The Cosmos

Planets & Life
PHYS 214

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Please start all class related emails with "214:"

Today's Lecture

- Pop Quiz 2
- Habitable zones

- Note the book likes to use T_e to represent the temperature of an Earth-like planet, instead I'll use T_p in my derivations

Motivation

- Before we look at the formation of planets let’s look at the impact of the star’s luminosity on the surrounding planets
- As the basis for defining the habitable zone we will assume we want to have liquid water on a planets surface
- Note this modelling is actually highly non-trivial even the comparatively complex model we will look at is an over-simplification

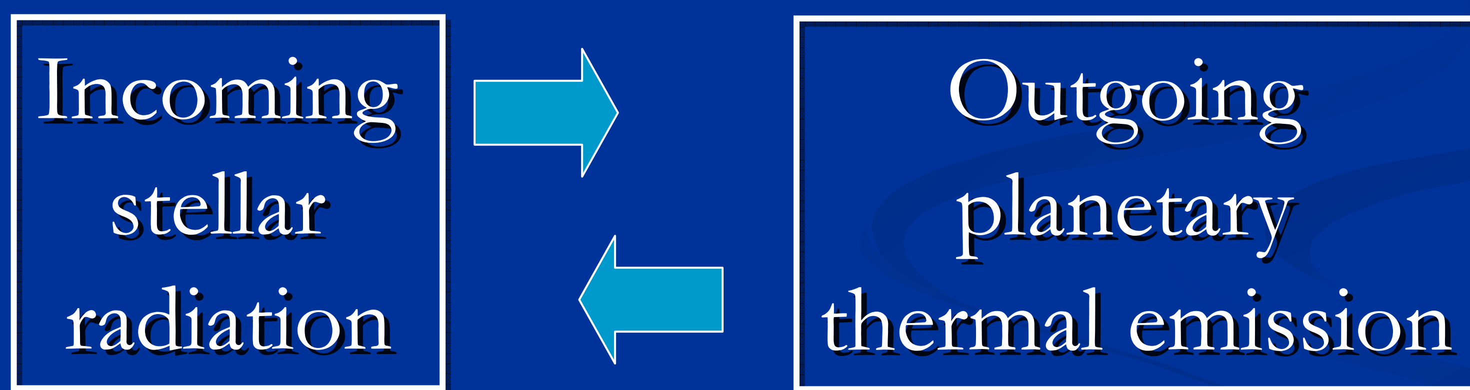
Circumstellar Habitable Zone

- A *circumstellar habitable zone* is defined as encompassing the range of distances from a star for which liquid water can exist on a planetary surface

- Note: clearly this is a very broad definition and many types of life require a far narrower temperature range to survive

Energy balance

- At the very simplest level, the two competing effects in determining the temperature on the surface of a planet are



- For there not to be a net change in temperature on the planet the two effects *must balance*

What if there isn't a balance?

- Suppose incoming radiation exceeds outgoing thermal emission
  - Planet must heat up – this is the beginning stage of a greenhouse effect
- Suppose incoming radiation is less than outgoing thermal emission
  - Planet must cool down – this is the beginning stage of a snowball-earth type event

Estimates based on stellar luminosity

- Full problem (including atmospheric effects) was first studied by Hart (1978 *Icarus*, **33**, 23-39)
- We'll look at a simpler model using just concerns about the radiation balance that originated in the 1950s
- The first stage is thus to look at the total luminosity of the Sun
  - Note, we already looked at this when studying the HR diagram in the last lecture

Stellar Luminosity

- To calculate the luminosity L, recall we used the Stefan-Boltzman law, multiplied by the total surface area of the star:

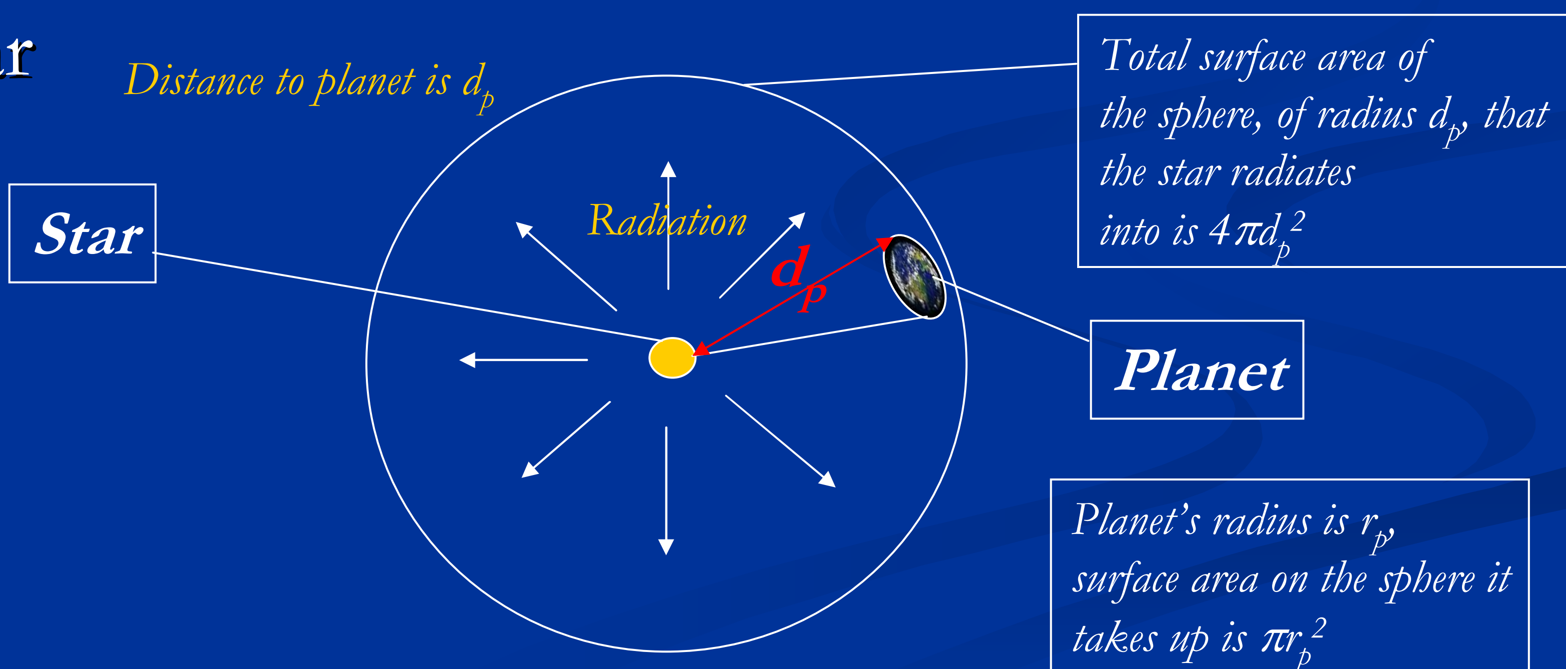
$$L = \text{surface area} \times \text{luminosity per unit of surface area}$$

$$L_* = 4\pi R_*^2 \sigma T_*^4$$

where R_* is the radius of the star, T_* the temperature, σ is the Stefan-Boltzman constant and L_* is the total luminosity of the star

Amount of radiation arriving at the planet

- We must now evaluate what fraction of that total luminosity is arriving at the planet
- The radiation is emitted in all directions by the star



Fraction of radiation arriving at planet

- To get this we just divide the area of the planet, by the total area of the sphere

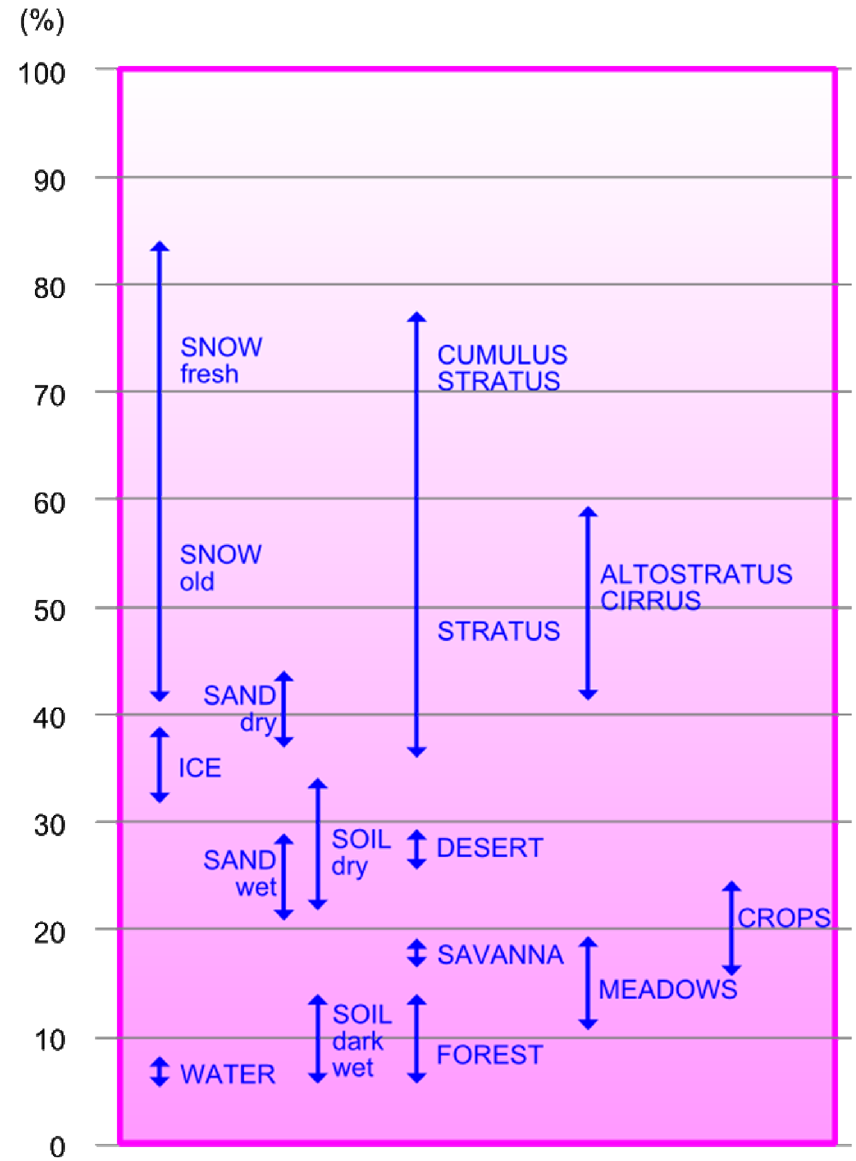
$$fraction = \frac{\pi r_p^2}{4\pi d_p^2}$$

- So the total amount of radiation arriving at the planet is

$$\text{Radiation reaching planet} = 4\pi R_*^2 \sigma T_*^4 \times \frac{\pi r_p^2}{4\pi d_p^2}$$

Reflected radiation: albedo

- Not all radiation reaching a plane will be absorbed by the surface – some will be reflected back into space
- This phenomena is called *albedo* and is often quoted as a percentage value
  - The value is strongly affected by cloud cover as well as snow covering (fresh snow can reflect up to 85% of incoming radiation)
- We will represent the amount of reflected energy by the letter a, so that the fraction or radiation reaching the planet is $(1-a)$



Stellar heating: putting it all together

- Thus when we include the albedo factor, the total amount of radiation heating the planet is

$$\text{Radiation heating planet} = 4\pi R_*^2 \sigma T_*^4 \times \frac{\pi r_p^2}{4\pi d_p^2} \times (1-a)$$

- We now have to equate this value to the amount of radiation emitted by the planet
- We use the Stefan-Boltzman law again, this time using the planetary temperature and radius

Planetary thermal emission

- So again we use,

  L = surface area × luminosity per unit of surface area

$$L_p = 4\pi r_p^2 \sigma T_p^4$$

where r_p is the radius of the planet, T_p is the temperature and L_p is the planetary luminosity

Equilibrium temperature

- We now equate the incoming radiation from the star to the thermal emission of the planet

$$\frac{4\pi R_*^2 \sigma T_*^4 \pi r_p^2}{4\pi d_p^2}(1-a) = 4\pi r_p^2 \sigma T_p^4$$

- Cancel factors of π, σ and r_p^2

$$\frac{R_*^2 T_*^4}{d_p^2}(1-a) = 4T_p^4$$

T_p: planetary temperature

- We can multiply both sides of the equation by ¼, and then take the fourth root to get T_p

$$T_p = \frac{1}{\sqrt{2}}(1-a)^{1/4}\sqrt{\frac{R_*}{d_p}}\;T_*$$

- If instead when equating the incoming radiation to the outgoing emission we re-insert the total luminosity of star, $L_* = 4\pi R_*^2 \sigma T_*^4$

$$\frac{L_* \pi r_p^2}{4\pi d_p^2}(1-a) = 4\pi r_p^2 \sigma T_p^4$$

$$\text{Hence } \frac{L_*}{4 d_p^2}(1-a) = 4\pi\sigma T_p^4$$

After cancelling factors of r_p and π

T_p as a function of stellar luminosity

- We can rearrange our previous formula to get

$$T_p = \frac{1}{2}\frac{1}{\sqrt{d_p}}\left(\frac{L_*(1-a)}{\pi\sigma}\right)^{1/4}$$

- This is helpful because it allows us to write down a relationship as a function of solar values, without going into details of how we do this, the final result is

$$T_p = 278\frac{1}{\sqrt{d_{p\,\text{in AU}}}}\left(\frac{L_*}{L_{SUN}}\right)^{1/4}(1-a)^{1/4}\ \text{Kelvins} \quad \text{——}\textbf{(A)}$$

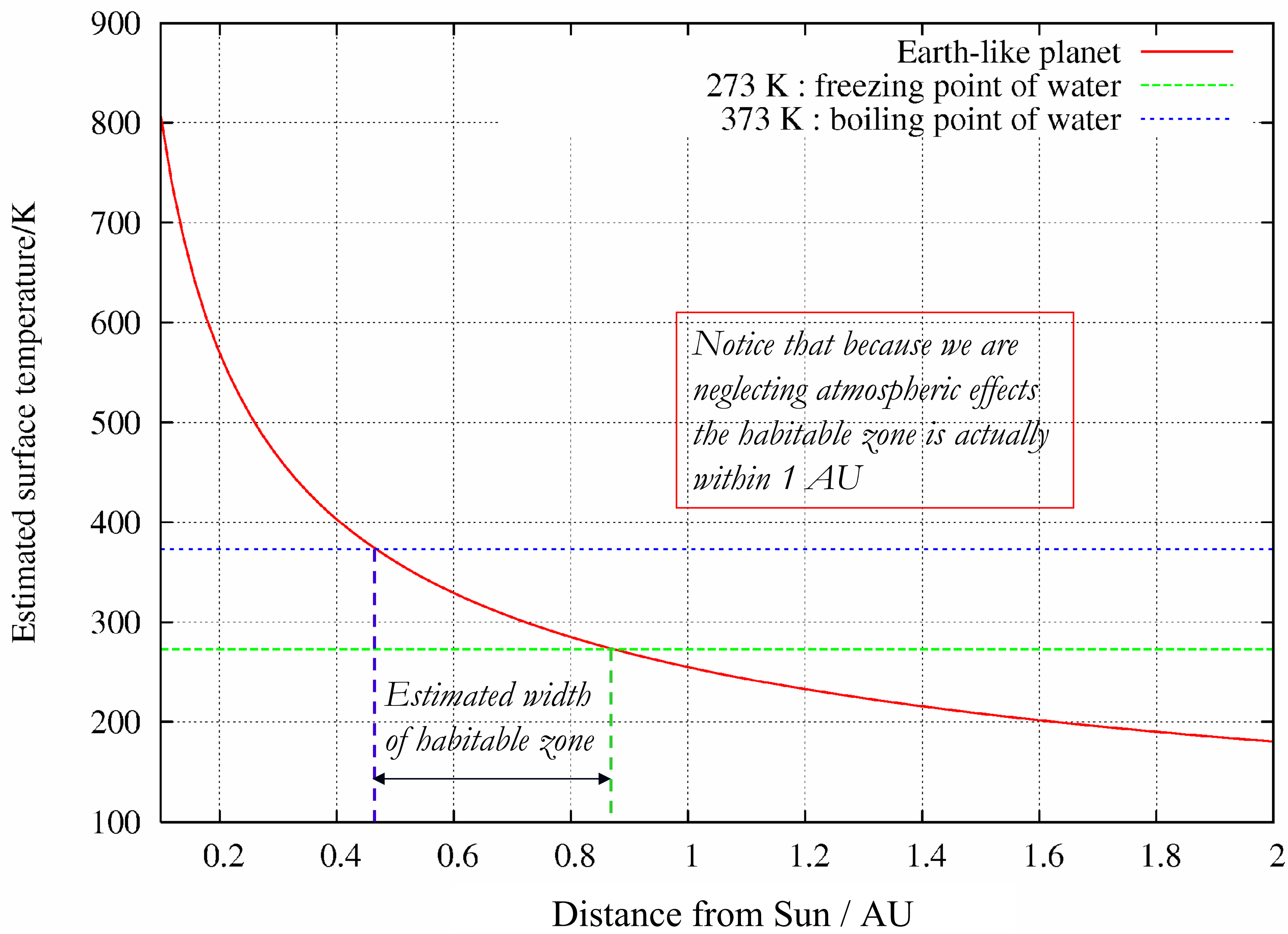
Where d_p must be given in astronomical units, and L_{SUN} is the luminosity of the Sun

Estimated temperature for Earth

- For an albedo of around 30% (a=0.3) for the Earth, we get an estimated temperature of about 255 K
- The average temperature for the Earth is actually much closer to 290 K
- Atmospheric greenhouse effect serves to keep the temperature higher (also a very tiny heating effect due to geothermal heat)

Estimating the width of the habitable zone

- To calculate the width of the habitable zone we need to find the distances for which T_p=273 K and T_p=373 K (freezing and boiling points of water)
- Plot up a graph for the T_p relationship as a function of planetary distance from the Sun and then read off values
  - Or we could just solve the equation labelled (A) for T_p=373 & 273 K as well



Earth-like planet
273 K : freezing point of water
373 K : boiling point of water
Notice that because we are neglecting atmospheric effects the habitable zone is actually within 1 AU
Estimated width of habitable zone
Estimated surface temperature/K
Distance from Sun / AU
900
800
700
600
500
400
300
200
100
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

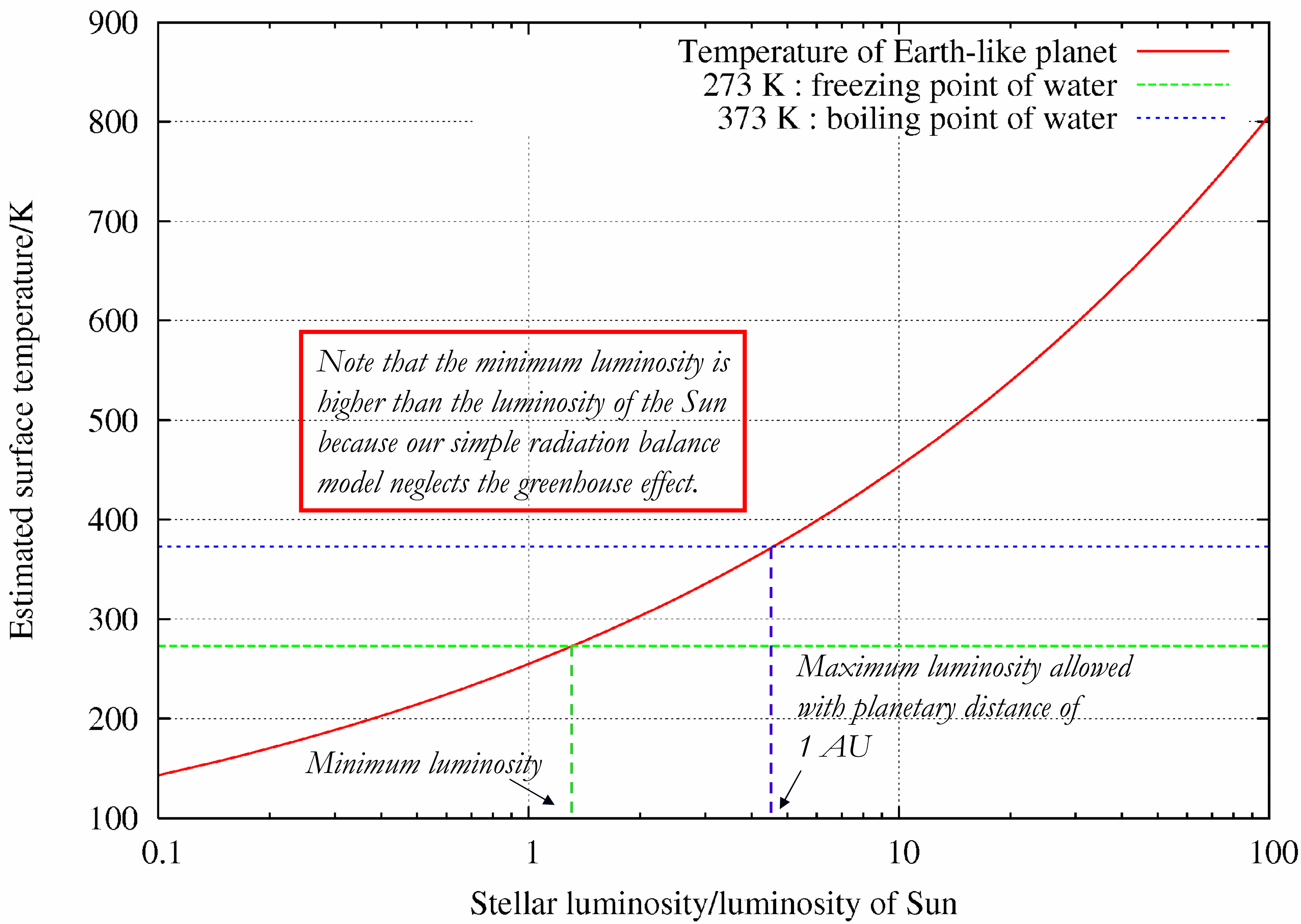
How does the estimated habitable zone change with stellar luminosity

- In the previous graph we looked at how the estimated planetary temperature

$$T_p = 278 \frac{1}{\sqrt{d_{p \text{ in AU}}}} \left(\frac{L_*}{L_{SUN}} \right)^{1/4} (1-a)^{1/4} \text{ Kelvins}$$

changed according to distance

- We can also fix the distance at the Earth’s orbit (1 AU) and vary the luminosity of star to see how that changes the habitable zone


Temperature of Earth-like planet
273 K : freezing point of water
373 K : boiling point of water
Note that the minimum luminosity is higher than the luminosity of the Sun because our simple radiation balance model neglects the greenhouse effect.
Minimum luminosity
Maximum luminosity allowed with planetary distance of 1 AU
Estimated surface temperature/K
Stellar luminosity/luminosity of Sun
900
800
700
600
500
400
300
200
100
0.1
1
10
100

Summary of lecture 10

- Habitable zones are broadly defined by a temperature range that supports liquid water
- Estimating a planetary temperature using a radiation balance model requires
  - The luminosity of the star
  - The distance from the star
  - The albedo of the planet
- The estimated temperature of the Earth is about 255 K, about 35 K lower than the average of 290 K
- The difference in these two values is caused by the greenhouse effect

Next lecture

- The Sun’s habitable zone & other habitable zone definitions (p 49-54)