ASTR 5500

Assignment 3.

1. From online databases it was possible to derive [Fe/H] values for 42 globular clusters with BV colour-magnitude diagrams in the Alcaino Atlas. The databases also provided updated reddenings E_{B-V} for the clusters. A summary of the slopes of the red giant branch for the clusters in combination with their other parameters is tabulated on the next page. A plot of the giant branch slope as a function of cluster metallicity is given in the accompanying graph. A correlation does appear in the data although it appears to be a quadratic and not a linear correlation. The RGB slopes appear to reach identical values near ~3.0 for low metallicity globulars [Fe/H] ~ −2.0. The slopes decrease in almost linear fashion for [Fe/H] > −1.5.

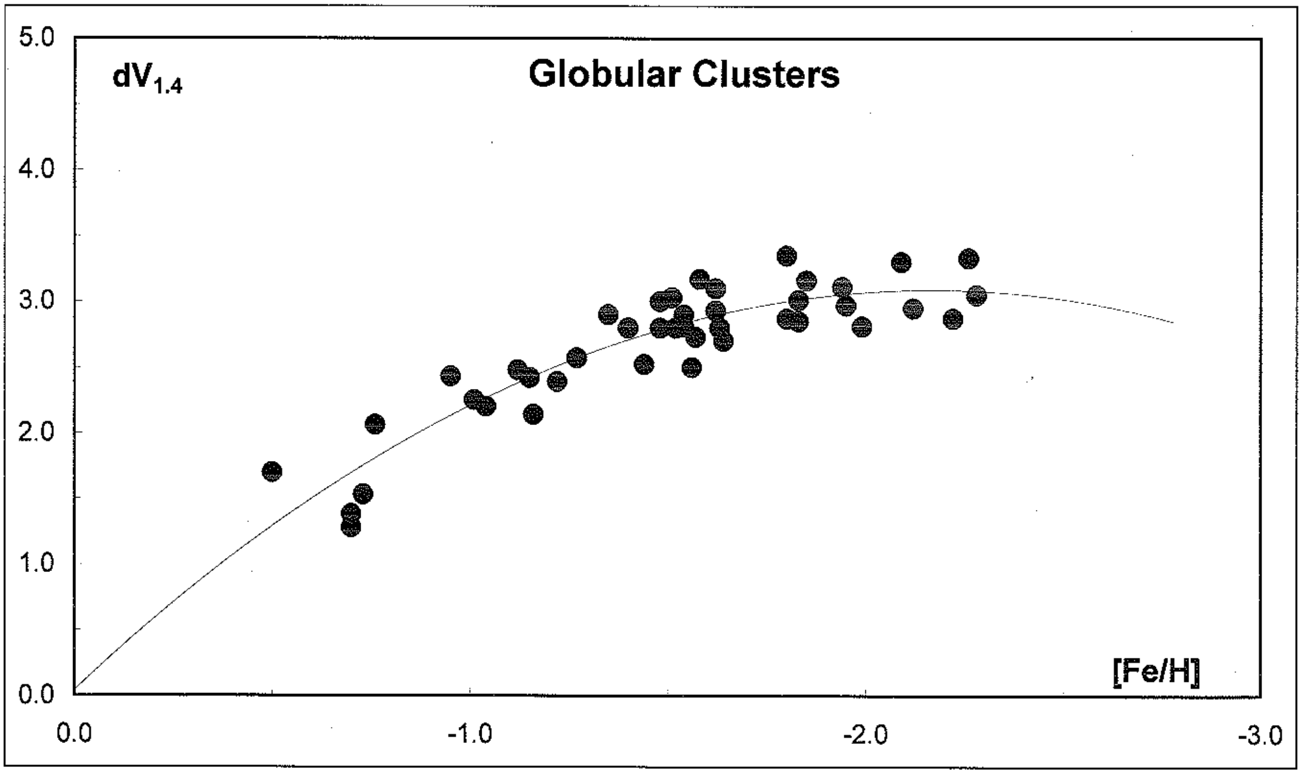
2. The transformed equations of solar motion can be written as:

$$\begin{bmatrix} \Pi_\odot \\ \Theta_\odot \\ Z_\odot \end{bmatrix} * \begin{bmatrix} \sum_i \cos^2 b \cos^2 l & \sum_i \cos^2 b \cos l \sin l & \sum_i \cos b \sin b \cos l \\ \sum_i \cos^2 b \cos l \sin l & \sum_i \cos^2 b \sin^2 l & \sum_i \sin b \cos b \sin l \\ \sum_i \cos b \sin b \cos l & \sum_i \cos b \sin b \sin l & \sum_i \sin^2 b \end{bmatrix}$$

$$= \begin{bmatrix} -\sum_i V_R \cos b \cos l \\ -\sum_i V_R \cos b \sin l \\ -\sum_i V_R \sin b \end{bmatrix}$$

and solved for $\Pi_\odot$, $\Theta_\odot$, $Z_\odot$ by matrix substitution.

Globular	Eb-v	log Fe/H	Vhb	V1.4	dM
NGC 104	0.04	-0.76	14.06	12.00	2.06
NGC 362	0.05	-1.16	15.44	13.30	2.14
NGC 1261	0.01	-1.35	16.70	13.80	2.90
NGC 1851	0.02	-1.22	16.09	13.70	2.39
NGC 2298	0.14	-1.85	16.11	12.95	3.16
NGC 2808	0.22	-1.15	16.22	13.80	2.42
NGC 3201	0.23	-1.58	14.77	11.60	3.17
Pal 4	0.01	-1.48	20.80	17.80	3.00
NGC 4147	0.02	-1.83	17.01	14.00	3.01
NGC 4372	0.39	-2.09	15.50	12.20	3.30
NGC 4833	0.32	-1.80	15.60	12.25	3.35
NGC 5024	0.02	-1.99	16.81	14.00	2.81
NGC 5139	0.12	-1.62	14.53	11.60	2.93
NGC 5272	0.01	-1.57	15.68	12.95	2.73
NGC 5466	0.00	-2.22	16.47	13.60	2.87
NGC 5897	0.09	-1.80	16.27	13.40	2.87
NGC 5904	0.03	-1.27	15.07	12.50	2.57
NGC 6171	0.33	-1.04	15.70	13.50	2.20
NGC 6205	0.02	-1.54	15.05	12.15	2.90
NGC 6218	0.19	-1.48	14.60	11.80	2.80
NGC 6254	0.28	-1.52	14.65	11.85	2.80
NGC 6341	0.02	-2.28	15.10	12.05	3.05
NGC 6352	0.21	-0.70	15.13	13.85	1.28
NGC 6356	0.28	-0.50	17.50	15.80	1.70
NGC 6362	0.09	-0.95	15.33	12.90	2.43
NGC 6397	0.18	-1.95	12.87	9.90	2.97
NGC 6522	0.48	-1.44	16.52	14.00	2.52
NGC 6541	0.14	-1.83	15.20	12.35	2.85
NGC 6637	0.16	-0.70	15.98	14.60	1.38
NGC 6656	0.34	-1.64	14.15	11.45	2.70
NGC 6712	0.45	-1.01	16.25	14.00	2.25
NGC 6723	0.05	-1.12	15.48	13.00	2.48
NGC 6752	0.04	-1.56	13.70	11.20	2.50
NGC 6779	0.20	-1.94	16.16	13.05	3.11
NGC 6838	0.25	-0.73	14.48	12.95	1.53
NGC 6934	0.10	-1.54	16.86	14.05	2.81
NGC 6981	0.05	-1.40	16.90	14.10	2.80
NGC 7006	0.05	-1.63	18.80	16.00	2.80
NGC 7078	0.10	-2.26	15.83	12.50	3.33
NGC 7089	0.06	-1.62	16.05	12.95	3.10
NGC 7099	0.03	-2.12	15.10	12.15	2.95
NGC 7492	0.00	-1.51	17.63	14.60	3.03



$$\begin{bmatrix} X_\odot \\ Y_\odot \\ Z_\odot \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{N}\cos^2\delta_i\cos^2\alpha_i & \sum_{i=1}^{N}\cos^2\delta_i\cos\alpha_i\sin\alpha_i & \sum_{i=1}^{N}\cos\delta_i\sin\delta_i\cos\alpha_i \\ \sum_{i=1}^{N}\cos^2\delta_i\cos\alpha_i\sin\alpha_i & \sum_{i=1}^{N}\cos^2\delta_i\sin^2\alpha_i & \sum_{i=1}^{N}\sin\delta_i\cos\delta_i\sin\alpha_i \\ \sum_{i=1}^{N}\cos\delta_i\sin\delta_i\cos\alpha_i & \sum_{i=1}^{N}\cos\delta_i\sin\delta_i\sin\alpha_i & \sum_{i=1}^{N}\sin^2\delta_i \end{bmatrix}$$

$$= \begin{bmatrix} -\sum_{i=1}^{N} V_{R_i}\cos\delta_i\cos\alpha_i \\ -\sum_{i=1}^{N} V_{R_i}\cos\delta_i\sin\alpha_i \\ -\sum_{i=1}^{N} V_{R_i}\sin\delta_i \end{bmatrix}$$

, which can be solved for $X_\odot$, $Y_\odot$,, and $Z_\odot$.

The equations involving the proper motions are somewhat more complicated, since they involve the unknown distances to the stars in the group. A solution in this case yields the values $X_\odot/K$, $Y_\odot/K$, and $Z_\odot/K$, which are also of considerable value (see Mihalas for details).

In the case of a radial velocity solution or a proper motion solution for the solar motion, the **apex of the solar motion** (direction of motion of the sun relative to the group) is given by:

$$\tan\alpha_{apex} = \frac{Y_\odot}{X_\odot} = \frac{Y_\odot/K}{X_\odot/K},$$

$$\tan\delta_{apex} = \frac{Z_\odot}{(X_\odot^2 + Y_\odot^2)^{1/2}} = \frac{Z_\odot/K}{[(X_\odot/K)^2 + (Y_\odot/K)^2]^{1/2}},$$

and the velocity of the solar motion is given by:

$$S_\odot = (X_\odot^2 + Y_\odot^2 + Z_\odot^2)^{1/2} = K \times [(X_\odot/K)^2 + (Y_\odot/K)^2 + (Z_\odot/K)^2]^{1/2}.$$

Note that the velocity of the sun relative to a group cannot be determined using only proper motion data unless the distances to the stars in the group are also known so that K is specified.

All results for the solar motion which make use of least squares solutions for these equations are **kinematic estimates** for the solar motion. The problem of deriving the **dynamical motion** of the sun relative to the LSR makes use of such kinematic results in conjunction with mathematical expectations for the rotation of the galactic disk.

From strictly qualitative arguments it is expected that the asymmetric drift for any group of stars must depend directly upon the nature of the orbits for the stars in the group. For **stars in nearly circular orbits no asymmetric drift is expected**, while for stars having a mix of very eccentric orbits the asymmetric drift should be fairly significant. A group of stars having the latter properties will also exhibit a fairly large dispersion in the component of their orbital motions directed along the line-of-sight to the galactic centre, the Π velocities, whereas stars in strictly circular orbits have no such component of their orbital motion. This **correlation of asymmetric drift with the dispersion in Π velocities,** $\sigma_\Pi{}^2$, for various kinematic groups is also predicted from quantitative arguments, and proves to be a valuable tool for determining the exact parameters for the sun's LSR velocity.

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      PROGRAM SOLARM
C     FINDS COMPONENTS OF SOLAR MOTION USING INPUTTED COORDINATES
C     AND VELOCITIES OF A GROUP OF STARS
C     ENTER DATA IN UNITS OF RA, DEC, & RADIAL VELOCITY
C     RA AND DEC IN DECIMAL UNITS OF DEGREES
C     TAKES INPUT FILE LABELLED solarm.dat
      REAL  RA,DEC,VR
      REAL  A1,A2,A3,B1,B2,B3,C1,C2,C3,D1,D2,D3,E1,E2,E3,Q
      REAL  DET,DETX,DETY,DETZ,X,Y,Z,DETDX,DETDY,DETDZ,DX,DY,DZ
      REAL  DETEX,DETEY,DETEZ,EX,EY,EZ,F1,F2,F3
      DATA  A1,A2,A3,B1,B2,B3 / 0.0,0.0,0.0,0.0,0.0,0.0 /
      DATA  C1,C2,C3,D1,D2,D3 / 0.0,0.0,0.0,0.0,0.0,0.0 /
      DATA  E1,E2,E3,I / 0.0,0.0,0.0,0 /
      DIMENSION  A(100),D(100),V(100),P(100)
      OPEN (UNIT=7, FILE='solarm.dat', STATUS='OLD')
10    READ (7,*,END=12) RA, DEC, VR
      I = I + 1
      A(I) = RA
      D(I) = DEC
      V(I) = VR
      A1 = A1 + COSD(A(I))*COSD(A(I))*COSD(D(I))*COSD(D(I))
      A2 = A2 + COSD(A(I))*SIND(A(I))*COSD(D(I))*COSD(D(I))
      A3 = A3 + COSD(A(I))*COSD(D(I))*SIND(D(I))
      B1 = A2
      B2 = B2 + SIND(A(I))*SIND(A(I))*COSD(D(I))*COSD(D(I))
      B3 = B3 + SIND(A(I))*COSD(D(I))*SIND(D(I))
      C1 = A3
      C2 = B3
      C3 = C3 + SIND(D(I))*SIND(D(I))
      D1 = D1 - V(I)*COSD(A(I))*COSD(D(I))
      D2 = D2 - V(I)*SIND(A(I))*COSD(D(I))
      D3 = D3 - V(I)*SIND(D(I))
      GOTO 10
12    DET = A1*(B2*C3 - B3*C2) - A2*(B1*C3 - B3*C1) + A3*(B1*C2 - B2*C1)
      DETX = D1*(B2*C3 - B3*C2) - A2*(D2*C3 - B3*D3) + A3*(D2*C2 - B2*D3)
      DETY = A1*(D2*C3 - B3*D3) - D1*(B1*C3 - B3*C1) + A3*(B1*D3 - D2*C1)
      DETZ = A1*(B2*D3 - D2*C2) - A2*(B1*D3 - D2*C1) + D1*(B1*C2 - B2*C1)
      X = DETX/DET
      Y = DETY/DET
      Z = DETZ/DET
      DO 13 J = 1,I
      Q = Z*SIND(D(J))
      P(J) = 0.0 - X*COSD(A(J))*COSD(D(J)) - Y*SIND(A(J))*COSD(D(J)) - Q
      E1 = E1 - ABS(V(J) - P(J))*COSD(A(J))*COSD(D(J))
      E2 = E2 - ABS(V(J) - P(J))*SIND(A(J))*COSD(D(J))
      E3 = E3 - ABS(V(J) - P(J))*SIND(D(J))
13    CONTINUE
      F1 = E1/REAL(I - 1)
      F2 = E2/REAL(I - 1)
      F3 = E3/REAL(I - 1)
14    DETDX = E1*(B2*C3 - B3*C2) - A2*(E2*C3 - B3*E3) + A3*(E2*C2 - B2*E3)
      DETDY = A1*(E2*C3 - B3*E3) - E1*(B1*C3 - B3*C1) + A3*(B1*E3 - E2*C1)
      DETDZ = A1*(B2*E3 - E2*C2) - A2*(B1*E3 - E2*C1) + E1*(B1*C2 - B2*C1)
      DETEX = F1*(B2*C3 - B3*C2) - A2*(F2*C3 - B3*F3) + A3*(F2*C2 - B2*F3)
      DETEY = A1*(F2*C3 - B3*F3) - F2*(B1*C3 - B3*C1) + A3*(B1*F3 - F2*C1)
      DETEZ = A1*(B2*F3 - F2*C2) - A2*(B1*F3 - F2*C1) + F1*(B1*C2 - B2*C1)
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      DX = DETDX/DET
      DY = DETDY/DET
      DZ = DETDZ/DET
      EX = DETEX/DET
      EY = DETEY/DET
      EZ = DETEZ/DET
      WRITE (6,15) X, DX, EX
15    FORMAT (1X,'X = ',F13.5,' DX = +-',F13.5,' SE = +-',F13.5)
      WRITE (6,16) Y, DY, EY
16    FORMAT (1X,'Y = ',F13.5,' DY = +-',F13.5,' SE = +-',F13.5)
      WRITE (6,17) Z, DZ, EZ
17    FORMAT (1X,'Z = ',F13.5,' DZ = +-',F13.5,' SE = +-',F13.5)
      STOP
      END
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X =	-0.88410	DX = +-	21.36244	SE = +-	1.12434
Y =	116.87366	DY = +-	-36.56865	SE = +-	-1.02901
Z =	-7.93466	DZ = +-	-21.52102	SE = +-	-1.13269

The solution for the select sample of 20 Mira variables is:

$$\Pi_{\odot} = -0.88 \pm 1.12 \text{ s.e.} = -u_{\odot}$$
$$\Theta_{\odot} = +116.87 \pm 1.03 \text{ s.e.} = V_{\odot}$$
$$Z_{\odot} = -7.93 \pm 1.13 \text{ s.e.} = W_{\odot}$$

The drift in θ velocities is very large, but not quite as large (i.e. ~ 200 - 250 km/s) as Halo Population II. The Π and Z velocities imply a mix of disk stars, so the best population type for the stars is Intermediate Disk-Population II.

3. For this situation, $\omega(R) = \frac{\theta}{R} = K$, and $\frac{d\theta}{dR} = \frac{d}{dR}(KR) = K$.

$\therefore$ Oort's $A = \frac{1}{2}\left[\frac{\theta_0}{R_0} - \left(\frac{d\theta}{dR}\right)_{R_0}\right] = \frac{1}{2}(K - K) = 0$ km/s/kpc.

And Oort's $B = -\frac{1}{2}\left[\frac{\theta_0}{R_0} + \left(\frac{d\theta}{dR}\right)_{R_0}\right] = -\frac{1}{2}(K+K) = \frac{-2K}{2} = -K$.

For $\theta_0 = 251$ km/s and $R_0 = 8.5$ kpc, $K = \frac{\theta_0}{R_0} = \frac{251}{8.5}$ km/s/kpc

$= 29.53$ km/s/kpc

So $A = 0$ km/s/kpc and $B = -29.53$ km/s/kpc.

The radial velocities of other bar members can be established from the general relations for Galactic rotation, namely from:

$$v_R = R_0\left[\frac{\theta}{R} - \frac{\theta_0}{R_0}\right]\sin l = R_0(K-K)\sin l = 0 \text{ km/s}.$$

$\therefore$ Other bar members should not display and radial velocity variation as a function of Galactic longitude, l, which makes sense for a fixed Galactic feature such as a bar.

4. Access was made of the online Dias, Alessi, Moitinho & Lapine catalogue DAML02, and all data for open clusters were downloaded in Galactic coordinates. Clusters without radial velocities were omitted and calculations were made for v_R(LSR) using the solar motion from Turner (2014, CJP, 92, 959). The parameters d (kpc) and $v_R / d(\text{kpc}) \cos^2 b$ were calculated, the latter plotted as a function of $\sin 2l$ (see graph). There was considerable scatter, but that was reduced by eliminating all values of $|v_R / d \cos^2 b| > 50$ km/s, then all values of $v_R / d \cos^2 b < -30$ km/s for positive $\sin 2l$ and $v_R / d \cos^2 b > 30$ km/s for negative $\sin 2l$. That left the data in the plotted figure.

The best fitting value of A from this analysis was:

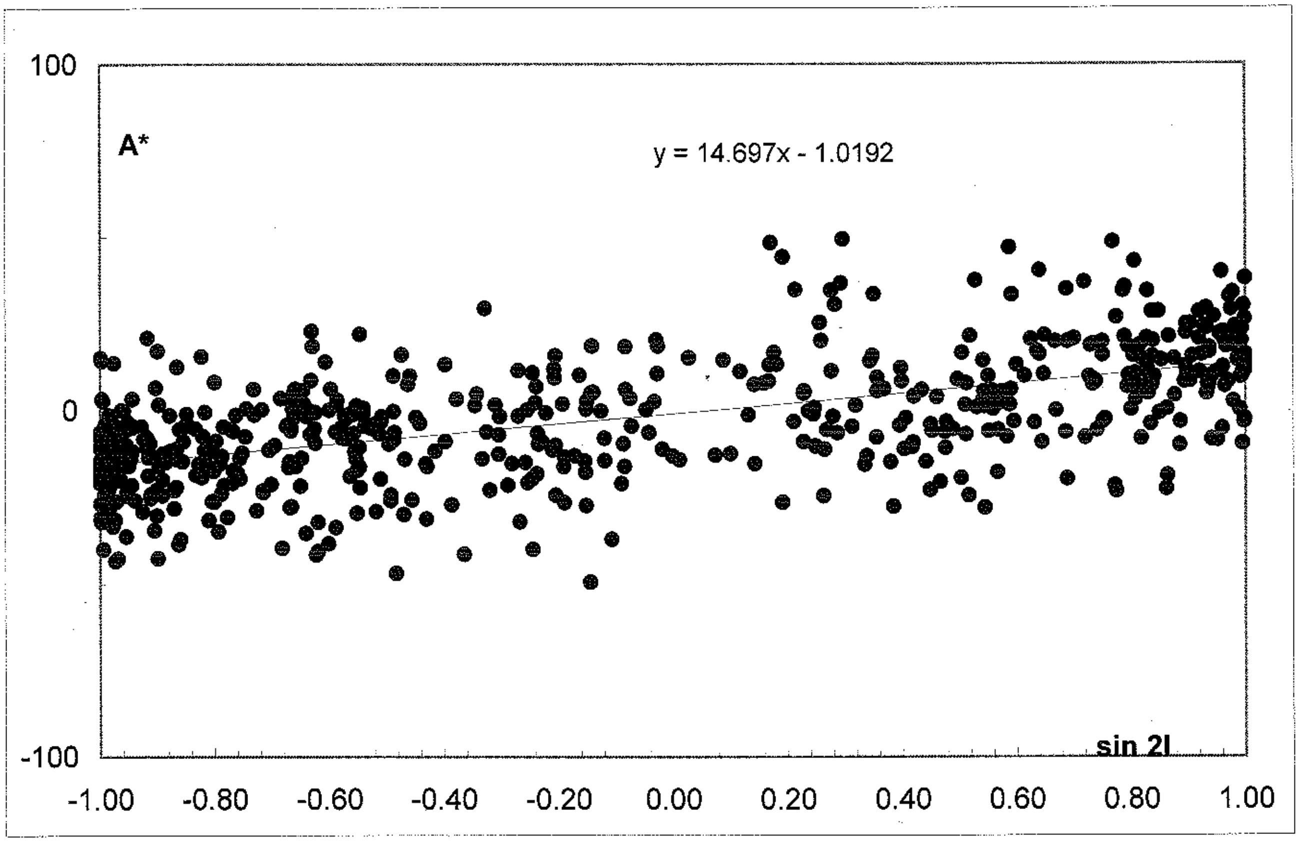
$$A = 14.6 \pm 0.6 \text{ km/s/kpc}$$

Clusters with turnoff pts younger than B5, corresponding to $(B-V)_0 = -0.20$, have ages of $\log t \lesssim 7.5$. When the data are restricted in such fashion, one finds:

$$A = 15.5 \text{ km/s/kpc for clusters with } \log t \leq 7.5$$

$$A = 14.1 \text{ km/s/kpc for clusters with } \log t > 7.5$$

The scatter is about the same in both cases. Recall that $A - B = \Theta_0 / R_0 = 251 / 8.5 = 29.53$ km/s/kpc. For a galaxy with a flat rotation curve $A = -B$, which is ½ (29.53 km/s/kpc) i.e. $A = 14.76$ km/s/kpc. The derived value matches expectations.



	value	uncertainty	wgt	valuexwgt
ALS	-1.0199	0.60696	2.714438	-2.76845
BLS	14.69427	0.85079	1.381514	20.30034
ANP	-1.41667	0.73243	1.864094	-2.64081
BNP	14.55249	0.91786	1.18699	17.27366
n	577			
		Awgt	4.578531	-5.40926
		Bwgt	2.568504	37.574
		pm		
A=	-1.18144	0.467344		
B=	14.62875	0.623964		