Position Space Interpretation for Generalized Parton Distributions

or: GPDs (at low t) and hadron tomography

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(brief) Motivation

- DIS $\stackrel{opt.theorem}{\longrightarrow}$ forward Compton amplitude $\stackrel{Bj-limit}{\longrightarrow}$ $q(x_{Bj})$

$$q(x_{Bj}) = \int \frac{dx^-}{2\pi} \langle p|\bar{q}\left(-\frac{x^-}{2}, \mathbf{0}_\perp\right) \gamma^+ q\left(\frac{x^-}{2}, \mathbf{0}_\perp\right)|p\rangle\, e^{ix^- x_{Bj} P^+}$$

  - Light-cone coordinates $x^\pm = \frac{1}{\sqrt{2}}\left(x^0 \pm x^1\right)$
  - $q(x) =$ light-cone momentum distribution of quarks in the target; $x =$ (light-cone) momentum fraction
  - no information about position of partons!

(brief) Motivation

- generalization to $p' \neq p \quad \Rightarrow \quad$ **G**eneralized **P**arton **D**istributions

$$GPD(x,\xi,t) \equiv \int \frac{dx^-}{2\pi} \langle p'|\bar{q}\left(-\frac{x^-}{2},\mathbf{0}_\perp\right)\gamma^+ q\left(\frac{x^-}{2},\mathbf{0}_\perp\right)|p\rangle\, e^{ix^- xP^+}$$

with $\Delta = p - p'$, $t = \Delta^2$, and $\xi(p^+ + p^{+\prime}) = -2\Delta^+$.

- can be probed e.g. in **D**eeply **V**irtual **C**ompton **S**cattering (DVCS) (HERMES, JLab@12GeV, eRHIC, ...)
- Interesting observation: X.Ji, PRL**78**,610(1997)

$$\langle J_q \rangle = \frac{1}{2}\int_0^1 dx\, x\left[H_q(x,0,0) + E_q(x,0,0)\right]$$

$$\boxed{\text{DVCS}} \quad \Leftrightarrow \quad \boxed{\text{GPDs}} \quad \Leftrightarrow \quad \boxed{\vec{J}_q}$$

- But: what other "physical information" about the nucleon can we obtain by measuring/calculating GPDs?

Outline

- Probabilistic interpretation of GPDs as Fourier transforms of impact parameter dependent PDFs
  - $H(x,0,-\mathbf{\Delta}_\perp^2) \stackrel{FT}{\longrightarrow} q(x,\mathbf{b}_\perp)$
  - $\tilde{H}(x,0,-\mathbf{\Delta}_\perp^2) \stackrel{FT}{\longrightarrow} \Delta q(x,\mathbf{b}_\perp)$
  - $E(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow \perp$ distortion of PDFs when the target is transversely polarized

Generalized Parton Distributions (GPDs)

$$\int \frac{dx^-}{2\pi} e^{ix^-\bar{p}^+x} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right) \right| p \right\rangle = H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,\Delta^2)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p)$$

$$\int \frac{dx^-}{2\pi} e^{ix^-\bar{p}^+x} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right) \gamma^+\gamma_5 q\left(\frac{x^-}{2}\right) \right| p \right\rangle = \tilde{H}(x,\xi,\Delta^2)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}(x,\xi,\Delta^2)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p)$$

where $\Delta = p - p'$ is the momentum transfer and ξ measures the longitudinal momentum transfer on the target $\Delta^+ = \xi(p^+ + p^{+\prime})$.

Parton Interpretation

$$\int \frac{dx^-}{2\pi} e^{ix^-\bar{p}^+x} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right)\right| p \right\rangle = H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,\Delta^2)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p)$$

- $x =$ mean long. momentum fraction carried by active quark
- $\xi =$ longitudinal (p^+) momentum transfer
- In general no probabilistic interpretation since initial and final state not the same
- Instead: interpretation as transition amplitude
- $\int dx H(x,\xi,\Delta^2) = F_1(\Delta^2)$ and $\int dx E(x,\xi,\Delta^2) = F_2(\Delta^2)$

$\hookrightarrow$ GPDs provide a decomposition of form factor w.r.t. the momentum fraction (in IMF) carried by the active quark

- Actually $GPD = GPD(x,\xi,\Delta^2,q^2)$, but will not discuss q^2 dependence of GPDs today! ($\longrightarrow$ resolution in $\perp$ direction)

What is Physics of GPDs ?

- Definition of GPDs resembles that of form factors

$$\left\langle p' \left| \hat{O} \right| p \right\rangle = H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,\Delta^2)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p)$$

with $\hat{O} \equiv \int \frac{dx^-}{2\pi} e^{ix^-\bar{p}^+x}\bar{q}\left(-\frac{x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right)$

$\hookrightarrow$ relation between PDFs and GPDs similar to relation between a charge and a form factor

$\hookrightarrow$ If form factors can be interpreted as Fourier transforms of charge distributions in position space, what is the analogous physical interpretation for GPDs ?

Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi} \bar{q}\left(\frac{-x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,\xi,t)$	?

Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi} \bar{q}\left(\frac{-x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,0,t)$	$q(x,\mathbf{b}_\perp)$

$q(x,\mathbf{b}_\perp)$ $=$ impact parameter dependent PDF

Impact parameter dependent PDFs

- define state that is localized in $\perp$ position:

$$\left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \lambda\right\rangle \equiv \mathcal{N} \int d^2\mathbf{p}_\perp \left|p^+, \mathbf{p}_\perp, \lambda\right\rangle$$

  Note: $\perp$ boosts in IMF form Galilean subgroup $\Rightarrow$ this state has $\mathbf{R}_\perp \equiv \frac{1}{P^+} \int dx^- d^2\mathbf{x}_\perp\, \mathbf{x}_\perp T^{++}(x) = \mathbf{0}_\perp$
  (cf.: working in CM frame in nonrel. physics)

- define impact parameter dependent PDF

$$q(x, \mathbf{b}_\perp) \equiv \int \frac{dx^-}{4\pi} \langle p^+, \mathbf{0}_\perp | \, \bar{q}\left(-\frac{x^-}{2}, \mathbf{b}_\perp\right) \gamma^+ q\left(\frac{x^-}{2}, \mathbf{b}_\perp\right) \left|p^+, \mathbf{0}_\perp\right\rangle e^{ixp^+x^-}$$

Impact parameter dependent PDFs

- use translational invariance to relate to same matrix element that appears in def. of GPDs

$$q(x,\mathbf{b}_\perp) \equiv \int dx^- \langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp | \bar{q}(-\frac{x^-}{2}, \mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2}, \mathbf{b}_\perp) | p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \rangle e^{ixp^+x^-}$$

$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \int dx^- \langle p^+, \mathbf{p}'_\perp | \bar{q}(-\frac{x^-}{2}, \mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2}, \mathbf{b}_\perp) | p^+, \mathbf{p}_\perp \rangle e^{ixp^+x^-}$$

Impact parameter dependent PDFs

- use translational invariance to relate to same matrix element that appears in def. of GPDs

$$q(x,\mathbf{b}_\perp) \equiv \int dx^- \left\langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right| \bar{q}(-\frac{x^-}{2}, \mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2}, \mathbf{b}_\perp) \left| p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right\rangle e^{ixp^+x^-}$$

$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \int dx^- \left\langle p^+, \mathbf{p}'_\perp \right| \bar{q}(-\frac{x^-}{2}, \mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2}, \mathbf{b}_\perp) \left| p^+, \mathbf{p}_\perp \right\rangle e^{ixp^+x^-}$$

$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \int dx^- \left\langle p^+, \mathbf{p}'_\perp \right| \bar{q}(-\frac{x^-}{2}, \mathbf{0}_\perp)\gamma^+ q(\frac{x^-}{2}, \mathbf{0}_\perp) \left| p^+, \mathbf{p}_\perp \right\rangle e^{ixp^+x^-}$$

$$\times e^{i\mathbf{b}_\perp \cdot (\mathbf{p}_\perp - \mathbf{p}'_\perp)}$$

Impact parameter dependent PDFs

- use translational invariance to relate to same matrix element that appears in def. of GPDs

$$q(x,\mathbf{b}_\perp) \equiv \int dx^- \left\langle p^+,\mathbf{R}_\perp=\mathbf{0}_\perp\right| \bar{q}(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{b}_\perp) \left|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp\right\rangle e^{ixp^+x^-}$$

$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \int dx^- \left\langle p^+,\mathbf{p}'_\perp\right| \bar{q}(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{b}_\perp) \left|p^+,\mathbf{p}_\perp\right\rangle e^{ixp^+x^-}$$

$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \int dx^- \left\langle p^+,\mathbf{p}'_\perp\right| \bar{q}(-\frac{x^-}{2},\mathbf{0}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{0}_\perp) \left|p^+,\mathbf{p}_\perp\right\rangle e^{ixp^+x^-} \times e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp-\mathbf{p}'_\perp)}$$

$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp H\left(x,0,-\left(\mathbf{p}'_\perp-\mathbf{p}_\perp\right)^2\right) e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp-\mathbf{p}'_\perp)}$$

$$\hookrightarrow \quad \boxed{q(x,\mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H(x,0,-\mathbf{\Delta}_\perp^2) e^{i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}}$$

Impact parameter dependent PDFs

- $$q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H(x, 0, -\mathbf{\Delta}_\perp^2) e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

  $(\mathbf{\Delta}_\perp = \mathbf{p}_\perp - \mathbf{p}'_\perp,\ \xi = 0)$
- $q(x, \mathbf{b}_\perp)$ has physical interpretation of a density

$$q(x, \mathbf{b}_\perp) \geq 0 \quad \text{for} \quad x > 0$$

$$q(x, \mathbf{b}_\perp) \leq 0 \quad \text{for} \quad x < 0$$

Discussion: $GPD \leftrightarrow q(x, \mathbf{b}_\perp)$

- GPDs allow simultaneous determination of longitudinal momentum and transverse position of partons

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H(x, 0, -\mathbf{\Delta}_\perp^2) e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

- very similar results for impact parameter dependent polarized quark distributions (nucleon longitudinally polarized)

$$\Delta q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} \tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2) e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

- $q(x, \mathbf{b}_\perp)$ has interpretation as density (positivity constraints!)

$$\begin{aligned} q(x, \mathbf{b}_\perp) &\sim \left\langle p^+, \mathbf{0}_\perp \left| b^\dagger(xp^+, \mathbf{b}_\perp) b(xp^+, \mathbf{b}_\perp) \right| p^+, \mathbf{0}_\perp \right\rangle \\ &= \left| b(xp^+, \mathbf{b}_\perp) |p^+, \mathbf{0}_\perp\rangle \right|^2 \geq 0 \end{aligned}$$

$\hookrightarrow$ probabilistic interpretation!

Discussion: $GPD \leftrightarrow q(x, \mathbf{b}_\perp)$

- Nonrelativistically, connection

$$H(x, 0, -\mathbf{\Delta}_\perp^2) \stackrel{FT}{\longleftrightarrow} q(x, \mathbf{b}_\perp)$$

not surprising!
Absence of relativistic corrections to identification
$H(x, 0, -\mathbf{\Delta}_\perp^2) \stackrel{FT}{\longleftrightarrow} q(x, \mathbf{b}_\perp)$ due to Galilean subgroup in IMF

Discussion: $GPD \leftrightarrow q(x, \mathbf{b}_\perp)$

- $\mathbf{b}_\perp$ distribution measured w.r.t. $\mathbf{R}_\perp^{CM} \equiv \sum_i x_i \mathbf{r}_{i,\perp}$
  $\hookrightarrow$ width of the $\mathbf{b}_\perp$ distribution should go to zero as $x \rightarrow 1$, since the active quark becomes the $\perp$ center of momentum in that limit!
  $\hookrightarrow$ $H(x, 0, t)$ must become t-indep. as $x \rightarrow 1$.

- Use intuition about nucleon structure in position space to make predictions for GPDs:
  large x ($x \gg \frac{m_\pi}{M_N}$): quarks from localized valence 'core',
  small x ($x \sim \frac{m_\pi}{M_N}$ or less): contributions from larger 'pion cloud' (C.Weiss et al.)

$\hookrightarrow$ size of hadrons in impact parameter space expected to increase as x decreases

$\hookrightarrow$ expect decrease of the t-dependence ($\perp$ size) in $H(x, 0, t)$ as x increases (recently confirmed in LGT calcs. by J.W.Negele et al.)

- model: $H_q(x, 0, -\mathbf{\Delta}_\perp^2) = q(x) e^{-a \mathbf{\Delta}_\perp^2 (1-x) \ln \frac{1}{x}}$.

The physics of $E(x,0,-\mathbf{\Delta}_\perp^2)$

- So far: only unpolarized (or long. polarized) nucleon

  In general, use ($\Delta^+ = 0$)

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \left\langle P{+}\Delta,\uparrow \left| \bar{q}\left(\frac{-x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right)\right| P,\uparrow\right\rangle = H(x,0,-\mathbf{\Delta}_\perp^2)$$

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \left\langle P{+}\Delta,\uparrow \left| \bar{q}\left(\frac{-x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right)\right| P,\downarrow\right\rangle = -\frac{\Delta_x - i\Delta_y}{2M} E(x,0,-\mathbf{\Delta}_\perp^2).$$

- Consider nucleon polarized in x direction (in IMF)
  $|X\rangle \equiv |p^+,\mathbf{R}_\perp = \mathbf{0}_\perp,\uparrow\rangle + |p^+,\mathbf{R}_\perp = \mathbf{0}_\perp,\downarrow\rangle$.

$\hookrightarrow$ unpolarized quark distribution for this state:

$$q_X(x,\mathbf{b}_\perp) = q(x,\mathbf{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} E(x,0,-\mathbf{\Delta}_\perp^2) e^{i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}$$

The physics of $E(x,0,-\Delta_\perp^2)$

- $q_X(x,\mathbf{b}_\perp)$ in transversely polarized nucleon is transversely distorted compared to longitudinally polarized nucleons !
- mean displacement of flavor q ($\perp$ flavor dipole moment)

$$d_y^q \equiv \int dx \int d^2\mathbf{b}_\perp q_X(x,\mathbf{b}_\perp) b_y = \frac{1}{2M}\int dx E_q(x,0,0) = \frac{\kappa_q^p}{2M}$$

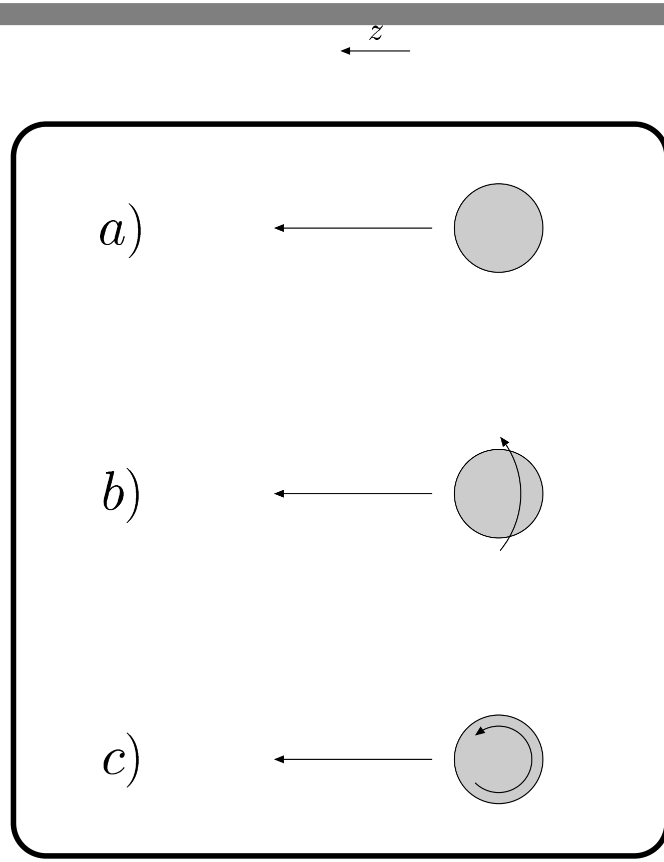
with $\kappa^p_{u/d} \equiv F_2^{u/d}(0) = \mathcal{O}(1-2) \quad \Rightarrow \quad d_y^q = \mathcal{O}(0.2 fm)$

- CM for flavor q shifted relative to CM for whole proton by

$$\int dx \int d^2\mathbf{b}_\perp q_X(x,\mathbf{b}_\perp) x b_y = \frac{1}{2M}\int dx\, x E_q(x,0,0)$$

$\hookrightarrow$ not surprising to find that second moment of E_q is related to angular momentum carried by flavor q

physical origin for $\perp$ distortion



Comparison of a non-rotating sphere that moves in z direction with a sphere that spins at the same time around the z axis and a sphere that spins around the x axis When the sphere spins around the x axis, the rotation changes the distribution of momenta in the z direction (adds/subtracts to velocity for $y > 0$ and $y < 0$ respectively). For the nucleon the resulting modification of the (unpolarized) momentum distribution is described by $E(x, 0, -\mathbf{\Delta}_\perp^2)$.

simple model for $E_q(x,0,-\mathbf{\Delta}_\perp^2)$

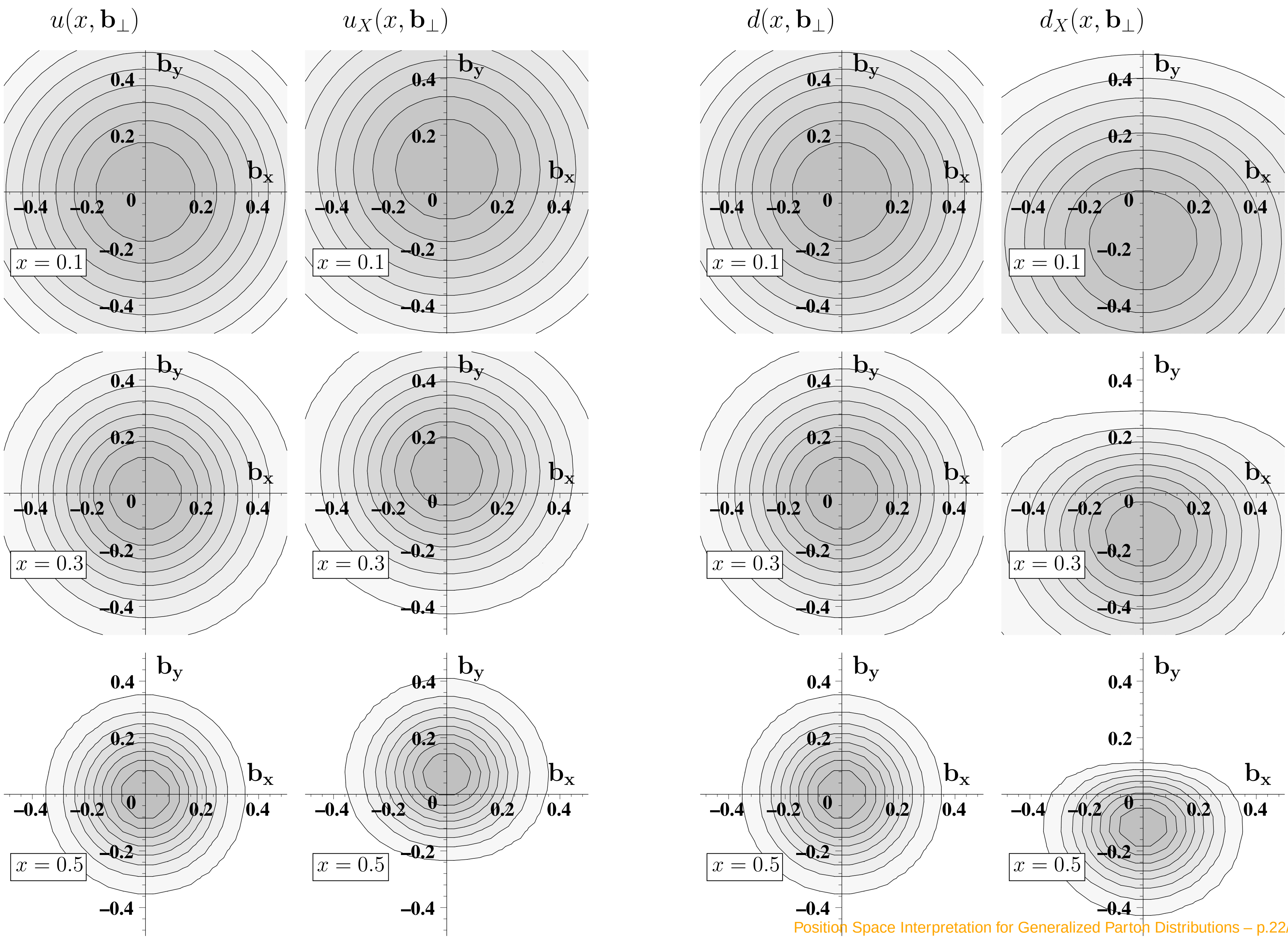
- For simplicity, make ansatz where $E_q \propto H_q$

$$E_u(x,0,-\mathbf{\Delta}_\perp^2) = \frac{\kappa_u^p}{2} H_u(x,0,-\mathbf{\Delta}_\perp^2)$$
$$E_d(x,0,-\mathbf{\Delta}_\perp^2) = \kappa_d^p H_d(x,0,-\mathbf{\Delta}_\perp^2)$$

with

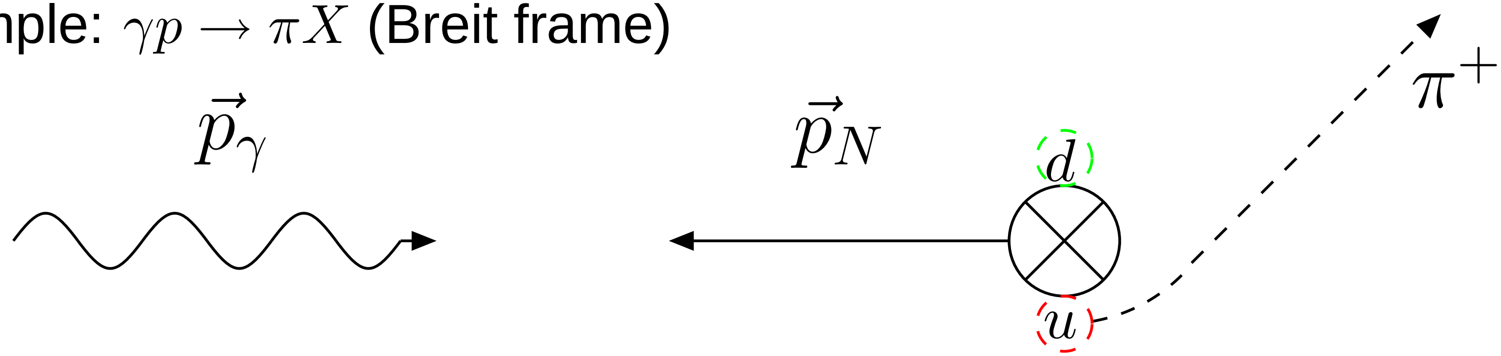
$$\kappa_u^p = 2\kappa_p + \kappa_n = 1.673 \qquad \kappa_d^p = 2\kappa_n + \kappa_p = -2.033.$$

- Satisfies: $\int dx E_q(x,0,0) = \kappa_q^P$
- Model too simple but illustrates that anticipated distortion is very significant since κ_u and κ_d known to be large!


$u(x, \mathbf{b}_\perp)$
$u_X(x, \mathbf{b}_\perp)$
$d(x, \mathbf{b}_\perp)$
$d_X(x, \mathbf{b}_\perp)$
$x = 0.1$
$x = 0.3$
$x = 0.5$
$\mathbf{b_x}$
$\mathbf{b_y}$
−0.4
−0.2
0
0.2
0.4

connection with $\perp$ SSA

- example: $\gamma p \to \pi X$ (Breit frame)



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign determined by κ_u & κ_d
- attractive FSI deflects active quark towards the center of momentum

$\hookrightarrow$ FSI converts left-right position space asymmetry of leading quark into right-left asymmetry in momentum

- compare: convex lens that is illuminated asymmetrically

$\hookrightarrow$ semi-classical picture for recent results by Brodsky et al.

- natural explanation for correlation between sign of κ_q and sign of SSA

Other topics

- QCD evolution
- extrapolating to $\xi = 0$
- transverse hyperon polarization

Summary

- DVCS allows probing GPDS

$$\int \frac{dx^-}{2\pi} e^{ixp^+x^-} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right) \right| p \right\rangle$$

- GPDs resemble both PDFs and form factors: defined through matrix elements of light-cone correlation, but $\Delta \equiv p' - p \neq 0$.
- t-dependence of GPDs at $\xi = 0$ (purely $\perp$ momentum transfer) $\Rightarrow$ Fourier transform of impact parameter dependent PDFs $q(x, \mathbf{b}_\perp)$

$\hookrightarrow$ knowledge of GPDs for $\xi = 0$ provides novel information about nonperturbative parton structure of nucleons: distribution of partons in $\perp$ plane

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H(x, 0, -\mathbf{\Delta}_\perp^2) e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

$$\Delta q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} \tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2) e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

- $q(x, \mathbf{b}_\perp)$, $\Delta q(x, \mathbf{b}_\perp)$ have probabilistic interpretation, e.g. $q(x, \mathbf{b}_\perp) > 0$ for $x > 0$

Summary

- $\frac{\Delta_\perp}{2M} E(x, 0, -\boldsymbol{\Delta}_\perp^2)$ describes how the momentum distribution of unpolarized partons in the $\perp$ plane gets transversely distorted when is nucleon polarized in $\perp$ direction.
- (attractive) final state interaction converts $\perp$ position space asymmetry into $\perp$ momentum space asymmetry

$\hookrightarrow$ simple physical explanation for sign of Sivers asymmetry

- Similar mechanism also applicable to many other semi-inclusive events, such as transverse polarizations in hyperon production.
- published in: M.B., PRD **62**, 71503 (2000), hep-ph/0105324, and hep-ph/0207047; see also D. Soper, PRD **15**, 1141 (1977).
- Connection to SSA in M.B., PRD **66**, 114005 (2002); hep-ph/0302144.

extrapolating to $\xi = 0$

- bad news: $\xi = 0$ not directly accessible in DVCS since long. momentum transfer necessary to convert virtual γ into real γ
- good news: moments of GPDs have simple ξ-dependence (polynomials in ξ)
  $\hookrightarrow$ should be possible to extrapolate!

even moments of $H(x,\xi,t)$:

$$
\begin{aligned}
H_n(\xi,t) &\equiv \int_{-1}^{1} dx x^{n-1} H(x,\xi,t) = \sum_{i=0}^{\left[\frac{n-1}{2}\right]} A_{n,2i}(t)\xi^{2i} + C_n(t) \\
&= A_{n,0}(t) + A_{n,2}(t)\xi^2 + ... + A_{n,n-2}(t)\xi^{n-2} + C_n(t)\xi^n,
\end{aligned}
$$

i.e. for example

$$\int_{-1}^{1} dx x H(x,\xi,t) = A_{2,0}(t) + C_2(t)\xi^2.$$

- For n^{th} moment, need $\frac{n}{2}+1$ measurements of $H_n(\xi,t)$ for same t but different ξ to determine $A_{n,2i}(t)$.
- GPDs @ $\xi = 0$ obtained from $H_n(\xi = 0, t) = A_{n,0}(t)$
- similar procedure exists for moments of $\tilde{H}$

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QCD evolution

So far ignored! Can be easily included because

- For $t \ll Q^2$, leading order evolution t-independent
- For $\xi = 0$ evolution kernel for GPDs same as DGLAP evolution kernel

likewise:

- impact parameter dependent PDFs evolve such that different $\mathbf{b}_\perp$ do not mix (as long as $\perp$ spatial resolution much smaller than Q^2)

$\hookrightarrow$ above results consistent with QCD evolution:

$$H(x,0,-\mathbf{\Delta}_\perp^2,Q^2) = \int d^2b_\perp q(x,\mathbf{b}_\perp,Q^2)e^{i\mathbf{b}_\perp \mathbf{\Delta}_\perp}$$
$$\tilde{H}(x,0,-\mathbf{\Delta}_\perp^2,Q^2) = \int d^2b_\perp \Delta q(x,\mathbf{b}_\perp,Q^2)e^{i\mathbf{b}_\perp \mathbf{\Delta}_\perp}$$

where QCD evolution of $H, \tilde{H}, q, \Delta q$ is described by DGLAP and is independent on both $\mathbf{b}_\perp$ and $\mathbf{\Delta}_\perp^2$, provided one does not look at scales in $\mathbf{b}_\perp$ that are smaller than $1/Q$.

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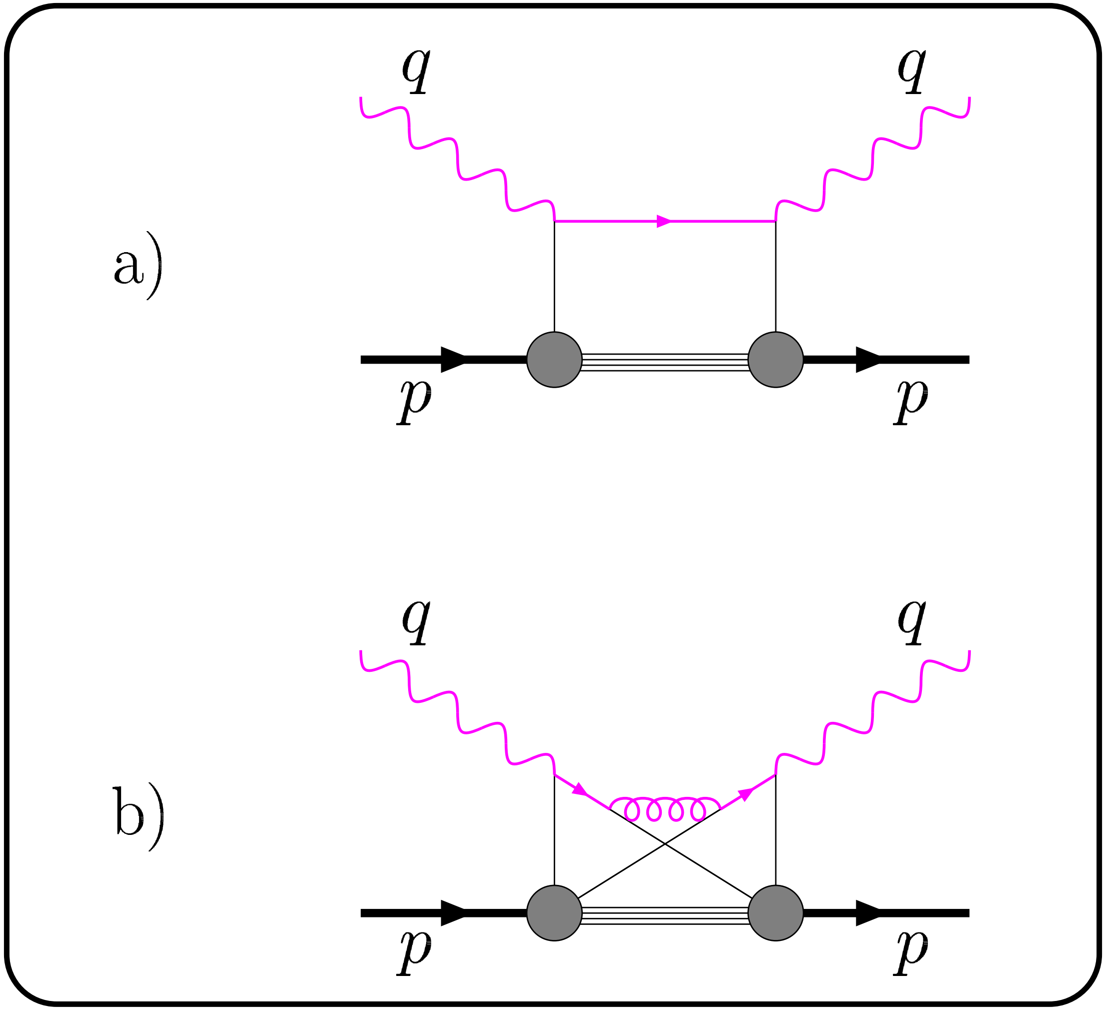
suppression of crossed diagrams



Flow of the large momentum q through typical diagrams contributing to the forward Compton amplitude. a) 'handbag' diagrams; b) 'cat's ears' diagram. Diagram b) is suppressed at large q due to the presence of additional propagators.

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Form factor vs. charge distribution (non-rel.)

- define state that is localized in position space (center of mass frame)

$$\left|\vec{R}=\vec{0}\right\rangle \equiv \mathcal{N}\int d^3\vec{p}\,|\vec{p}\rangle$$

- define charge distribution (for this localized state)

$$\rho(\vec{r}) \equiv \left\langle \vec{R}=\vec{0}\right| j^0(\vec{r}) \left|\vec{R}=\vec{0}\right\rangle$$

- use translational invariance to relate to same matrix element that appears in def. of form factor

$$\rho(\vec{r}) \equiv \left\langle \vec{R}=\vec{0} \right| j^0(\vec{r}) \left| \vec{R}=\vec{0} \right\rangle$$

$$= |\mathcal{N}|^2 \int d^3\vec{p} \int d^3\vec{p}' \, \langle \vec{p}' | \, j^0(\vec{r}) \, | \vec{p} \rangle$$

$$= |\mathcal{N}|^2 \int d^3\vec{p} \int d^3\vec{p}' \langle \vec{p}' | \, j^0(\vec{0}) \, | \vec{p} \rangle e^{i\vec{r}\cdot(\vec{p}-\vec{p}')},$$

$$= |\mathcal{N}|^2 \int d^3\vec{p} \int d^3\vec{p}' \, F\left(-(\vec{p}' - \vec{p})^2 \right) e^{i\vec{r}\cdot(\vec{p}-\vec{p}')}$$

$\hookrightarrow$

$$\rho(\vec{r}) = \int \frac{d^3\vec{\Delta}}{(2\pi)^3} F(-\vec{\Delta}^2) e^{i\vec{r}\cdot\vec{\Delta}}$$

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density interpretation of $q(x, \mathbf{b}_\perp)$

- express quark-bilinear in twist-2 GPD in terms of light-cone 'good' component $q_{(+)} \equiv \frac{1}{2}\gamma^-\gamma^+ q$

$$\bar{q}'\gamma^+ q = \bar{q}'_{(+)}\gamma^+ q_{(+)} = \sqrt{2} q'^{\dagger}_{(+)} q_{(+)}.$$

- expand $q_{(+)}$ in terms of canonical raising and lowering operators

$$q_{(+)}(x^-, \mathbf{x}_\perp) = \int_0^\infty \frac{dk^+}{\sqrt{4\pi k^+}} \int \frac{d^2\mathbf{k}_\perp}{2\pi} \sum_s \times \left[u_{(+)}(k,s) b_s(k^+, \mathbf{k}_\perp) e^{ikx} + v_{(+)}(k,s) d_s^\dagger(k^+, \mathbf{k}_\perp) e^{ikx}\right],$$

density interpretation of $q(x, \mathrm{b}_\perp)$

with usual (canonical) equal light-cone time x^+ anti-commutation relations, e.g.

$$\left\{b_r(k^+,\mathbf{k}_\perp), b_s^\dagger(q^+,\mathbf{q}_\perp)\right\} = \delta(k^+ - q^+)\delta(\mathbf{k}_\perp - \mathbf{q}_\perp)\delta_{rs}$$

and the normalization of the spinors is such that

$$\bar{u}_{(+)}(p,r)\gamma^+ u_{(+)}(p,s) = 2p^+\delta_{rs}.$$

Note: $\bar{u}_{(+)}(p',r)\gamma^+ u_{(+)}(p,s) = 2p^+\delta_{rs}$ for $p^+ = p'^+$, one finds for $x > 0$

$$q(x,\mathbf{b}_\perp) = \mathcal{N}' \sum_s \int \frac{d^2\mathbf{k}_\perp}{2\pi} \int \frac{d^2\mathbf{k}'_\perp}{2\pi} \left\langle p^+,\mathbf{0}_\perp \right| b_s^\dagger(xp^+,\mathbf{k}'_\perp) b_s(xp^+,\mathbf{k}_\perp) \left| p^+,\mathbf{0}_\perp \right\rangle \times e^{i\mathbf{b}_\perp\cdot(\mathbf{k}_\perp - \mathbf{k}'_\perp)}.$$

density interpretation of $q(x,\mathrm{b}_\perp)$

- Switch to mixed representation:
  momentum in longitudinal direction
  position in transverse direction

$$\tilde{b}_s(k^+, \mathbf{x}_\perp) \equiv \int \frac{d^2\mathbf{k}_\perp}{2\pi} b_s(k^+, \mathbf{k}_\perp) e^{i\mathbf{k}_\perp \cdot \mathbf{x}_\perp}$$

$\hookrightarrow$

$$\begin{aligned} q(x, \mathbf{b}_\perp) &= \sum_s \left\langle p^+, \mathbf{0}_\perp \right| \tilde{b}_s^\dagger(xp^+, \mathbf{b}_\perp) \tilde{b}_s(xp^+, \mathbf{b}_\perp) \left| p^+, \mathbf{0}_\perp \right\rangle . \\ &= \sum_s \left| \tilde{b}_s(xp^+, \mathbf{b}_\perp) \left| p^+, \mathbf{0}_\perp \right\rangle \right|^2 \\ &\geq 0. \end{aligned}$$

back

Boosts in nonrelativistic QM

$$\vec{x}' = \vec{x} + \vec{v}t \qquad\qquad t' = t$$

purely kinematical (quantization surface $t = 0$ inv.)

$\hookrightarrow$ 1. boosting wavefunctions very simple

$$q_{\vec{v}}(\vec{p}_1, \vec{p}_2) = q_{\vec{0}}(\vec{p}_1 - m_1\vec{v}, \vec{p}_2 - m_2\vec{v}).$$

2. dynamics of center of mass

$$\vec{R} \equiv \sum_i x_i \vec{r}_i \qquad \text{with} \qquad x_i \equiv \frac{m_i}{M}$$

decouples from the internal dynamics

Relativistic Boosts

$$t' = \gamma\left(t + \frac{v}{c^2}z\right), \qquad z' = \gamma\left(z + vt\right) \qquad \mathbf{x}'_\perp = \mathbf{x}_\perp$$

generators satisfy Poincaré algebra:

$$\begin{aligned}
[P^\mu, P^\nu] &= 0 \\
[M^{\mu\nu}, P^\rho] &= i\left(g^{\nu\rho}P^\mu - g^{\mu\rho}P^\nu\right) \\
\left[M^{\mu\nu}, M^{\rho\lambda}\right] &= i\left(g^{\mu\lambda}M^{\nu\rho} + g^{\nu\rho}M^{\mu\lambda} - g^{\mu\rho}M^{\nu\lambda} - g^{\nu\lambda}M^{\mu\rho}\right)
\end{aligned}$$

rotations: $M_{ij} = \varepsilon_{ijk}J_k$, boosts: $M_{i0} = K_i$.

Galilean subgroup of $\perp$ boosts

introduce generator of $\perp$ 'boosts':

$$B_x \equiv M^{+x} = \frac{K_x + J_y}{\sqrt{2}} \qquad B_y \equiv M^{+y} = \frac{K_y - J_x}{\sqrt{2}}$$

Poincaré algebra $\Longrightarrow$ commutation relations:

$$\begin{aligned} [J_3, B_k] &= i\varepsilon_{kl} B_l & \qquad [P_k, B_l] &= -i\delta_{kl} P^+ \\ \left[P^-, B_k\right] &= -iP_k & \qquad [P^+, B_k] &= 0 \end{aligned}$$

with $k, l \in \{x, y\}$, $\varepsilon_{xy} = -\varepsilon_{yx} = 1$, and $\varepsilon_{xx} = \varepsilon_{yy} = 0$.

Together with $[J_z, P_k] = i\varepsilon_{kl} P_l$, as well as

$$\begin{aligned} \left[P^-, P_k\right] &= \left[P^-, P^+\right] = \left[P^-, J_z\right] = 0 \\ \left[P^+, P_k\right] &= \left[P^+, B_k\right] = \left[P^+, J_z\right] = 0. \end{aligned}$$

Same as commutation relations among generators of nonrel. boosts, translations, and rotations in x-y plane, provided one identifies

$$\begin{aligned} P^- &\longrightarrow \text{Hamiltonian} \\ \mathbf{P}_\perp &\longrightarrow \text{momentum in the plane} \\ P^+ &\longrightarrow \text{mass} \\ L_z &\longrightarrow \text{rotations around } z\text{-axis} \\ \mathbf{B}_\perp &\longrightarrow \text{generator of boosts in the plane,} \end{aligned}$$

back to discussion

Consequences

- many results from NRQM carry over to $\perp$ boosts in IMF, e.g.
- $\perp$ boosts kinematical

$$\begin{aligned} q_{\boldsymbol{\Delta}_\perp}(x,\mathbf{k}_\perp) &= q_{\mathbf{0}_\perp}(x,\mathbf{k}_\perp - x\boldsymbol{\Delta}_\perp) \\ q_{\boldsymbol{\Delta}_\perp}(x,\mathbf{k}_\perp,y,\mathbf{l}_\perp) &= q_{\mathbf{0}_\perp}(x,\mathbf{k}_\perp - x\boldsymbol{\Delta}_\perp, y, \mathbf{l}_\perp - y\boldsymbol{\Delta}_\perp) \end{aligned}$$

- Transverse center of momentum $\mathbf{R}_\perp \equiv \sum_i x_i \mathbf{r}_{\perp,i}$ plays role similar to NR center of mass, e.g. $\int d^2\mathbf{p}_\perp \left|p^+, \mathbf{p}_\perp\right\rangle$ corresponds to state with $\mathbf{R}_\perp = \mathbf{0}_\perp$.

back

$\perp$ Center of Momentum

- field theoretic definition

$$p^+\mathbf{R}_\perp \equiv \int dx^- \int d^2\mathbf{x}_\perp T^{++}(x)\mathbf{x}_\perp = M^{+\perp}$$

- $M^{+\perp} = \mathbf{B}^\perp$ generator of transverse boosts
- parton representation:

$$\mathbf{R}_\perp = \sum_i x_i \mathbf{r}_{\perp,i}$$

($x_i =$ momentum fraction carried by i^{th} parton)

back

Poincaré algebra:

$$
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\end{aligned}
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rotations: $M_{ij} = \varepsilon_{ijk}J_k$, boosts: $M_{i0} = K_i$. back

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back

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back

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$$\begin{aligned}\psi_{\boldsymbol{\Delta}_\perp}(x,\mathbf{k}_\perp) &= \psi_{\mathbf{0}_\perp}(x,\mathbf{k}_\perp - x\boldsymbol{\Delta}_\perp)\\ \psi_{\boldsymbol{\Delta}_\perp}(x,\mathbf{k}_\perp,y,\mathbf{l}_\perp) &= \psi_{\mathbf{0}_\perp}(x,\mathbf{k}_\perp - x\boldsymbol{\Delta}_\perp,y,\mathbf{l}_\perp - y\boldsymbol{\Delta}_\perp)\end{aligned}$$

- Transverse center of momentum $\mathbf{R}_\perp \equiv \sum_i x_i \mathbf{r}_{\perp,i}$ plays role similar to NR center of mass, e.g. $|p^+,\mathbf{R}_\perp = \mathbf{0}_\perp\rangle \equiv \int d^2\mathbf{p}_\perp\, |p^+,\mathbf{p}_\perp\rangle$ corresponds to state with $\mathbf{R}_\perp = \mathbf{0}_\perp$.

back

Proof that $\mathrm{B}_\perp |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp\rangle = 0$

- Use

$$e^{-i\mathbf{v}_\perp\cdot\mathbf{B}_\perp}|p^+,\mathbf{p}_\perp,\lambda\rangle = |p^+,\mathbf{p}_\perp + p^+\mathbf{v}_\perp,\lambda\rangle$$

$\hookrightarrow$

$$e^{-i\mathbf{v}_\perp\cdot\mathbf{B}_\perp}\int d^2\mathbf{p}_\perp|p^+,\mathbf{p}_\perp,\lambda\rangle = \int d^2\mathbf{p}_\perp|p^+,\mathbf{p}_\perp,\lambda\rangle$$

$\hookrightarrow$

$$\mathbf{B}_\perp\int d^2\mathbf{p}_\perp|p^+,\mathbf{p}_\perp,\lambda\rangle = 0$$

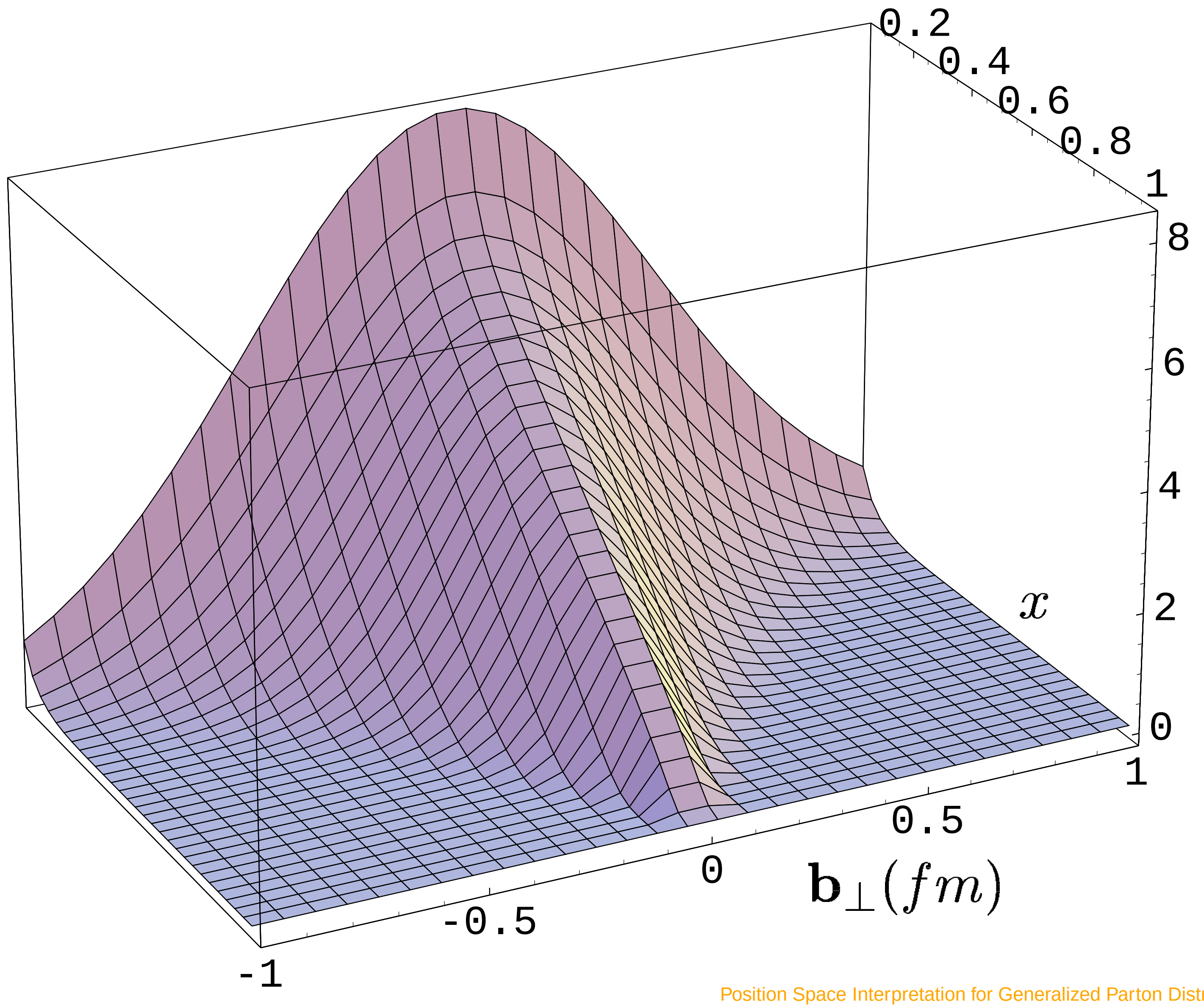
back

Example

- Ansatz: $H_q(x,0,-\mathbf{\Delta}_\perp^2) = q(x)e^{-a\mathbf{\Delta}_\perp^2(1-x)\ln\frac{1}{x}}$.

$$\hookrightarrow \quad q(x,\mathbf{b}_\perp) = q(x)\frac{1}{4\pi a(1-x)\ln\frac{1}{x}} e^{-\frac{\mathbf{b}_\perp^2}{4a(1-x)\ln\frac{1}{x}}}$$

simple model for $q(x, \mathrm{b}_\perp)$



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Application: $\perp$ hyperon polarization

model for hyperon polarization in $pp \to Y + X$ ($Y \in \Lambda, \Sigma, \Xi$) at high energy:

- peripheral scattering
- $s\bar{s}$ produced in overlap region, i.e. on "inside track"

$\hookrightarrow$ if Y deflected to left then s produced on left side of Y (and vice versa)

$\hookrightarrow$ if $\kappa_s > 0$ then intermediate state has better overlap with final state Y that has spin down (looking into the flight direction)

$\hookrightarrow$ remarkable prediction: $\boxed{\vec{P}_Y \sim -\kappa_s^Y \vec{p}_P \times \vec{p}_Y.}$

back

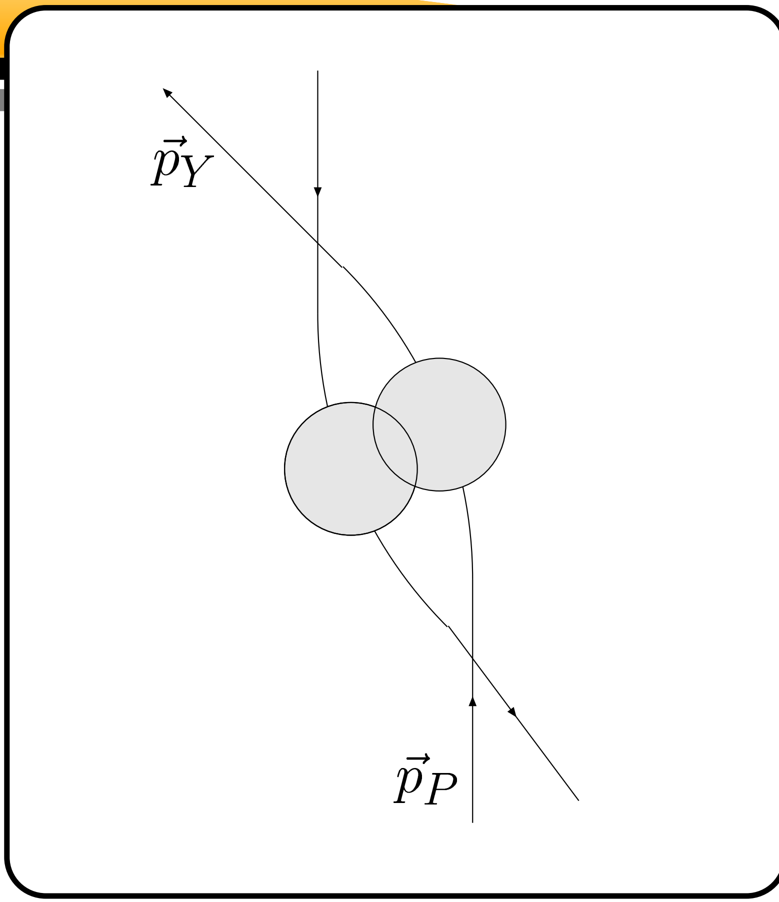

Figure 1: $P + P \longrightarrow Y + X$ where the incoming P (from bottom) is deflected to the left during the reaction. The $s\bar{s}$ pair is assumed to be produced in the overlap region, i.e. on the left 'side' of the Y.

back

- SU(3) analysis for κ_s^B yields (assuming $|\kappa_s^p| \ll |\kappa_u^p|, |\kappa_d^p|$)

$$\begin{aligned} \kappa_s^\Lambda &= \kappa^p + \kappa_s^p = 1.79 + \kappa_s^p \\ \kappa_s^\Sigma &= \kappa^p + 2\kappa^n + \kappa_s^p = -2.03 + \kappa_s^p \\ \kappa_s^\Xi &= 2\kappa^p + \kappa^n + \kappa_s^p = 1.67 + \kappa_s^p . \end{aligned}$$

$\hookrightarrow$ expect (polarization $\mathcal{P}$ w.r.t. $\vec{p}_P \times \vec{P}_Y$)

$$\mathcal{P}_\Lambda < 0 \qquad \mathcal{P}_\Sigma > 0 \qquad \mathcal{P}_\Xi < 0$$

- exp. result:

$$0 < \mathcal{P}_{\Sigma^0} \approx \mathcal{P}_{\Sigma^-} \approx \mathcal{P}_{\Sigma^+} \approx -\mathcal{P}_\Lambda \approx -\mathcal{P}_{\Xi^0} \approx -\mathcal{P}_{\Xi^-}$$

back

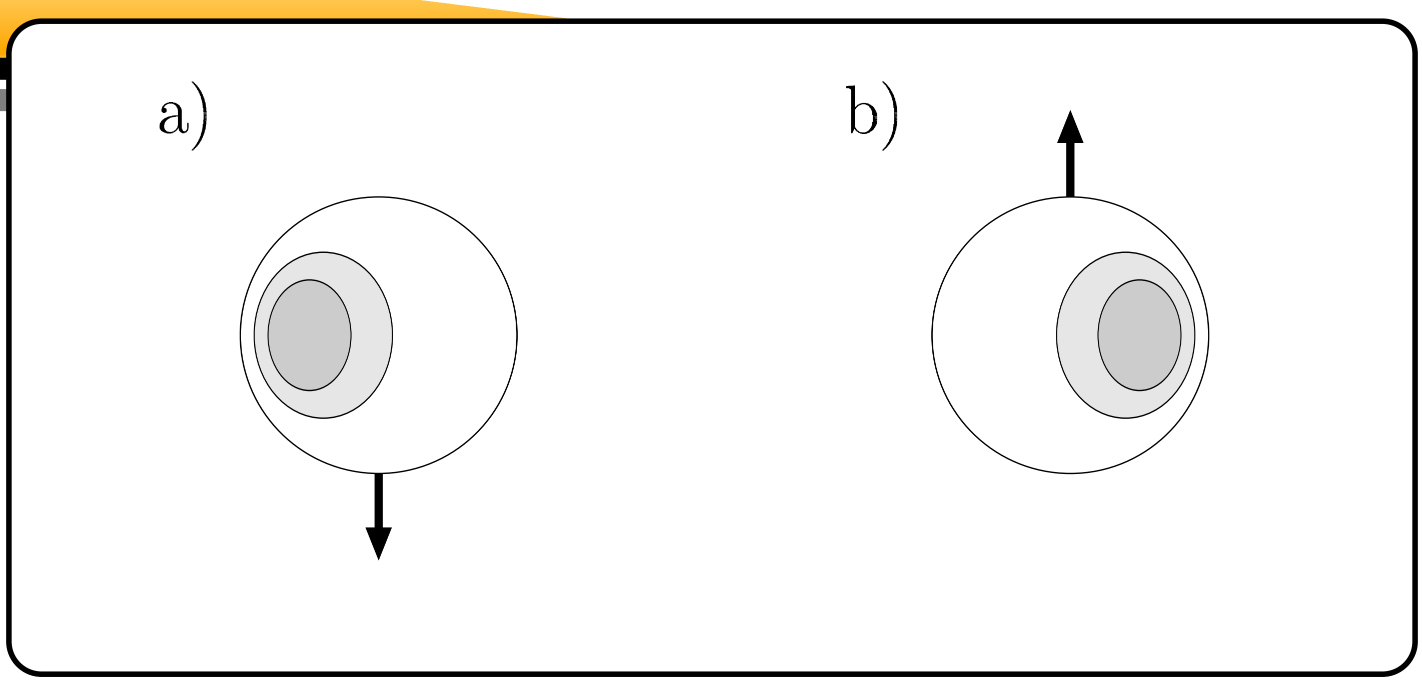

Figure 2: Schematic view of the transverse distortion of the s quark distribution (in grayscale) in the transverse plane for a transversely polarized hyperon with $\kappa_s^Y > 0$. The view is (from the rest frame) into the direction of motion (i.e. momentum into plane) for a hyperon that moves with a large momentum. In the case of spin down (a), the s-quarks get distorted towards the left, while the distortion is to the right for the case of spin up (b).

physical origin for $\perp$ distortion

- anomalous magnetic moment coupling in Dirac eq:

$$\frac{i\kappa}{2M}\bar{q}\sigma^{\mu\nu}qF_{\mu\nu} = \frac{i\kappa}{2M}\left[\bar{q}\sigma^{ij}qF_{ij} + 2\bar{q}\sigma^{0\nu}qF_{0\nu}\right]$$
$$\hookrightarrow \kappa\left[\vec{\sigma}\cdot\vec{B} + (\sigma\times\vec{p})\cdot\vec{E}\right]$$

  - moving spin $\frac{1}{2}$ particle with anomalous magnetic moment has (viewed from observer at rest) transverse electric dipole moment, which is perp. to both its spin and momentum.

  $\hookrightarrow$ $\perp$ distortion of $q(x, \mathbf{b}_\perp)$ is consequence of Lorentz invariance for Dirac particle with anomalous magnetic moment.

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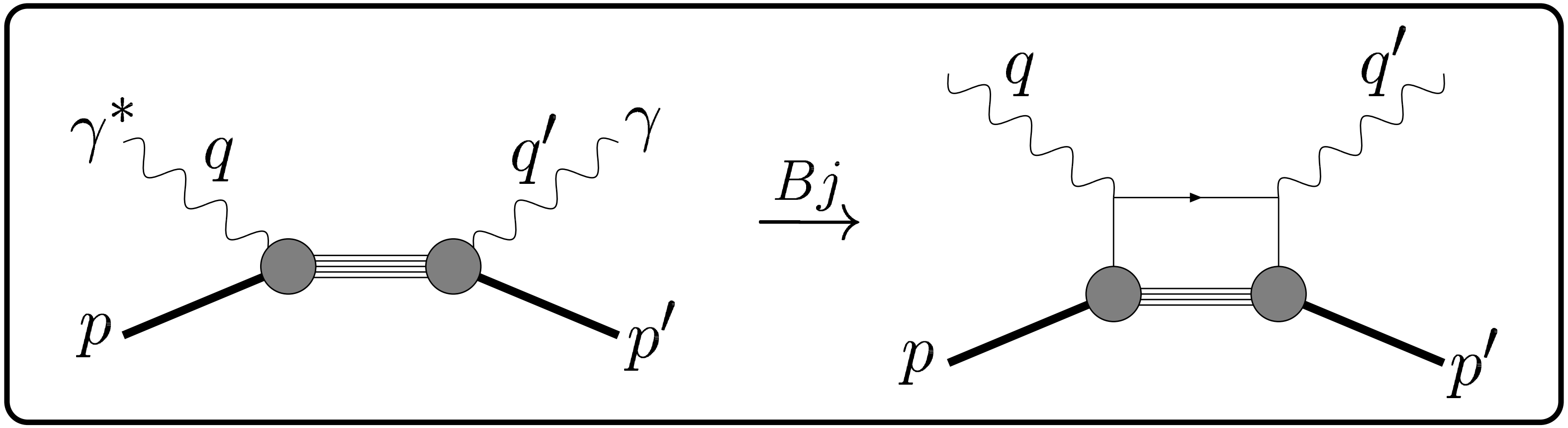
Deeply Virtual Compton Scattering (DVCS)



$$T^{\mu\nu} = i\int d^4z\, e^{i\bar{q}\cdot z} \left\langle p' \left| T J^\mu\left(-\frac{z}{2}\right) J^\nu\left(\frac{z}{2}\right) \right| p \right\rangle$$

$$\stackrel{Bj}{\hookrightarrow} \frac{g_\perp^{\mu\nu}}{2}\int_{-1}^{1} dx \left(\frac{1}{x-\xi+i\varepsilon} + \frac{1}{x+\xi-i\varepsilon}\right) H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+ u(p) \quad + \quad \dots$$

back

$\bar{q} = (q+q')/2 \qquad \Delta = p' - p \qquad x_{Bj} \equiv -q^2/2p\cdot q = 2\xi(1+\xi)$