Physics 214 - Problem Set 2

Due in Class on Wednesday, Feb 28, 2007 (Time allowed=three weeks)

Numbers like the mass of the Sun, distance of the Earth, number of meters in a light year, etc, can be found in the text or on the Internet.
Show all working.

(1) Using $E = mc^2$, we can calculate the amount of mass lost in the Sun due to the fusion of 4 protons into one helium nuclei (the p-p chain).
(a) If 0.7% of the mass of the 4 protons is released as energy, calculate how many Joules this single fusion reaction releases.
(b) Given the total luminosity of the Sun is 3.85×10^{26} J s^{-1}, calculate the total mass loss that is occuring every second.
(c) The total mass of the Sun is 2.0×10^{30} kg and its main sequence lifetime is 10 billion years, calculate the fraction of the Sun's mass that is converted from hydrogen to helium over the Sun's main sequence lifetime.

(2) (a) Using the Stephan-Boltzman Law (with $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}) and the surface area of a sphere, estimate the luminosity of a white dwarf star of diameter 11,700 km and temperature 24,800 K.
(b) Do the same for the Sun, diameter of 1.39×10^6 km, and temperature 5800 K.
(c) How do these ratios compare, especially given that the size of the white dwarf is very similar to that of Earth?
(d) For the Sun the peak emission of the black-body curve appears at 480 nm. Given the above surface temperature of the white dwarf, at what wavelength will its peak emission occur?

(3) In the lecture on habitable zones we derived an equation for average planetary temperature based upon the "radiation balance model". While a reasonable starting point, this model neglects the greenhouse effect and thus under estimates the surface temperature of planets with greenhouse-type atmospheres. Suppose we change the model to include a greenhouse parameter ϵ that can take a range on 0 to 1, although in practice it will be between 0.2 to 1. In the lectures I showed that the the albedo factor (1-a) just multiplied the amount of radiation arriving at the planet so that the radiation balance gives

$$\frac{L_* \pi r_p^2}{4\pi d_p^2} \times (1 - a) = 4\pi r_p^2 \sigma T_p^4$$

(see notes for the meaning of each variable).
(a) Include the greenhouse parameter ϵ in this equation such that now it divides the incoming radiation. Then rearrange this formula to give T_p as a function of all the other variables to arrive at a formula similar to that in slide 16 of lecture 10. Rewrite this formula in the form given in the lower equation of slide 16, and then rearrange it to give the distance in astronomical units as a function of the other variables.
(b) Suppose we have an albedo of $a = 0.3$ and a greenhouse parameter of $\epsilon = 0.5$, use the equation you just derived to calculate the position of the inner and outer edges of the HZ and thus the width of the habitable zone for a planet orbiting around a solar luminosity star.
(c) Suppose over time the greenhouse parameter changes from 0.5 to 1, what happens to the habitable zone?
(d) Finally, suppose the albedo of the planet orbiting a solar luminosity star is initially 0.5 and the greenhouse parameter is 0.4. After a billion years the albedo has fallen to 0.25, while the greenhouse parameter is now 0.6. How has the habitable zone in this system changed?

(4) We discussed in class that during the collapse of the solar nebula the angular momentum is conserved. Suppose the initial solar nebula was about 0.25 light years in diameter, had a mass of 1 solar mass and a rotation rate of about once every million years.
(a) Calculate the speed of rotation of the outer edge in km s^{-1}.
(b) Under gravitational collapse the solar nebula shrinks down to a diameter of 100 AU. Assuming that angular momentum is conserved what is the rotation speed of the outer edge now?
(c) What is the initial density of nebula and how many hydrogen molecules per cubic centimetre is it?