Huygens-Fresnel Principle

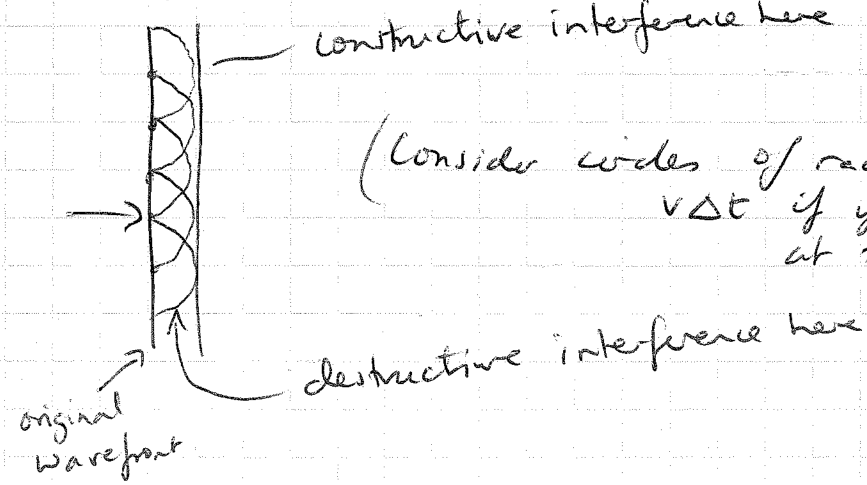
Q: How can we figure out how a wave will propagate through/around a material in 2D?

A: We need to look at how waves propagate – the Huygens-Fresnel Principle outlines a procedure that lets us do exactly this.

Key idea:

Each point in an advancing wave front is the source of new outgoing disturbance, with the resulting wave being a sum of all these outgoing disturbances.

ie. for a plane wave:



(Consider circles of radius $v\Delta t$ if you're looking at time propagation)

Q: There is something slightly weird about this idea – can you see it?

A: The fact we only considered the forward moving wave – wouldn't there be one behind?

Actually we really need to consider the forward propagation only – after all a wave won't suddenly go backward without reflecting.
So we consider semi-circles in the direction of propagation.

This idea probably came from looking at water waves. If you put in a small gap in a sheet then the outgoing disturbance is a semi-circle regardless of the waveform hitting the gap. See Fig 8.6.

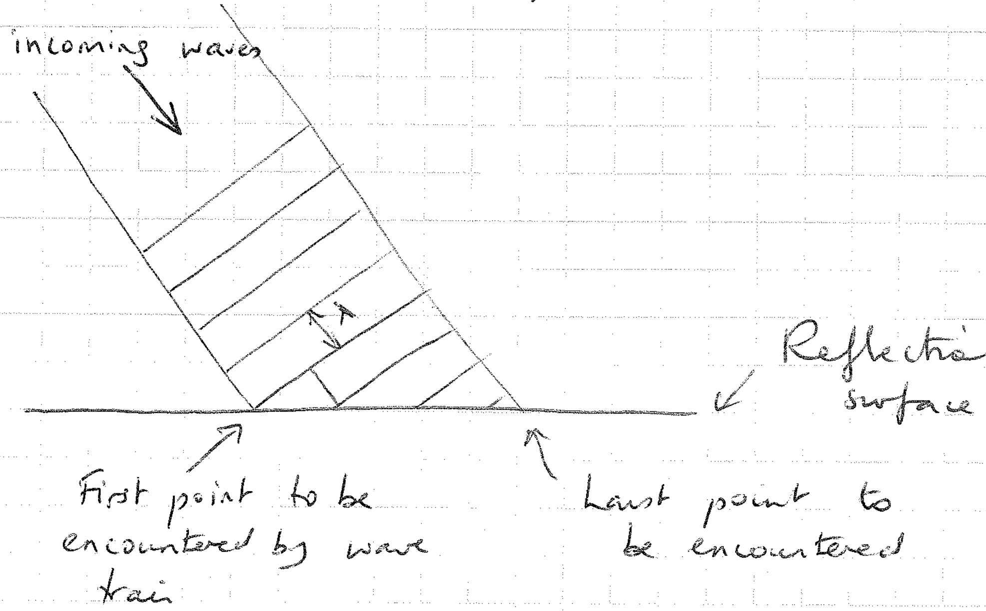
While this approach might feel slightly "a qualitative" "artificial" it does give us a method of determining what happens when waves meet a barrier or obstacle.

Reflection & Refraction of Plane Waves

We saw what happens in 1-d when a wave hits a discontinuity - reflection & partial transmission

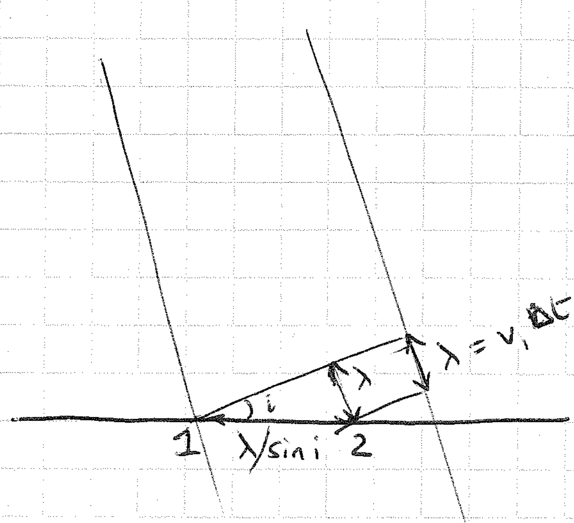
Q: What can happen in 2-d that's different?
A: Change in the direction of propagation!

To analyse we'll just consider a train of incoming waves & we'll mark the peaks.



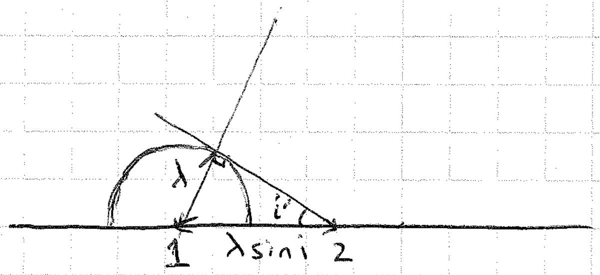
Separation of wavefronts = $\lambda = v\Delta t$

So now consider two (in phase) wavefronts & how far their disturbances will have propagated.



Now consider the wavefronts propagating from 1 & 2.

The wavefront 2 hasn't moved anywhere since it is the last of the two points. The wavefront from 1 will have moved $v_1 \Delta t = \lambda$



If we draw the tangent to the wavefront that will mark the front of the outgoing wave

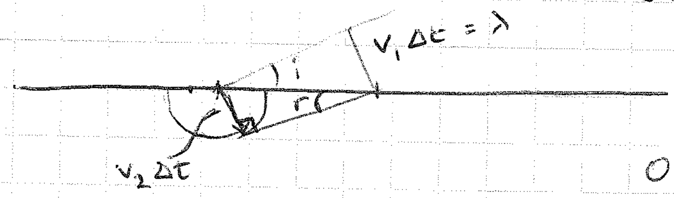
But the shape of this triangle is basically the same as what we had before
- opposite side length $= \lambda$
- hypotenuse " " $= \lambda/\sin i$

$\therefore$ $i = i'$ and the incoming angle of incidence matches the outgoing.

Exactly what we'd expect!

What about if we propagate into a different medium?

In this case the situation considers a forward moving wave rather than the reflected one. This medium will also have a different speed of propagation, say v_2



Incoming is still the same as before

Separation of points on surface is again $\lambda/\sin i$

Outgoing wavefront travels $v_2 \Delta t$

How can we calculate angle r?

It's again constructed as the tangent to the outgoing wavefront of radius $v_2 \Delta t$

The hypotenuse is again $\lambda/\sin i$ but using $\lambda = v_1 \Delta t$ we must have

$$\sin r = \frac{opp}{hyp} = \frac{v_2 \Delta t}{\lambda/\sin i} = \frac{v_2 \Delta t}{v_1 \Delta t/\sin i}$$

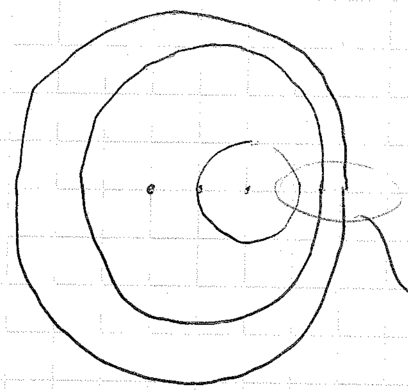
$$\Rightarrow \quad \frac{\sin r}{\sin i} = \frac{v_2}{v_1}$$

Note: evaluating the amplitudes is considerably harder. For transverse waves it even depends upon the state of polarization!

Doppler Effect & Related Phenomena

This is the situation where the source of the effect is moving through the medium, at say some speed u.

Case 1 $u <$ sound speed



Consider wavefronts exactly one wavelength apart, ie time τ between them where $\lambda_0 = v\tau$

(τ is of course $\frac{1}{\nu}$)

BUT source moves $u\tau$ in the time between wavefronts.

wavefronts are compressed together

So separation between fronts is:

$$v\tau - u\tau = \lambda_{new}$$

$$\Rightarrow \quad (v-u)\tau = \lambda_{new}$$

& for situation at rest. $\lambda_0 = v\tau$

$$\Rightarrow \quad \frac{(v-u)\tau}{v\tau} = \frac{\lambda_{new}}{\lambda_0}$$

$$\Rightarrow \quad \lambda_{new} = \left(1 - \frac{u}{v}\right)\lambda_0$$

(Usually people just write λ rather than λ_{new})

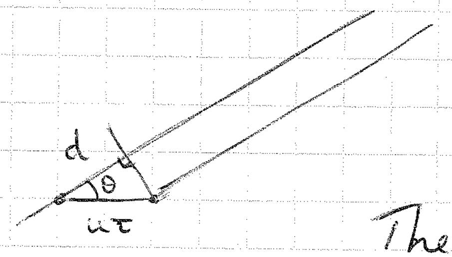
You can do the same analysis on the other side where the wavelength will be <u>stretched out</u>

$$\Rightarrow \quad (v+u)\tau = \lambda_{new}$$

$$\Rightarrow \quad \lambda_{new} = \left(1 + \frac{u}{v}\right)\lambda_0$$

So if you're behind the source you hear a longer wavelength, and ahead you hear a shorter wavelength.

What happens at an angle?



At rest we again have
$\lambda_0 = v\tau$

The wave fronts propagate in circular directions as before

But net distance between wavefronts is

$$v\tau - d$$
$$= v\tau - u\tau\cos\theta$$

Quickly check this makes sense:
if straight ahead we have $v\tau - u\tau$ ✓
if directly behind " " $v\tau + u\tau$ ✓

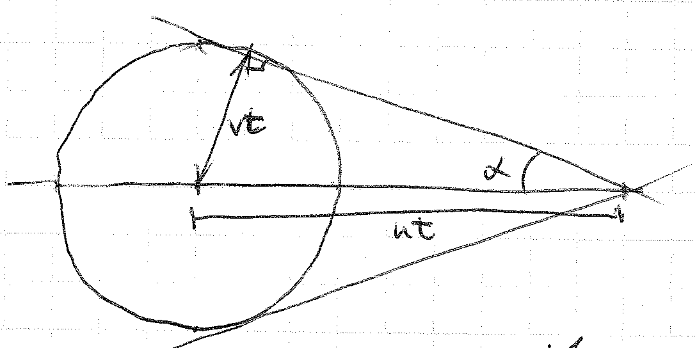
Thus $\lambda(\theta) = (v - u\cos\theta)\tau$

$$\Rightarrow \quad \frac{\lambda(\theta)}{\lambda_0} = \frac{(v - u\cos\theta)\tau}{v\tau} = \left(1 - \frac{u}{v}\cos\theta\right)$$

$$\therefore \quad \lambda(\theta) = \left(1 - \frac{u}{v}\cos\theta\right)\lambda_0$$

Situation 2: when $u > v$

Now the system moves faster than the sound speed.



$$\sin\alpha = \frac{v\tau}{u\tau} = \frac{v}{u}$$

Note $\frac{u}{v}$ = "Mach number"

α = "mach angle"

If these were sound waves this would be the "sonic boom" cone.