ASTR 3500.1

(10 marks for each question.)

Assignment 1.

1. The apparent distance modulus of an object is given by:

$$V - M_V = 5\log d - 5 + A_V$$

We are given that: $d = 9.0\,kpc = 9000\,pc$
$V = +13.0$
$M_V = -4.15$

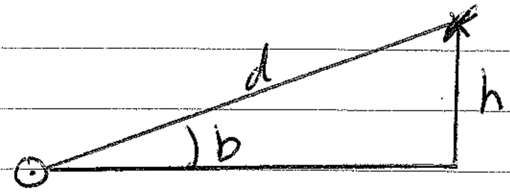
$$\therefore A_V = (V - M_V) + 5 - 5\log d$$
$$= [13.0 - (-4.15)] + 5 - 5\log_{10}(9000)$$
$$= 22.15 - 5(3.9542425)$$
$$= 22.15 - 19.77$$
$$= 2.38 \text{ magnitudes}$$

a. The amount of visual interstellar extinction affecting UKS 2 is 2.38 magnitudes (A_V).

b. The amount of extinction / kpc is $2^m.38 / 9\,pc$
$= 0^m.26 / kpc$.

That amount is somewhat small, given that values closer to $1^m/kpc$ are more common along lines of sight in the plane of the Milky Way. UKS 2, however, is located in a line of sight lying slightly below the plane $\sin\theta = -0.5\,kpc / 9.0\,kpc = -0.05555$
i.e. $\theta = -3^\circ.18$, which means it suffers less dust extinction, and probably all local, than other distant objects exactly in the Galactic plane.

2. The height of an object above the Galactic plane is established geometrically from its Galactic latitude b and distance d (see diagram) as: $h = d \sin b$.



For M13, $b = 40.9^\circ$, $d = 7000$ pc (7 kpc)

$$\therefore h(M13) = 7000 \text{ pc} \sin(40.9^\circ)$$
$$= 7010 \text{ pc} \times 0.6547408$$
$$= 4583 \text{ pc}$$

For the Orion Nebula, $b = -19.4^\circ$, $d = 450$ pc.

$$\therefore h(ON) = 450 \text{ pc} \sin(-19.4^\circ)$$
$$= 450 \text{ pc} \times -0.3321611$$
$$= -149 \text{ pc}$$

The results indicate that the Orion Nebula qualifies as belonging to the Thin Disk, maximum heights above the plane of ~350 pc, whereas M13 lies well outside the range of the Thick Disk, maximum heights above the plane of ~1000 pc. M13 is therefore a member of the Galactic halo, and the Orion Nebula the thin disk.

3. The standard formula for escape speed in a Keplerian system is given by: $v_{esc} = \sqrt{\frac{2GM_G}{R}}$, where M_G is actually $M_G + M_*$.

Take $G = 6.6726 \times 10^{-11}\ Nm^2/kg^2$

$R = 8.5$ kpc (locally) $= 8.5 \times 10^3 \times 206265 \times 1.4960 \times 10^{11}$ m

$= 2.6229 \times 10^{20}$ m

(a) $v_{esc} = 310$ km/s $= 3.10 \times 10^5$ m/s

$\therefore M_G = \frac{R v_{esc}^2}{2G} = \frac{(2.6229 \times 10^{20}\ m)(3.10 \times 10^5\ m/s)^2}{2 \times 6.6726 \times 10^{-11}\ Nm^2/kg^2}$

$= 1.8888 \times 10^{41}$ kg $/ 1.989 \times 10^{30}$ kg$/M_\odot$

$= 9.5 \times 10^{11}\ M_\odot$ (9.4961×10^{11})

This value is reasonably close to the value of $1.25 \times 10^{11}\ M_\odot$ obtained from assuming Keplerian motion for the LSR about the Galactic centre.

(b) For $v_{esc} = 500$ km/s $= 5.0 \times 10^5$ m/s, we have:

$M_G = \frac{R v_{esc}^2}{2G} = \frac{(2.6229 \times 10^{20}\ m)(5.0 \times 10^5\ m/s)^2}{2 \times 6.6726 \times 10^{-11}\ Nm^2/kg^2}$

$= 4.9136 \times 10^{41}$ kg $/ 1.989 \times 10^{30}$ kg$/M_\odot$

$= 2.5 \times 10^{11}\ M_\odot$ (2.4704×10^{11})

This value is reasonably close to the value of $\sim 2 \times 10^{11}\ M_\odot$ obtained from assuming Keplerian motion for disk stars at 16 kpc from the Galactic centre.

Since Keplerian motion does not apply locally, it seems likely that result (b) may be closer to the true mass of the Galaxy.

4. For Keplerian orbits, $(m_1 + m_2) = \frac{4\pi^2}{G} \frac{a^3}{P^2}$, where $P = \frac{2\pi a}{v}$

$\therefore (m_1 + m_2) = \frac{4\pi^2}{G} \frac{R^3 v^2}{4\pi^2 R^2} = \frac{R v^2}{G}$, for $a = R$

Since $m_1 = m_G >>> m_2 = m_*$, $\therefore v^2 \simeq \frac{G m_G}{R}$

or: $v = \frac{G^{1/2} m_G^{1/2}}{R^{1/2}}$, where R = distance to the Galactic centre, m_G = mass of the Galaxy

$\therefore \frac{dv}{dR} = \frac{d}{dR}\left(\frac{G^{1/2} m_G^{1/2}}{R^{1/2}}\right) = -\frac{1}{2} \frac{G^{1/2} m_G^{1/2}}{R^{3/2}}$

Oort's $A = -\frac{1}{2}\left[\left(\frac{dv}{dR}\right)_{R_0} - \frac{v_{R_0}}{R_0}\right] = -\frac{1}{2}\left[-\frac{1}{2} - 1\right] \frac{G^{1/2} m_G^{1/2}}{R_0^{3/2}} = \frac{3}{4} \frac{G^{1/2} m_G^{1/2}}{R_0^{3/2}}$

Oort's $B = -\frac{1}{2}\left[\left(\frac{dv}{dR}\right)_{R_0} + \frac{v_{R_0}}{R_0}\right] = -\frac{1}{2}\left[-\frac{1}{2} + 1\right] \frac{G^{1/2} m_G^{1/2}}{R_0^{3/2}} = -\frac{1}{4} \frac{G^{1/2} m_G^{1/2}}{R_0^{3/2}}$

But $\frac{G^{1/2} m_G^{1/2}}{R_0^{3/2}} = \frac{\left[(6.6726 \times 10^{-11}\ \mathrm{N\,m^2/kg^2})(1.989 \times 10^{30}\ \mathrm{kg}/M_\odot \times 1.2451 \times 10^{11} M_\odot)\right]^{1/2}}{(8500\ \mathrm{pc} \times 206265\ \mathrm{AU/pc} \times 1.496 \times 10^{11}\ \mathrm{m/AU})^{3/2}}$

$= \frac{\left[(6.6726 \times 10^{-11}\ \mathrm{N\,m^2/kg^2})(1.989 \times 10^{30} \times 1.2451 \times 10^{11}\ \mathrm{kg})\right]^{1/2}}{(8500 \times 206265 \times 1.496 \times 10^{11}\ \mathrm{m})^{1/2}\ 8.5\ \mathrm{kpc}}$

for units in m/s/kpc

$= \frac{2.51 \times 10^5\ \mathrm{m/s/kpc}}{8.5} = 29.53\ \mathrm{km/s/kpc}$

$\therefore A = \frac{3}{4}(29.53\ \mathrm{km/s/kpc}) = 22\ \mathrm{km/s/kpc}$

$B = -\frac{1}{4}(29.53\ \mathrm{km/s/kpc}) = -7\ \mathrm{km/s/kpc}$

Both values lie outside the range of 11–14 km/s/kpc for A and $|B|$ found observationally, implying that Keplerian orbits do not apply in local regions of the Galaxy.

5. For NGC 2281, the best parameters with $E_{U-B}/E_{B-V} = 0.72$ are (see graphs):

$E_{B-V} = 0.09 \pm 0.01$

$V - M_V = 8.70 \pm 0.05 \quad = 5\log d - 5 + A_V$

$\therefore V_0 - M_V = 5 \log d - 5$.

$V_0 - M_V = V - M_V - A_V = 8.70 - (3 \times 0.09)$

$= 8.70 - 0.27$

$= 8.43$

$\therefore d = 10^{(8.43+5)/5}$ pc

$= 485$ pc.

$$\frac{\Delta d}{d} = \frac{\Delta(V - M_V)}{2.1714724} = \frac{\pm[(0.05)^2 + (0.03)^2]^{1/2}}{2.1714724} = \frac{\pm 0.0583095}{2.1714724}$$

$= \pm 0.0268531$

$\therefore \Delta d = \pm 0.0268531 \times 485$ pc

$= \pm 13$ pc.

i.e. $d = 485 \pm 13$ pc.

Hipparcos parallaxes yield (only 3 stars)

Star	π (mas)	$\Delta\pi$ (mas)	wgt $\left(\frac{1}{\Delta\pi^2}\right)$	$\pi \times$ wgt
1	1.63	1.24	0.6503642	1.0600937
2	4.10	1.66	0.3628973	1.4878792
3	0.91	1.40	0.5102040	0.4642857
		Sum	1.5234657	3.0122586

$\therefore \langle\pi\rangle$(weighted) $= \frac{3.01\ldots}{1.52\ldots} = 1.9772409$ mas ± 0.810184

Gives $d = 1/\pi = 1/1.977..$ pc $= 505.76$ pc ± 207.2pc (from $\frac{\Delta\pi}{\pi}$)

$d = 485 \pm 13$ pc (ZAMS fitting) versus 506 ± 207pc (Hipparcos πs)

ZAMS gives more precise results, although both agree

From results for Bica 6 (notes) at similar l, expect V_R(LSR) ≈ -1 km/s consistent with observations

RA=06 48 17 174.9
DC=+41 04 42 16.88
NGC 2281

Star	Ref.	V	B-V	U-B
5	1305	11.99	1.39	1.41
7	1305	11.74	0.41	0.09
8				
14				
16				
18				
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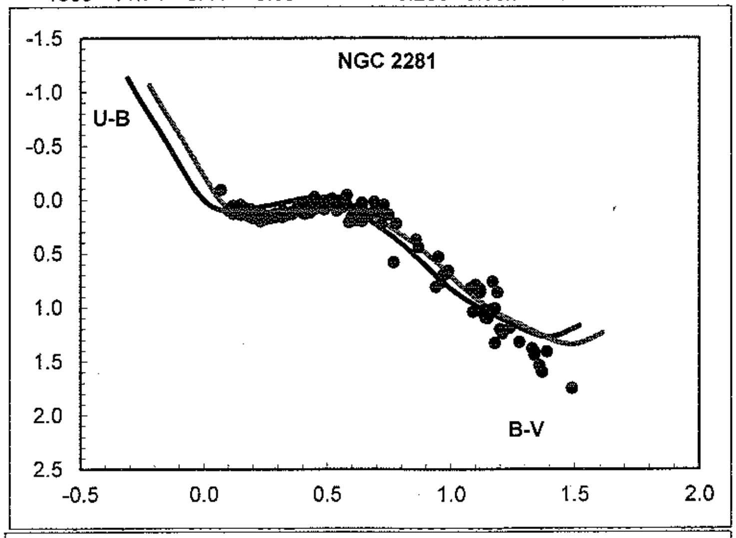
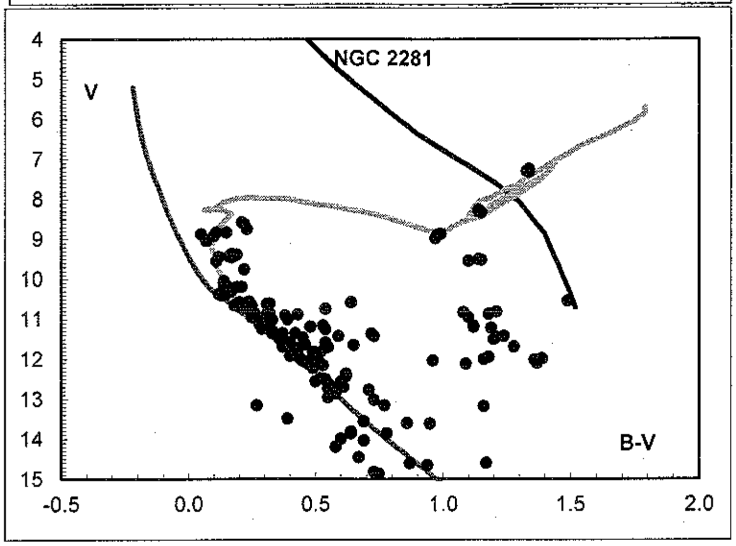
42	1305	12.74	0.55	0.01
43	48	11.14	0.29	0.15
43	69	11.12	0.29	0.15
43	1305	11.24	0.29	0.13
44	48	10.86	0.26	0.16
44	69	10.84	0.26	0.15
44	1305	10.95	0.25	0.18
45	48	10.25	0.15	0.12
45	69	10.20	0.19	0.09
45	1305	10.34	0.15	0.12
46	48	11.70	0.55	0.04
46	1305	11.83	0.52	-0.01
48	48	11.73	0.42	0.08
48	69	11.75	0.40	0.12
48	1305	11.83	0.41	0.06
49	48	10.56	0.24	0.16
49	69	10.62	0.24	0.18
49	1305	10.66	0.25	0.16
50	48	11.17	1.12	0.86
50	69	11.22	1.19	0.86
50	1305	11.19	1.12	0.83
51	48	11.58	0.37	0.10
51	69	11.58	0.40	0.09
51	1305	11.69	0.37	0.10
52	48	10.91	0.38	0.09
52	69	10.89	0.43	0.08





(B-V)o	(U-B)o	Mv	B-V	U-B	V
-0.310	-1.133	-3.50	-0.22	-1.06	5.20
-0.300	-1.089	-3.10	-0.21	-1.02	5.60
-0.290	-1.047	-2.70	-0.20	-0.98	6.00
-0.280	-1.008	-2.40	-0.19	-0.94	6.30
-0.270	-0.969	-2.13	-0.18	-0.90	6.57
-0.260	-0.932	-1.90	-0.17	-0.86	6.80
					7.00
					7.18
					7.35
					7.53
					7.70
					7.87
					8.05
					8.22
					8.38
					8.53
					8.67
					8.80
					8.93
					9.06
					9.19
					9.31
					9.43
					9.55
					9.65
					9.76
					9.85
					9.95
					10.04
					10.12
					10.20
					10.26
					10.31
					10.36
					10.41
					10.45
					10.49
					10.54
					10.59
					10.64
					10.69
					10.74
0.110	0.105	2.09	0.20	0.17	10.79
0.120	0.104	2.14	0.21	0.17	10.84
0.130	0.103	2.18	0.22	0.17	10.88
0.140	0.100	2.23	0.23	0.17	10.93
0.150	0.098	2.27	0.24	0.17	10.97
0.160	0.095	2.31	0.25	0.16	11.01
0.170	0.091	2.35	0.26	0.16	11.05
0.180	0.088	2.39	0.27	0.16	11.09
0.190	0.084	2.43	0.28	0.15	11.13
0.200	0.080	2.48	0.29	0.15	11.18
0.210	0.076	2.51	0.30	0.15	11.21
0.220	0.072	2.55	0.31	0.14	11.25
0.230	0.068	2.59	0.32	0.14	11.29
0.240	0.064	2.64	0.33	0.13	11.34
0.250	0.059	2.69	0.34	0.13	11.39
0.260	0.055	2.73	0.35	0.12	11.43
0.270	0.050	2.78	0.36	0.12	11.48
0.280	0.046	2.82	0.37	0.12	11.52
0.290	0.041	2.87	0.38	0.11	11.57
0.300	0.037	2.93	0.39	0.11	11.63
0.310	0.032	2.98	0.40	0.10	11.68
0.320	0.027	3.03	0.41	0.10	11.73
0.330	0.022	3.08	0.42	0.09	11.78
0.340	0.017	3.14	0.43	0.09	11.84
0.350	0.013	3.19	0.44	0.08	11.89
0.360	0.008	3.26	0.45	0.08	11.96

Eb-v=	0.090	0.01	log t = 8.6
V-Mv=	8.70	0.05	
Vo-Mv	8.43		
d=	485		
pm	13		
Mv	-3.09		
dm-M	0.06		
S Sge			
V	5.614		
B	6.416		
U	6.983		
B-V	0.802		
U-B	0.567		
pm	8.38	0.923	
(B-V)o	0.7		
Ebv(B0)	-0.76		
Eb-v	0.102		
Vo	5.344		
Mv	-3.09		

B-V	Mv	B-Vred	Vred
1.70	-3.00	1.790	5.700
1.70	-2.84	1.790	5.860
1.65	-2.62	1.740	6.080
1.60	-2.47	1.690	6.230
1.57	-2.37	1.660	6.330
1.50	-2.18	1.590	6.520
1.45	-2.04	1.540	6.660
1.40	-1.83	1.490	6.870
1.35	-1.67	1.440	7.030
1.30	-1.45	1.390	7.250
1.25	-1.28	1.340	7.420
1.20	-1.04	1.290	7.660
1.15	-0.94	1.240	7.760
1.10	-0.74	1.190	7.960
1.05	-0.65	1.140	8.050
1.03	-0.52	1.120	8.180
1.00	-0.17	1.090	8.530
1.05	-0.24	1.140	8.460
1.10	-0.42	1.190	8.280
1.15	-0.62	1.240	8.080
1.20	-0.79	1.290	7.910
1.25	-1.00	1.340	7.700
1.30	-1.22	1.390	7.480
1.35	-1.58	1.440	7.120
1.30	-1.24	1.390	7.460
1.25	-1.09	1.340	7.610
1.20	-0.91	1.290	7.790
1.15	-0.73	1.240	7.970
1.10	-0.55	1.190	8.150
1.05	-0.40	1.140	8.300
1.00	-0.18	1.090	8.520
0.95	-0.02	1.040	8.680
0.90	0.16	0.990	8.860
0.85	0.08	0.940	8.780
0.80	-0.02	0.890	8.680
0.75	-0.12	0.840	8.580
0.70	-0.23	0.790	8.470
0.65	-0.31	0.740	8.390
0.60	-0.38	0.690	8.320
0.55	-0.43	0.640	8.270
0.50	-0.50	0.590	8.200
0.45	-0.55	0.540	8.150
0.40	-0.58	0.490	8.120
0.35	-0.65	0.440	8.050
0.30	-0.71	0.390	7.990
0.25	-0.72	0.340	7.980
0.20	-0.73	0.290	7.970
0.15	-0.74	0.240	7.960
0.10	-0.71	0.190	7.990
0.05	-0.63	0.140	8.070
0.00	-0.42	0.090	8.280
-0.03	-0.43	0.060	8.270
0.00	-0.46	0.090	8.240
0.05	-0.42	0.140	8.280
0.08	-0.34	0.170	8.360
0.05	-0.15	0.140	8.550
0.03	0.00	0.120	8.700
0.00	0.50	0.090	9.200
0.02	1.00	0.110	9.700
0.05	1.33	0.140	#####
0.10	1.83	0.190	#####
0.15	2.29	0.240	#####
0.20	2.50	0.290	#####

_r	_RAJ2000	_DEJ2000	HIP	RAhms	DEdms	Vmag	RA(ICRS)	DE(ICRS)	Plx	pmRA	pmDE	e_Plx	VTmag	B-V	pi	unc	wgt	pi x wgt
arcmin	deg	deg				mag	deg	deg	mas	mas/yr	mas/yr	mas	mag	mag				
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4.551	102.170206	41.066498	32654	06 48 40.85	+41 03 59.5	8.64	102.170218	41.0665167	1.63	-3.79	-7.79	1.24	8.616	0.214	1.63	1.24	0.6504	1.0601
6.8415	101.995275	41.177132	32592	06 47 58.87	+41 10 37.7	10.25	101.995289	41.1771451	4.1	-4.24	-5.3	1.66	10.225	0.162	4.1	1.66	0.3629	1.4879
7.0141	102.092158	40.962527	32626	06 48 22.12	+40 57 45.2	8.86	102.092163	40.9625452	0.91	-1.46	-7.68	1.4	8.834	0.113	0.91	1.4	0.5102	0.4643
																4.3	1.5235	3.0123
															<pi>	1.977		
															unc	0.656		
															d=	506	pc	
															unc	168		