THERMODYNAMIC STATE & IONIZATION:

PRESSURE EQUATION OF STATE

$$P_{gas} = P(T, \rho, \mu \ldots) \quad \text{(EOS)}$$

STELLAR ATMOSPHERE: ρ LOW

∴ IDEAL GAS LAW GOOD

$$P(\tau) = N(\tau)\, k_B T_{kin}(\tau) \quad (\text{dyne/cm}^2)$$

$$= \frac{\rho(\tau)\, k_B T_{KIN}(\tau)}{(\mu(\tau)\, m_H)}$$

- N (cm^{-3})
- μ = MEAN "MOLECULAR" "WEIGHT" OF MIXTURE (a.m.u.) – RUTTEN
- m_H = 1 a.m.u. (g)

N_Z = NO. DENSITY OF ELEMENT Z

"SPECIES" k : IONIZATION STATE OF ELEMENT Z

- e.g. Ca II (Ca^{+1})

$$\therefore N_Z = \sum_{k}^{\text{STAGES}} N_k$$

$$N = \sum_{Z}^{\text{ELEMENTS}} N_Z + N_e$$

PARTIAL PRESSURE OF SPECIES, k:

$$P_k(\tau) = N_k(\tau)\, k_B T_{KIN}(\tau)$$

N_e = NO. DENSITY OF FREE ELECTRONS (e^-)

$$P_e(\tau) = N_e(\tau)\, k_B T_{KIN}(\tau)$$

$$P(\tau) = \sum_{z}^{\text{ELEMENTS}} \sum_{k}^{\text{STAGES}} N_{z,k}(\tau)\, k_B T_{KIN}(\tau) + N_e(\tau)\, k_B T(\tau)$$

ATOMIC MASS NUMBER OF ELEMENT Z, a_z

$$\rho = \sum_{z}^{\text{ELEMENTS}} (a_z m_H) N_z$$

$$\mu(\tau) = \frac{1}{m_H} \frac{\rho(\tau)}{N(\tau)} \quad \text{(a.m.u.)}$$

$$= \frac{1}{m_H} \frac{\sum_z m_H a_z N_z(\tau)}{\sum_z N_z(\tau) + N_e(\tau)}$$

ELECTRON (e^-) DONORS

- ELEMENTS, Z, OF:
  - 1^{ST} IONIZATION E, $\chi_I < k_B T$
  - RELATIVELY LARGE A_Z

- F, G, K STARS ($k_B T < 13.6$ eV):

  Na, Mg, Ca, Al, Si, Fe, (H^-)

- O, B, A STARS ($k_B T \gtrsim 13.6$ eV):

  H

DETERMINING N_e:

- ASSUME
  - $T_{KIN}(\tau)$, $N(\tau)$ KNOWN
  - NEUTRAL (I), SINGLE (II), DOUBLE (III) +ve ION. STAGES ONLY

    $$\therefore N_Z = N_I + N_{II} + N_{III}$$

  - LTE: $T_{IONIZATION} = T_{KIN}$

FOR IONIZATION STAGE k OF ELEMENT Z:

(k = I, II, OR III)

- IONIZATION FRACTION, f_k:

$$f_k \equiv \frac{N_k}{N_Z} = \frac{N_k}{N_I + N_{II} + N_{III}}$$

SAHA DISTRIBUTION:

FOR $k \rightleftarrows (k+1) + e^-$:

$$\frac{N_{k+1}}{N_k} = \frac{1}{N_e} C \, T_{KIN}^{3/2} \, e^{-\chi_{I,k}/k_B T_{KIN}}$$

$$= f(T_{KIN}, N_e)$$

Eg. $k = \text{II}$:

$$f_{\text{II}} = \frac{N_{\text{II}}}{N_{\text{I}} + N_{\text{II}} + N_{\text{III}}}$$

$$= \frac{N_{\text{II}}/N_{\text{I}}}{1 + N_{\text{II}}/N_{\text{I}} + \left(N_{\text{III}}/N_{\text{II}}\right)\left(N_{\text{II}}/N_{\text{I}}\right)}$$

$$= f(N_e)$$

ITERATIVE PROCEDURE:

- INITIAL GUESS $N_e^{(0)}(T)$

- FOR EACH e^--DONOR, Z:

$$f_{II}(T) = \frac{N_{II}(T)}{N_Z(T)} = f(N_e(T))$$

$$f_{III}(T) = \frac{N_{III}(T)}{N_Z(T)} = f(N_e(T))$$

- $$N_e^{(1)}(T) = \sum_Z N_Z(T) f_{II,Z}(T) + 2\sum_Z N_Z(T) f_{III,Z}(T)$$

UNTIL:

$$\frac{N_e^{(n)}(T) - N_e^{(n-1)}(T)}{N_e^{(n)}(T)} < \varepsilon \ll 1$$

- ALL T

$$\frac{N}{N_e} = \frac{\sum_Z N_Z + N_e}{N_e} = \frac{\sum_Z N_Z}{N_e} + 1$$

$$= \frac{\sum_Z N_Z}{\sum_Z N_Z f_{II,Z} + 2 \sum_Z N_Z f_{III,Z}} + 1$$

RECALL: $N_Z = A_Z N_H$

$$\frac{N}{N_e} = \frac{\sum_Z \underline{A_Z}}{\sum_Z \underline{A_Z} f_{II,Z} + 2 \sum_Z \underline{A_Z} f_{III,Z}} + 1$$

$\{A_Z\}$ VALUES ARE <u>INPUT</u> PARAMETERS

Eg.

GAS COMPOSED OF H
+ ONE "METAL", M

- $A_M \ll 1$
- STAGES I & II ONLY

STUDY N_e/N IN LIMITS OF
HIGH- & LOW-T_{KIN}