Physics 214 - Problem Set 3

Due in Class on Wednesday, March 21, 2007 (Time allowed=three weeks)

Numbers like the mass of the Sun, distance of the Earth, number of meters in a light year, etc, can be found in the text or on the Internet.

To receive full credit you must show all working.

(1) (a) Although we know the distance of the Earth from the Sun is 1 AU, this question examines precisely how we determine exactly how large 1 AU is. The method of determining the distance uses Kepler's Third Law of planetary motion, *in the form* $a^3/P^2 = constant$, the periods of the orbits of Earth and Mars, and the distance between Earth and Mars. When the Sun, Earth and Mars are aligned along a straight line (*in opposition*) an astronomer finds that time taken for the radar pulse to reach Mars and return to Earth is 8 minutes 45 seconds. For simplicity assume that the planets have circular orbits centered on the Sun. Using Kepler's Third Law, calculate the distance of the Earth from the Sun. (HINT: It may help to draw a diagram of the orbits which can then be used to derive the equations you will need to solve the problem. Explicitly, you will need two equations, one describing Kepler's 3rd Law for the Earth, and one describing Kepler's 3rd Law for Mars, which can then be used to solve for the distance to the Sun.)
(b) Using Kepler's Third Law *in the form* $(P/2\pi)^2 = a^3/(G(M+m))$, calculate the Mass of the Earth, given that the Moon takes 27.3 days to orbit the Earth, and that the Earth-Moon distance is 385,000 km. (You will need to make an assumption about the mass of the moon relative to the Earth, and also the nature of the orbit. You should clearly state what your assumptions are in both cases.)

(2) (a) Saturn has an orbital period of 29.46 years, and under the assumption of a circular orbit, an orbital radius of 9.53 AU. Calculate the velocity (in metres per second) of Saturn around the Sun.
(b) Suppose that we are viewing the Saturn-Sun system (ignore other planets) from outside the solar system. Assuming that we can measure light reflected by the Saturn in this system, what would be the change in wavelength associated with the Doppler shift caused by the movement in the orbit? (Assume the reference wavelength is 550 nm).
(c) Kepler's Third Law can be used to derive a relationship between the masses and velocities of the Sun and Saturn. The relationship is $M_{Sun}v_{Sun} = M_{Sat}v_{Sat}$, where M and v denote the velocities. Calculate the velocity of the Sun around the centre of mass and thus the Doppler shift expected for the motion of the Sun.
(d) Using experimental equipment (that detects motion caused by the Doppler shift) that has noise such that errors occur at around the level of ± 3 m s^{-1}, how well would you be able to detect the Doppler motion of the Sun in this case?

(3) The upcoming Space Interferometer Mission (SIM) will have a resolution of 4 micro arcseconds. In this question we'll calculate how this resolution determines the mass of planets that we can detect and the corresponding separation from the host star.
(a) For two object orbiting around a centre of mass, the relationship between the masses and orbital radii (given by the letter a) is given by $M_1 a_1 = M_2 a_2$. Assuming that the distance a_1 will correspond to the angular size we wish to detect when viewed from the Earth, use this relationship and the small angle formula, $\theta = D/d$, to derive an equation for the distance at which we can detect the movement of object 1 as a function of M_1, M_2, a_2 and the angular resolution of the SIM mission in *micro arcseconds*.
(b) Take M_1 to have the same mass as the Sun, M_2 to be the same as the Earth, and a_2 to be 1 AU. Calculate the distance (in ly) at which the motion of object 1 can still just be detected by SIM.
(c) Suppose the mass of object 2 is increased by a factor of three and its orbital radii decreased by a factor of four. What is the distance at which this new system can be detected by SIM?
(d) If we set a_2 equal to 0.5 AU, and keep M_2 equal to three Earth masses, calculate the mass of M_1 (in solar masses) if the system is still detectable at 100 ly. Given the mass of M_1 that you've found, what kind of object would you classify it as?