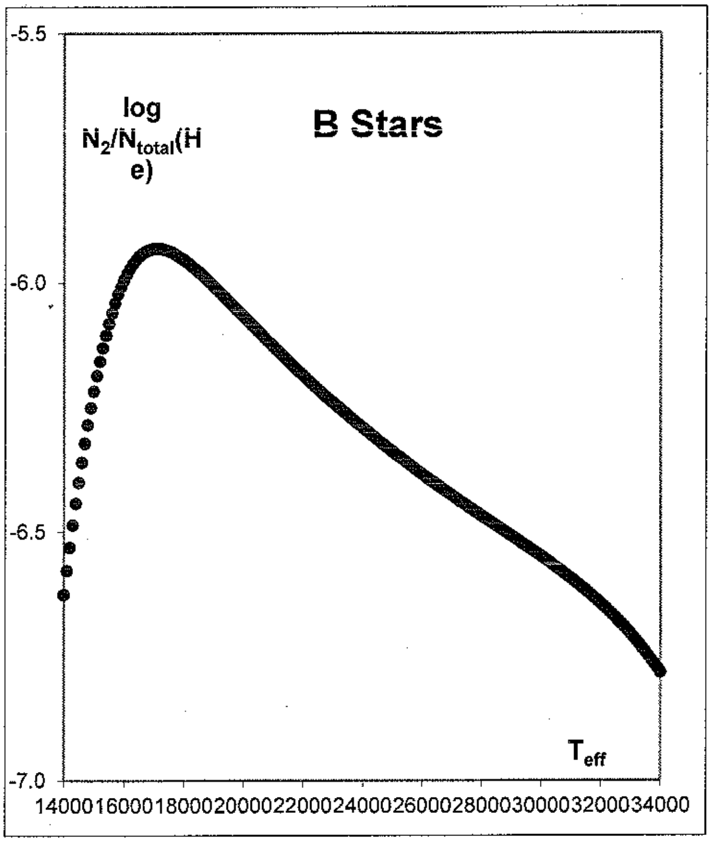
ASTRONOMY 3500

Assignment 2.

1. The Boltzmann equation is: $\log(N_n/N) = -\theta\chi_n + \log[g_n/u(T)]$
and the Saha equation is:
$$\log(N^{i+1}/N^{i}) = 2.5\log T - \theta I_i - \log P_e - 0.4772 + \log[2\,u_{i+1}(T)/u_i(T)],$$
where $\theta = 5040.3/T$ and $\log P_e = -3.5 + 1.5\log T$

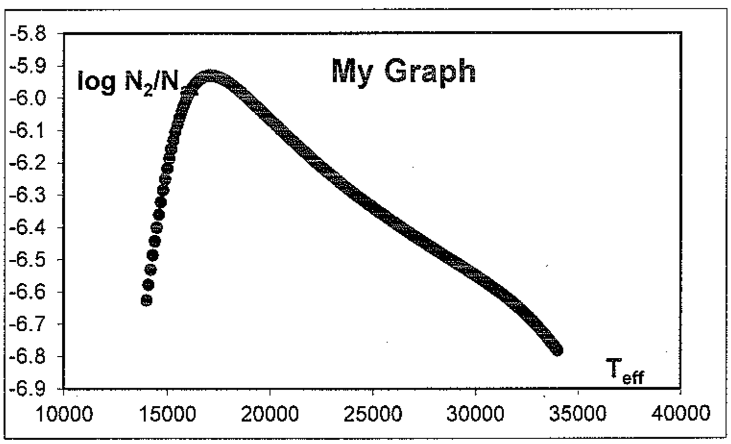
Here we need the ratios:
$$\log[N_2/N(\text{He I})] = -\frac{5040.3}{T}\times 20.96 + \log(9/1)$$
$$= -\frac{105{,}644.69}{T} + 0.9542425$$
$$\log[N(\text{He II})/N(\text{He I})] = -\frac{5040.3}{T}\times 24.58 + 2.5\log T + 3.5 - 1.5\log T - 0.4772 + \log[(2\times 2)/1]$$
(0.6020599)
$$= -\frac{123{,}890.57}{T} + \log T + 3.62486$$
$$\log[N(\text{He III})/N(\text{He II})] = -\frac{5040.3}{T}\times 54.40 + \log T + 3.5 - 0.4772 + \log[(2\times 1)/2]$$
(0.000000)
$$= -\frac{274{,}192.32}{T} + \log T + 3.0228$$
And to compute the ratio's:
$$\frac{N_2}{N_{total}} = \frac{N_2/N(\text{He I})}{N(\text{He I})/N(\text{He I}) + N(\text{He II})/N(\text{He I}) + N(\text{He III})/N(\text{He I})}$$

or
$$\frac{N_2}{N_{total}} = \frac{N_2/N(\text{He I})}{1 + N(\text{He II})/N(\text{He I}) + N(\text{He III})/N(\text{He II})\cdot N(\text{He II})/N(\text{He I})}.$$
Computations are presented on following page.



	log He II/He	g He III/He	g Hen2/He	Hen2/Hetota	Hen2/Hetotal
14000	-1.07771	-12.41497	-6.59136	2.36E-07	-6.62623
14100	-1.01186	-12.27299	-6.53784	2.64E-07	-6.57817
14200	-0.94692	-12.13298	-6.48508	2.94E-07	-6.53158
14300	-0.88286	-11.99491	-6.43306	3.26E-07	-6.48651
14400	-0.81968	-11.85874	-6.38176	3.61E-07	-6.44301
14500	-0.75734	-11.72442	-6.33116	3.97E-07	-6.40115
14600	-0.69584	-11.59193	-6.28126	4.36E-07	-6.36097
14700	-0.63515	-11.46121	-6.23204	4.76E-07	-6.32253
14800	-0.57526	-11.33224	-6.18349	5.18E-07	-6.28589
14900	-0.51616	-11.20499	-6.13558	5.61E-07	-6.25109
15000	-0.45783	-11.07941	-6.08832	6.05E-07	-6.21816
15100	-0.40025	-10.95547	-6.04168	6.5E-07	-6.18715
15200	-0.34341	-10.83315	-5.99565	6.95E-07	-6.15807
15300	-0.28729	-10.71241	-5.95023	7.4E-07	-6.13095
15400	-0.23188	-10.59322	-5.90539	7.84E-07	-6.10578
15500	-0.17717	-10.47554	-5.86114	8.27E-07	-6.08255
15600	-0.12315	-10.35936	-5.81745	8.68E-07	-6.06126
15700	-0.06979	-10.24464	-5.77432	9.08E-07	-6.04185
15800	-0.01709	-10.13135	-5.73173	9.46E-07	-6.02430
15900	0.03496	-10.01948	-5.68968	9.81E-07	-6.00854
16000	0.08638	-9.90898	-5.64816	1.01E-06	-5.99452
16100	0.13718	-9.79984	-5.60715	1.04E-06	-5.98216
16200	0.18736	-9.69203	-5.56665	1.07E-06	-5.97139
16300	0.23695	-9.58553	-5.52664	1.09E-06	-5.96211
16400	0.28595	-9.48030	-5.48712	1.11E-06	-5.95425
16500	0.33437	-9.37634	-5.44808	1.13E-06	-5.94772
16600	0.38222	-9.27362	-5.40952	1.14E-06	-5.94242
16700	0.42952	-9.17211	-5.37141	1.15E-06	-5.93827
16800	0.47627	-9.07179	-5.33376	1.16E-06	-5.93518
16900	0.52248	-8.97265	-5.29655	1.17E-06	-5.93307
17000	0.56816	-8.87465	-5.25978	1.17E-06	-5.93185
17100	0.61332	-8.77779	-5.22344	1.17E-06	-5.93145
17200	0.65798	-8.68204	-5.18752	1.17E-06	-5.93179
17300	0.70213	-8.58738	-5.15202	1.17E-06	-5.93281
17400	0.74579	-8.49379	-5.11693	1.16E-06	-5.93444
17500	0.78896	-8.40126	-5.08224	1.16E-06	-5.93661
17600	0.83165	-8.30977	-5.04794	1.15E-06	-5.93929
17700	0.87388	-8.21930	-5.01403	1.14E-06	-5.94241
17800	0.91565	-8.12983	-4.98050	1.13E-06	-5.94592
17900	0.95696	-8.04134	-4.94734	1.12E-06	-5.94979
18000	0.99783	-7.95383	-4.91456	1.11E-06	-5.95398
18100	1.03826	-7.86727	-4.88213	1.1E-06	-5.95845
18200	1.07826	-7.78164	-4.85007	1.09E-06	-5.96316
18300	1.11784	-7.69694	-4.81835	1.08E-06	-5.96809
18400	1.15700	-7.61315	-4.78697	1.06E-06	-5.97322
18500	1.19574	-7.53025	-4.75594	1.05E-06	-5.97851
18600	1.23409	-7.44823	-4.72524	1.04E-06	-5.98395
18700	1.27203	-7.36708	-4.69487	1.02E-06	-5.98952
18800	1.30959	-7.28677	-4.66482	1.01E-06	-5.99519
18900	1.34676	-7.20730	-4.63509	9.98E-07	-6.00096
19000	1.38355	-7.12866	-4.60567	9.84E-07	-6.00681
19100	1.41996	-7.05083	-4.57656	9.71E-07	-6.01273
19200	1.45601	-6.97380	-4.54776	9.58E-07	-6.01871
19300	1.49170	-6.89755	-4.51925	9.45E-07	-6.02473

Maximum



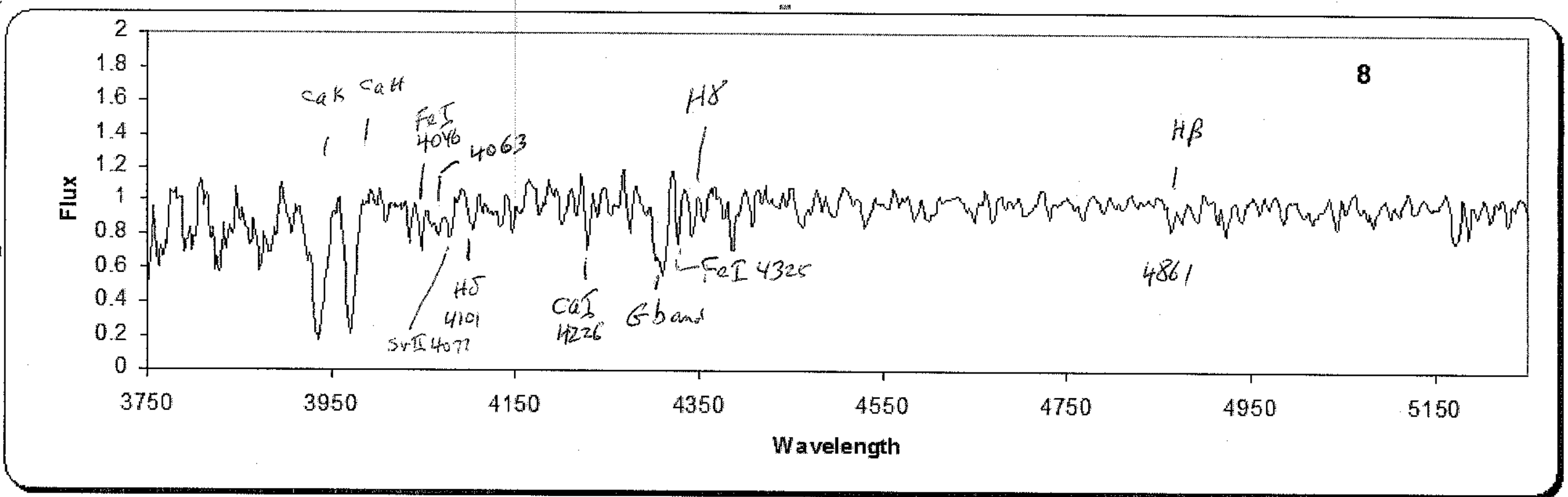
The temperature at which the lines of the neutral He triplets reach maximum strength should be ~17,100 K, corresponding to class B2 V. By comparison, the Böhm-Vitense scale lists a value of T = 21,500 K, while the Buser & Kurucz scale lists a value of T = 22,500 K for stars of $(B-V)_0 = -0.24$ (B2 V). The Turner scale (TURNER ZAMS) gives $T_{eff} = 24,257$ K for stars of spectral type B2 V, $T_{eff} = 19,253$ K for B3 V stars.

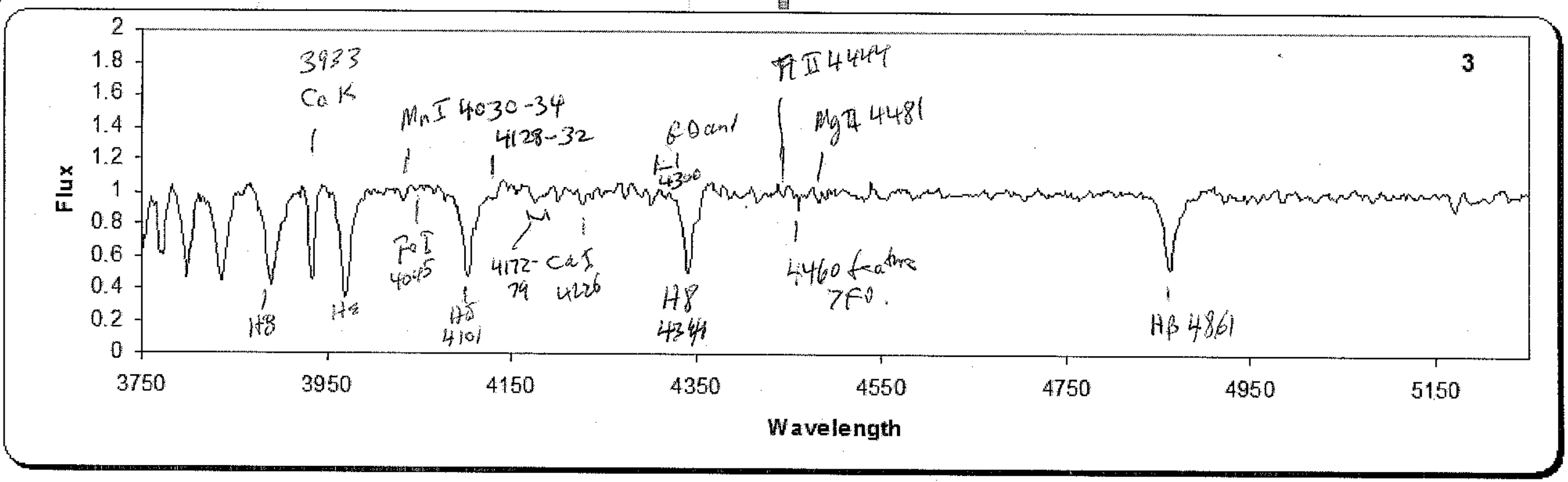
The value of T = 17,100 K derived here is smaller than all three T_{eff} estimates (Böhm-Vitense, Buser & Kurucz, Turner), which is consistent with lines formed high in the stellar atmosphere, well above the deeper continuum-forming regions responsible for T_{eff}. None of the scales can be considered high or low, but all scales are consistent in implying higher T_{eff}-values for B2 V stars than the value of T for which the He I triplet lines appear strongest.

Classification of Spectra illustrated.

Star 1. Fe I(4325) as strong or stronger than Hγ ⇒ G6–G8
Ca I (4226) ≊ Fe I (4325) ⇒ G6
Sr II (4077) / Fe I (4045) < 1 ⇒ luminosity class IV or V
λ(4063)/Sr II (4077) ≃ 1 ⇒ luminosity class III or IV
Best fit: G6 IV

Star 2. Ca K / Hε ratio ⇒ ~F0–F2
4172-79 / Ca I 4226 ~ F0
Mn I 4030-34 / Fe I 4045 ~ F0–F2
G band shape and 4312 Ti II suggest F0 III, no 4325 (Fe I) ⇒ F0 not F2
Ti II 3815, 4444, etc. suggest F0 ~ luminosity class V.
Best fit: F0 V

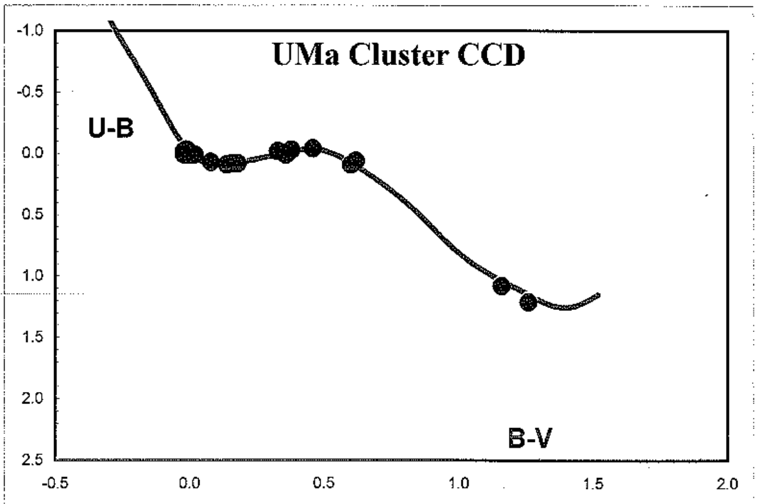

8
Flux
Wavelength
CaK CaH
FeI 4046
4063
SrII 4077
Hδ 4101
CaI 4226
G band
FeI 4325
Hγ
Hβ
4861

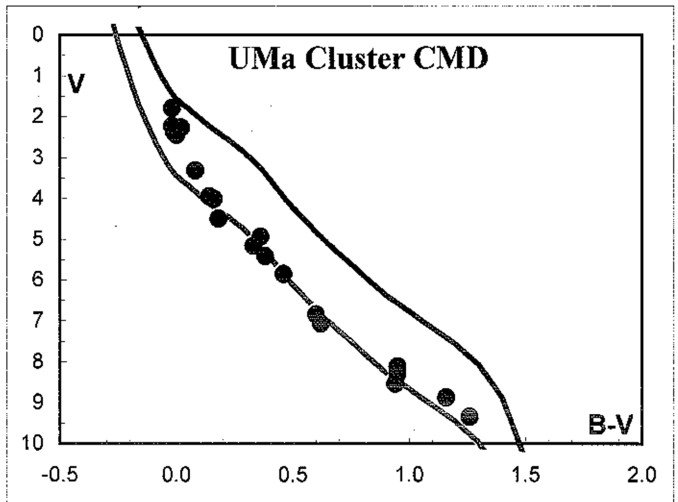

3
Flux
Wavelength
3933 Ca K
MnI 4030-34
4128-32
G band 4300
TiII 4444
MgII 4481
Hε
FeI 4045
Hδ 4101
4172-79
CaI 4226
Hγ 4344
4460 feature 7F0.
Hβ 4861

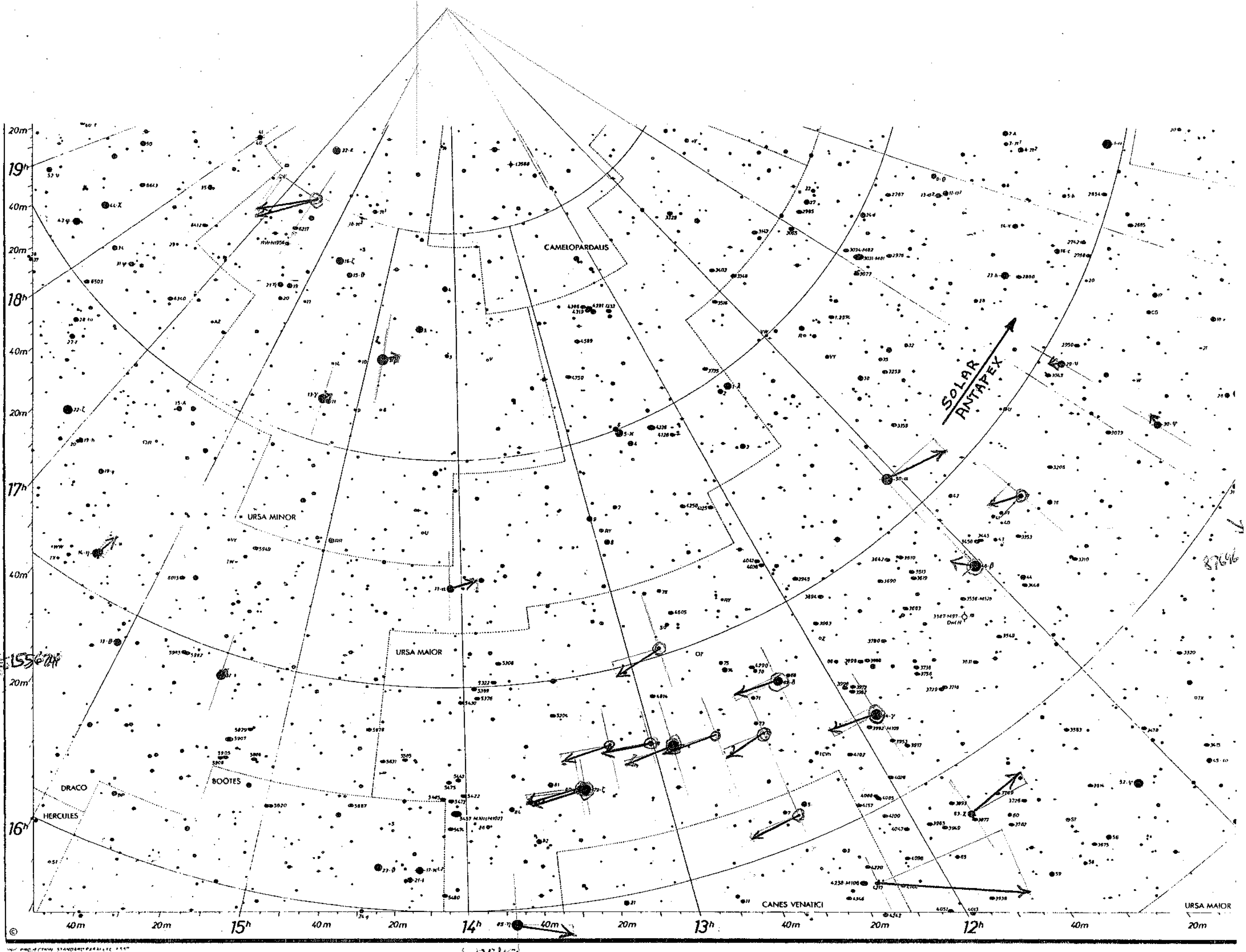
Star/HD	RA(1991.25)	DEC(1991.25)	muRA	muDEC	pi	unc	wgt	pixwgt	V	B-V	U-B	V-Mv
87696	10 07 25.762	+35 14 40.90	50.97	0.64	35.78	0.84	1.4172	50.7086	4.490	0.180	0.08	2.060
91480	10 35 09.693	+57 04 57.49	65.65	37.11	37.80	0.61	2.6874	101.5856	5.160	0.330	-0.02	2.080
95418	11 01 50.477	+56 22 56.74	81.66	33.74	41.07	0.60	2.7778	114.0833	2.380	-0.010	-0.03	0.880
103287	11 53 49.847	+53 41 41.14	107.76	11.16	38.99	0.68	2.1626	84.3209	2.440	0.000	0.01	0.880
106591	12 15 25.560	+57 01 57.42	103.56	7.81	40.05	0.60	2.7778	111.2500	3.310	0.080	0.07	1.370
109011	12 31 18.915	+55 07 07.72	99.70	-13.25	42.13	3.11	0.1034	4.3558	8.110	0.950		1.540
109647	12 35 51.288	+51 13 17.28	113.00	-8.06	38.08	1.11	0.8116	30.9066	8.530	0.940		2.000
110463	12 41 44.521	+55 43 28.83	121.53	-4.36	43.06	0.82	1.4872	64.0393	8.290	0.950		1.720
111456	12 48 39.464	+60 19 11.36	107.79	-30.60	41.39	3.20	0.0977	4.0420	5.850	0.460	-0.04	1.890
112185	12 54 01.749	+55 57 35.36	111.74	-8.99	40.30	0.62	2.6015	104.8387	1.790	-0.020	0.01	0.370
113139	13 00 43.700	+56 21 58.82	108.36	2.67	40.06	0.60	2.7778	111.2778	4.940	0.360	0.01	1.680
115043	13 13 37.009	+56 42 29.77	111.91	-17.88	38.92	0.67	2.2277	86.7008	6.840	0.600	0.09	2.020
116656	13 23 55.543	+54 55 31.30	121.23	-22.01	41.73	0.61	2.6874	112.1473	2.270	0.020	0.01	0.610
116657	13 23 55.543	+54 55 31.30	121.23	-22.01	41.73	0.61	2.6874	112.1473	3.95	0.14	0.09	1.720
116842	13 25 13.538	+54 59 16.65	120.35	-16.94	40.19	0.57	3.0779	123.6996	4.010	0.160	0.08	1.700
125451	14 19 16.280	+13 00 15.48	105.91	-32.22	38.33	0.81	1.5242	58.4210	5.410	0.380	-0.03	2.020
139006	15 34 41.268	+26 42 52.89	120.38	-89.44	43.65	0.79	1.6023	69.9407	2.230	-0.020	-0.02	0.810
150706	16 31 17.586	+79 47 23.19	95.83	-87.97	36.73	0.56	3.1888	117.1237	7.060	0.620	0.06	2.130
155674A	16 31 17.586	+79 47 23.19	86.06	-104.87	47.14	1.88	0.2829	13.3375	8.870	1.160	1.08	1.460
155674B	17 10 12.358	+54 29 24.50	87.93	-105.06	47.86	3.11	0.1034	4.9483	9.340	1.260	1.21	1.470
						Totals	37.084	1479.875			Avg	1.934
											s.d.	0.1643
											count	10
											s.e.	0.052

Eb-v=	0.000
V-Mv=	1.934
Vo-Mv	1.934
d=	24.367
unc	0.583

Weighted pi	39.9060
Uncertainty	0.1642
m-M	1.9948
Uncertainty	0.0089
d	25.0589
uncertainty	0.1031






SOLAR ANTAPEX
CAMELOPARDALIS
URSA MINOR
URSA MAIOR
CANES VENATICI
BOOTES
DRACO
HERCULES
19h
18h
17h
16h
15h
14h
13h
12h

2. Ursa Major Moving Cluster.

a. Solution for Hipparcos parallaxes: $\langle \pi \rangle = 39.91 \pm 0.16$ mas.

$m - M = 1.99 \pm 0.01$

$d = 25.06 \pm 0.10$ pc.

b. Best solution for likely single, unevolved cluster stars from $V - M_V$ and ZAMS fit is: $m - M = 1.93 \pm 0.05$

$d = 24.37 \pm 0.58$ pc.

c. See plot of proper motions on sky. UMa cluster stars are moving away from the solar antapex, unlike most stars in the field, which tend to move towards it, so cluster stars are straightforward to identify by their proper motions

d. $d = 24.16 \pm 1.85$ pc from proper motion gradients, not a very good estimate.

e.

Method	d
π	25.06 ± 0.10 pc
ZAMS fit	24.37 ± 0.58 pc
μ gradients	24.16 ± 1.85

All values just agree to within their uncertainties. Moving cluster method requires considerable motion in the line of sight for best results, but UMa stars display only a small component of their space velocities towards us. They are therefore not ideal for its application, and offers no advantages over the Hyades cluster as a ZAMS calibrator. It also has fewer stars to delineate the ZAMS completely.

f. If the Hipparcos parallaxes are to be believed, then there may be some evidence to indicate an increase to the Hyades cluster distance modulus from ~3.20, But the evidence is not strong. A stronger moving cluster distance estimate would help to decide the case.