CISC 810: Fundamentals of Computational Science, Assignment 5

Set September Nov 20th due Dec 1st

For all programming questions ensure code is commented well and appropriate function/subroutine calls are made.

email me a copy of your codes in all cases and provide a hard copy along with the additional requested information.

Q1. Use a composite version of the two-point Gaussian-Legendre quadrature rule discussed in the lecture to integrate

$$\int_0^{\pi/2} \sin^2(x) e^{-x/\pi} dx$$

for n=4,8,16,32 sub-intervals. You can map each of the sub-intervals into [1,-1] and then use the formulae for the integral estimate given in the lecture. Emperically compute the convergence rate with varying n.

Q1. The differential equation

$$y'(t) = \frac{y(1-y)}{2y-1}$$

with $y_0 = 5/6$ has the solution

$$y(t) = \frac{1}{2} + \left[\frac{1}{4} - \frac{5}{36}e^{-t}\right]^{1/2}.$$

(a) Write codes to solve the equation using 3 of the methods we discussed in the lectures, namely the forward Euler method, Runge-Kutta and 2-step Adams-Bashforth (you may utilize any prototype codes that I provided in the lecture).
(b) Measure the error at step t=2.0 for the different methods and plot up the error relative to the exact solution versus the step-size h. What order of convergence do you obtain for each method?

Q3. Sub-random sequences. Generate the first 10,000 points in the triplet series (x_1^i, x_2^i, x_3^i) where the x_j^i are given by Halton sequences for bases 3,5,7. Plot your results on a 3d graph.

Q4. (a) Write a code to integrate

$$I = \int_0^1 \frac{dx}{1+x^2} = \frac{\pi}{4}$$

using a Monte Carlo method. Recall that for a general multidimensional integral the sample variance defines the relative convergence relative to the mean of f via

$$\int f dV \simeq < f > \times V \pm \sqrt{\frac{< f^2 > - < f >^2}{N}} \times V.$$

Calculate the sample variance and plot it for N=$10^2, 10^3, 10^4, 10^5, 10^6$.
(b) Calculate the same integral this time using a sequence of points generated from the Halton's sequence (base 3) you developed in question 2. Plot up the new sample variance for the same N on the same plot as part (a).