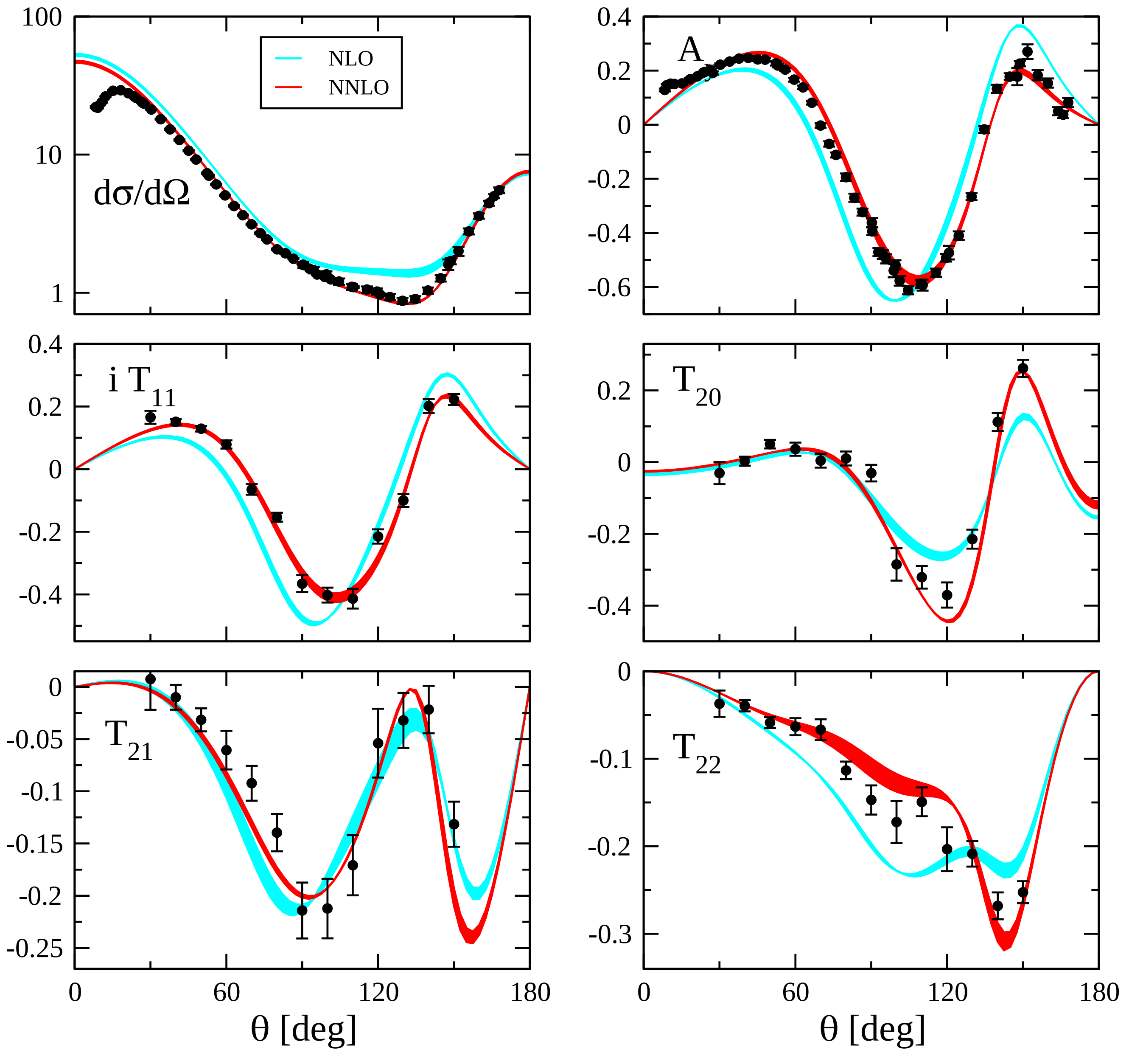
	NLO	NNLO	EXP
$E_{^3H}$ [MeV]	$-8.54\ldots-7.53$	$\mathbf{-8.68}$	-8.68
$E_{^4He}$ [MeV]	$-29.57\ldots-23.87$	$-29.51\ldots-29.98$	-29.6

Elastic *nd* scattering observables at 10 MeV



Filled circles are *nd* pseudo data based on K.Sagara et al., PRC 50 (1994) 576; G.Rauprich et al., Few–Body Systems 5 (1988) 67; F.Sperisen et al., NPA 422 (1984) 81 and Coulomb corrections calculated by A.Kievsky. The filled triangles are true *nd* data from C.R.Howell et al., Few–Body Systems 2 (1987) 19.

Elastic *nd* scattering observables at 65 MeV



Filled circles are *pd* data from S.Shimizu et al., PRC 52 (1995) 1193; H.Witała et al., Few–Body Systems 15 (1993) 67.

Improving the convergence of the chiral expansion

(see E.E., W.Glöckle, U.–G.Meißner, nucl-th/0304037; in preparation)

Look at **high NN partial waves** $(l \geq 2)$ which provide a good testing ground for chiral forces (see N.Kaiser et al., NPA 625 (1997) 758):

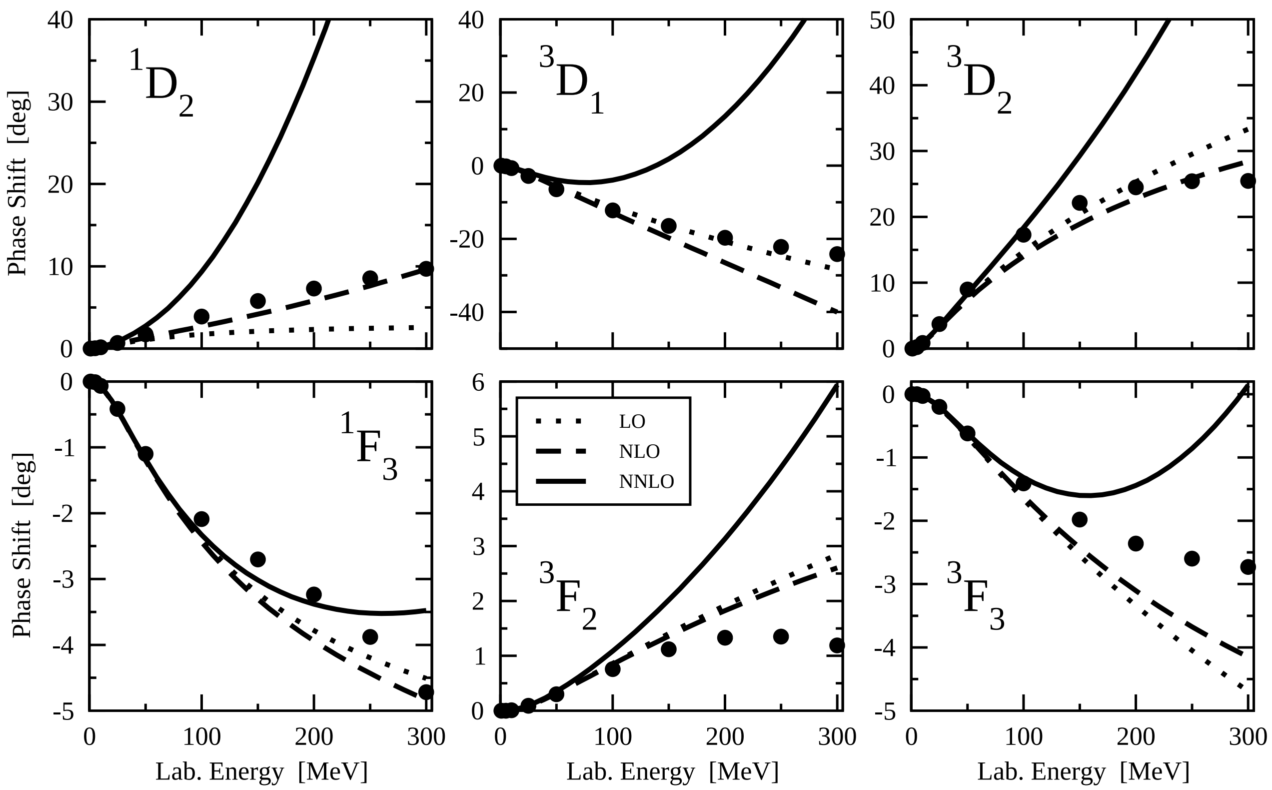
- **Perturbative regime**: $T^{sj}_{l,l'}(p',\ p) \sim V^{sj}_{l,l'}(p',\ p)$;
- **Contact interactions do not contribute** (up to NNLO).

$\Rightarrow$ **parameter–free predictions from chiral EFT!**

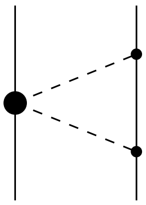
The LECs $c_{1,3,4}$ from the πN system:

$$c_1 = -0.81 \text{ GeV}^{-1},\ c_3 = -4.70 \text{ GeV}^{-1},\ c_4 = 3.40 \text{ GeV}^{-1}.$$

Peripheral NN waves using dimensional regularization



Why? The central part of chiral TPE at NNLO obtained using dimensional regularization shows an **unphysically strong attraction** at $r < 1.6$ fm.



$c_{1,3}$

$$V_C(q) = \frac{3g_A^2}{16F_\pi^4} \int \frac{d^3 l}{(2\pi)^3} \frac{\vec{l}^{\,2} - \vec{q}^{\,2}}{\omega_-^2 \omega_+^2} \left(8c_1 M_\pi^2 + c_3 (\vec{l}^{\,2} - \vec{q}^{\,2}) \right)$$

$$\xrightarrow{DR} \; -\frac{3g_A^2}{16\pi F_\pi^4} \Big\{ 2M_\pi^2 (2c_1 - c_3) - c_3 q^2 \Big\} (2M_\pi^2 + q^2) A(q) + \ldots ,$$

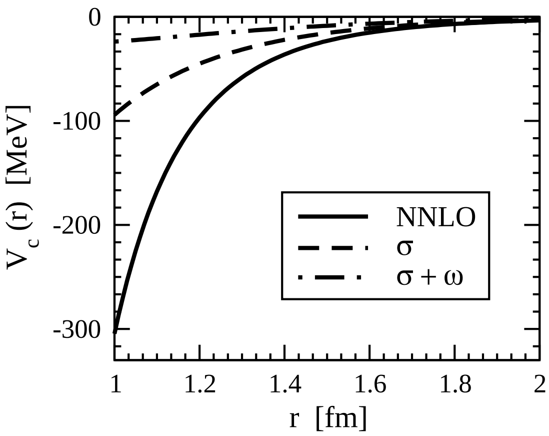
where $A(q) = \frac{1}{2q} \arctan \frac{q}{2M_\pi}$, $\quad \omega_\pm = \sqrt{(\vec{q} \pm \vec{l})^2 + 4M_\pi^2}$.

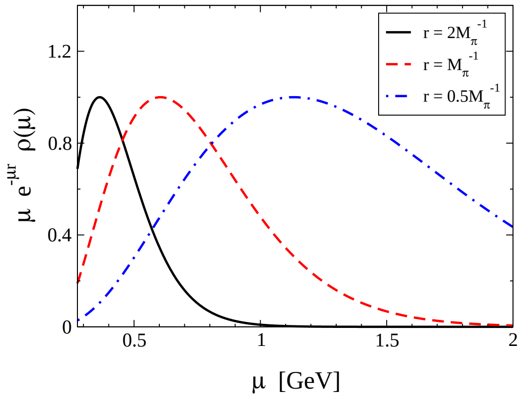
Spectral function representation of TPE:

$$V_C(q) = \frac{2q^4}{\pi} \int_{2M_\pi}^{\infty} d\mu \, \frac{1}{\mu^3} \frac{\rho(\mu)}{\mu^2 + q^2},$$

$\rho(\mu) = \mathrm{Im}\left[V_C(0^+ - i\mu) \right]$. TPE in r–space:

$$V_C(r) = \frac{1}{2\pi^2 r} \int_{2M_\pi}^{\infty} d\mu \, \mu \, e^{-\mu r} \rho(\mu)$$





The too strong attraction of V_C is due to large–μ components in the TPE spectrum, which cannot be described in chiral EFT.

Solution: regularized spectral function

$$\rho(\mu) \to \rho^\lambda(\mu) = \rho(\mu) \, \theta(\lambda - \mu) \, ,$$

where $\lambda \sim 500 \ldots 800$ MeV $\leq M_\rho$.

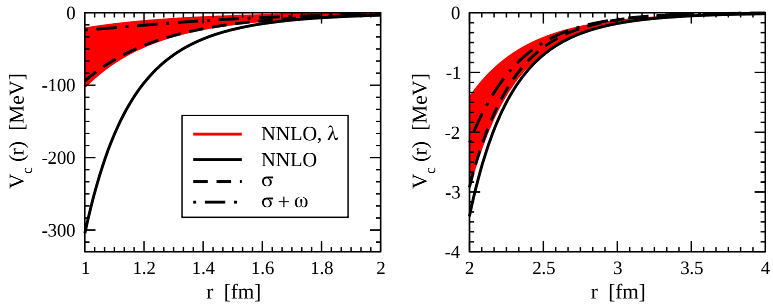
This spectral function regularization:

– is equivalent to DR ($\lambda \to \infty$) if terms in all orders in the EFT are kept;

– is equivalent to (finite) cutoff regularization of the pion loop integrals;

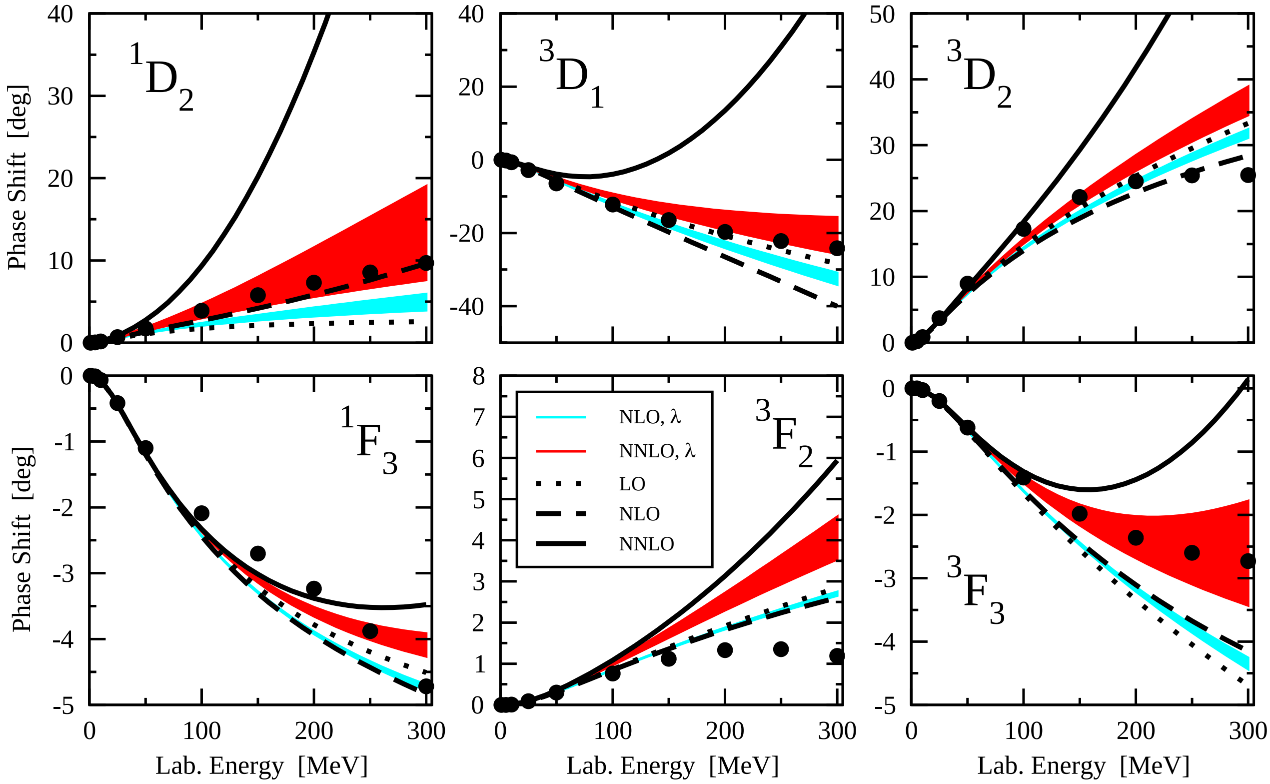
– **improves the convergence** of the chiral expansion for NN forces.

The resulting central potential (shown by the red band) is of the same order in magnitude as the one obtained in phenomenological boson–exchange models.



Here: $\lambda = 500 \ldots 800$ MeV.

Peripheral NN waves with the new regularization

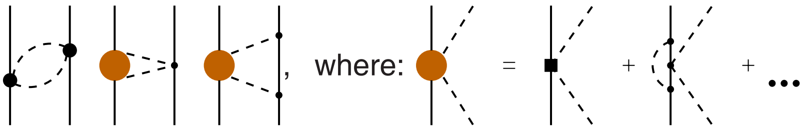


Similar idea has been applied to improve the convergence of the SU(3) baryon ChPT, see J.F.Donoghue, B.R.Holstein, PLB 136 (1998) 331; Borasoy et al., PRD 66 (2002) 094020.

Two nucleons at NNNLO (preliminary results)

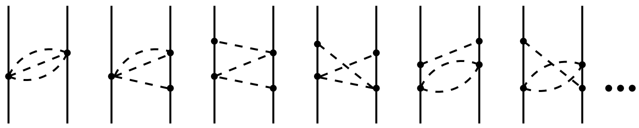
(E.E., W.Glöckle, U.–G.Meißner, in preparation)

- **two–pion exchange:**



  The expressions are given in N.Kaiser, PRC 64 (2001) 057001. We use the values of the LECs $c_{1,2,4}$, $\bar{d}_{3,5}$, $\bar{d}_1 + \bar{d}_2$, $\bar{d}_{14} + \bar{d}_{15}$ found in the πN system and $c_3 = -3.4$ GeV^{-1} (consistent with $c_3 = -4.69 \pm 1.34$ GeV^{-1} found in P.Büttiker, Ulf–G.Meißner, NPA 668 (2000) 97).

- **three–pion exchange:**



  Three–pion exchange turns out to be very small (numerically irrelevant), see N.Kaiser, PRC 61 (1999) 014003, PRC 62 (2000) 024001.

- **15 new short–range contact interactions** (12 are used).

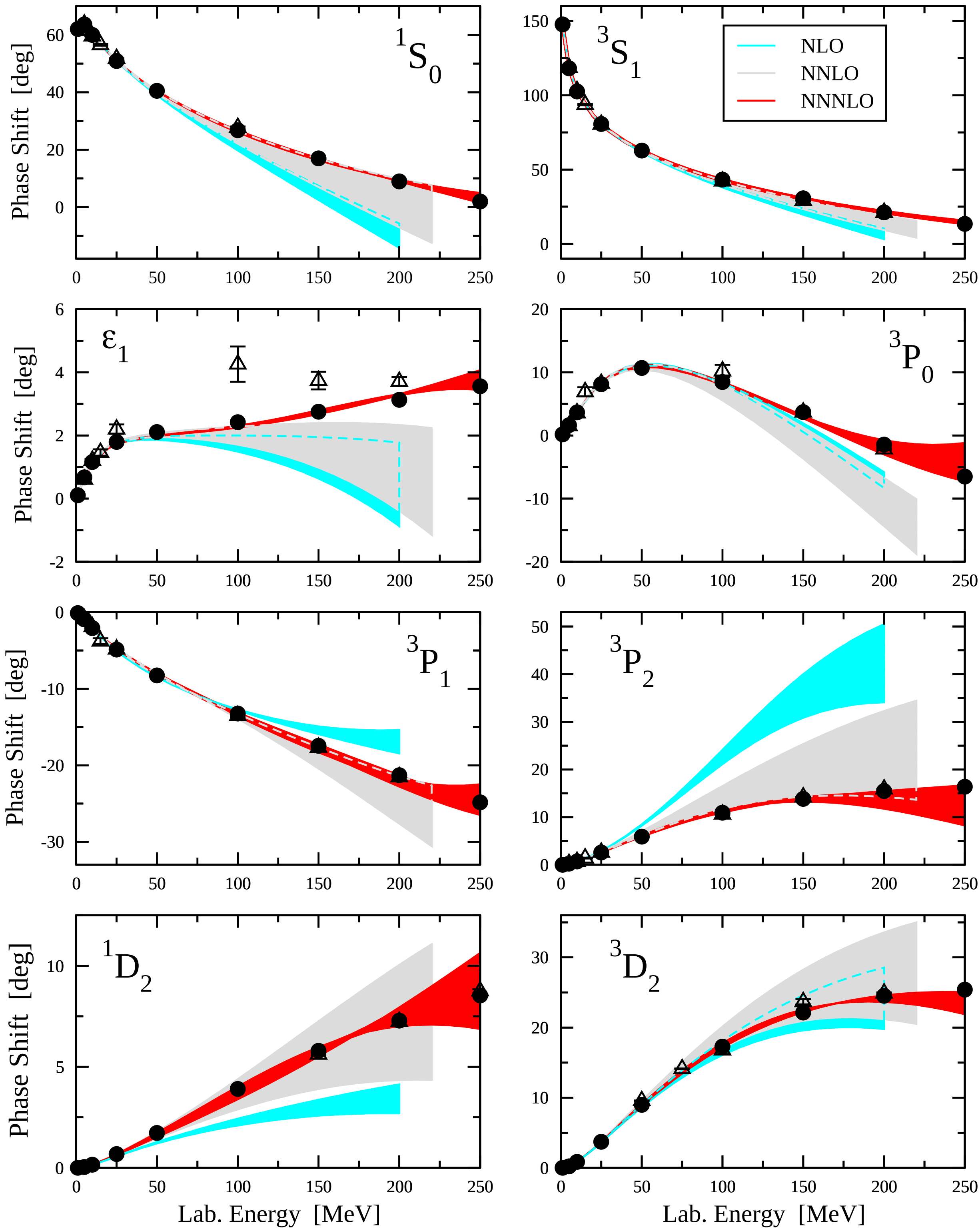
Basic features:

– **New regularization scheme** for the pion loop integrals. All c_i, $\bar{d}_i$ consistent with πN scattering.

– **Relativistic corrections** to H_0 ($H_0 = \sqrt{m^2 + p^2}$) and V (see J.L.Friar, PRC 60 (1999) 034002) are taken into account.

– **Cutoffs variation:** $\Lambda_{\rm LS} = 450 \ldots 650$ MeV; $\lambda_{\rm spectr} = 500 \ldots 700$ MeV.

NN phase shifts with the new regularization (preliminary)

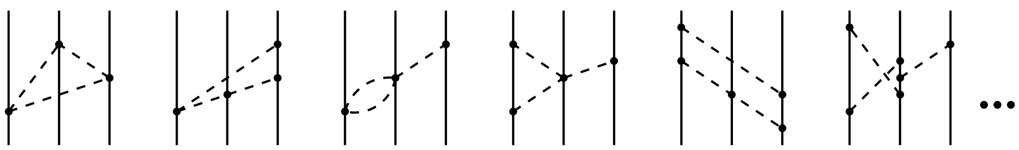


Summary

- Few–nucleon interactions can be described in chiral EFT in a **systematic** and **model independent** way.
- Complete analysis of the 2N and 3N systems at NNLO within chiral EFT has been performed. **At first time chiral 3NF has been included!** Results look promising.
- **New regularization scheme** is proposed, which allows to improve the convergence of the chiral expansion.
- First results for **two nucleons at NNNLO** with the new regularization scheme. All LECs are consistent with the πN –system.

Outlook

- **3N and 4N systems at NNLO with the new regularization scheme** and including isospin violating effects.
- **N^{3}LO corrections to the 3N force** (with V.Bernard, W.Glöckle, U.–G.Meißner, in progress).



No new contact 3N forces $\Longrightarrow$ **large predictive power!**

- **Nuclear current operators** (see: T.–S. Park, D.–P. Min, M. Rho, Phys. Rep. 233 (1993); NPA 596 (1996)) have to be rederived with the method of unitary transformation, which is used to calculate nuclear forces.
- **Electron scattering** on D and ^3He , **parity violation**, ...