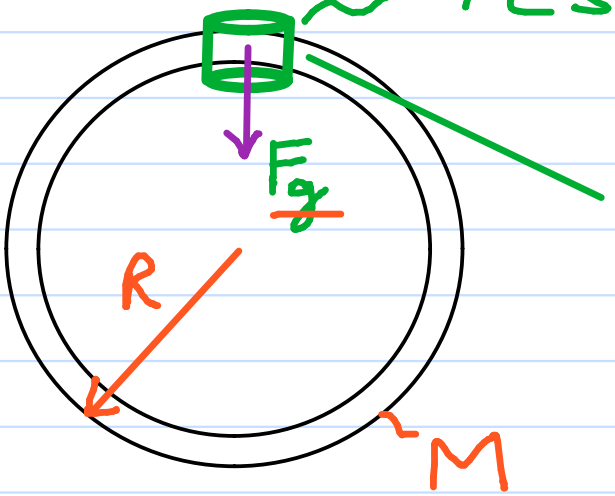
HYDROSTATIC EQUILIBRIUM (HSE):

→ TOTAL FLUID PRESSURE, $P(z)$

$$a_z(z) = 0 \rightarrow F_{NET\,z}(z) = 0$$

CYLINDRICAL TEST MASS, m



$$g(z) = g = \frac{GM}{R^2}$$

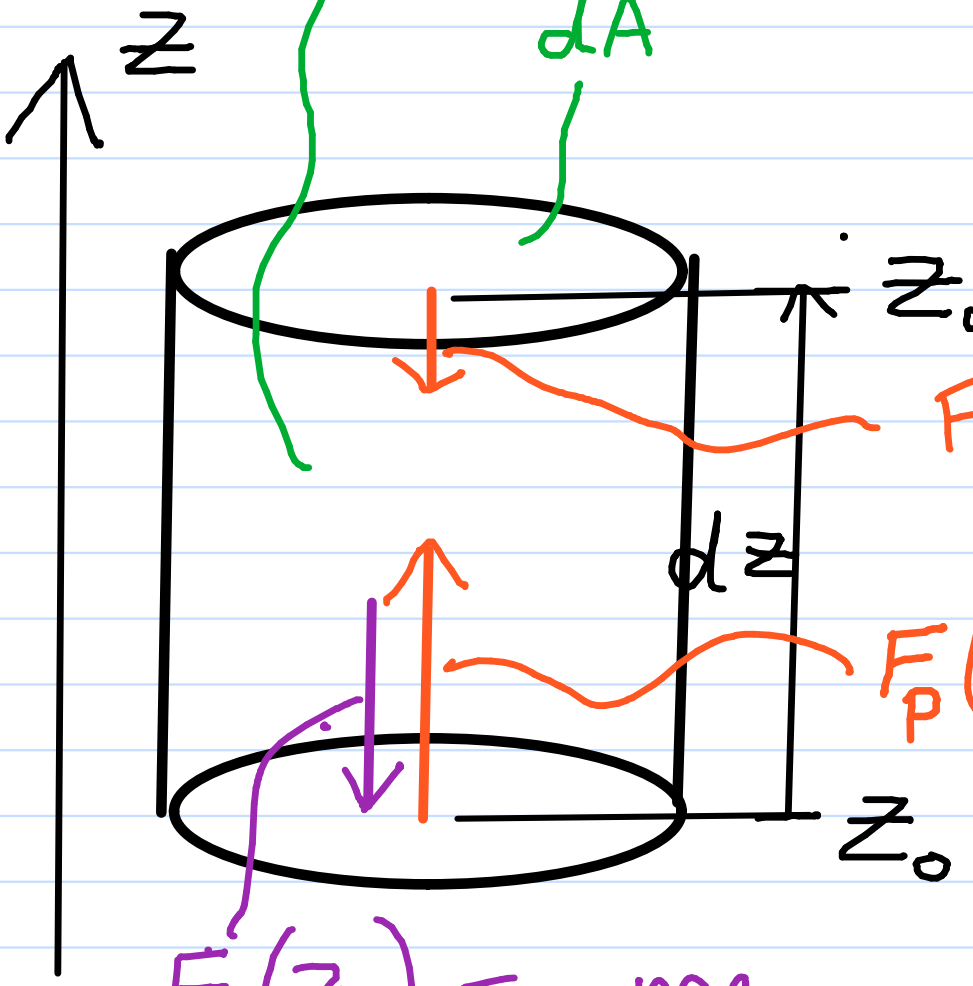
- INPUT PARAMETER

$$\therefore F_g = -mg$$

$dV = dz\,dA$

$m = \rho(z_0)\,dV$

$= \rho(z_0)\,dz\,dA$



$F_p(z_0+dz) = -P(z_0+dz)\,dA$

$F_p(z_0) = P(z_0)\cdot dA$

$F_g(z_0) = -mg$

HSE: $F_g + dF_p = 0$

$\therefore -mg + F_p(z_0) - F_p(z_0+dz) = 0$

$\therefore -\rho\,dA\,dz\,g + P(z_0)\,dA - P(z_0+dz)\,dA = 0$

$\therefore -\rho g\,dz + \{P(z_0) - P(z_0+dz)\} = 0$

$\{P(z_0) - P(z_0+dz)\} = -dP$

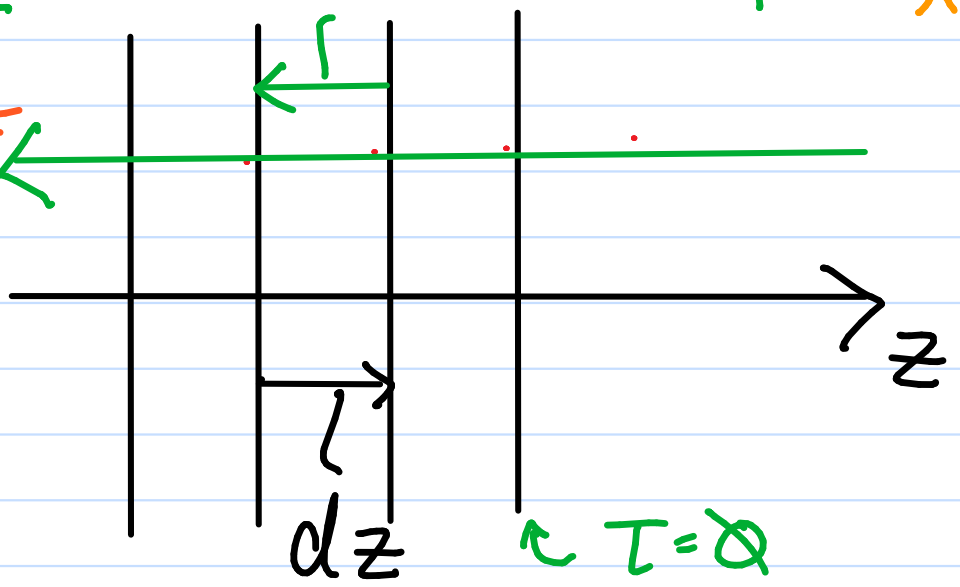
$$\therefore \boxed{\frac{dP(z)}{dz} = -g\rho(z)} \quad < 0$$

↳ HSE Eq.

GRAY

$$d\tau = \bar{K}(z)\rho(z)(-dz)$$

τ



↳ $\tau = 0$

UPPER BOUNDARY CONDITION

$$-\frac{dP(\tau)}{d\tau}\cdot\bar{K}(\tau)\rho(\tau) = -g\rho(\tau)$$

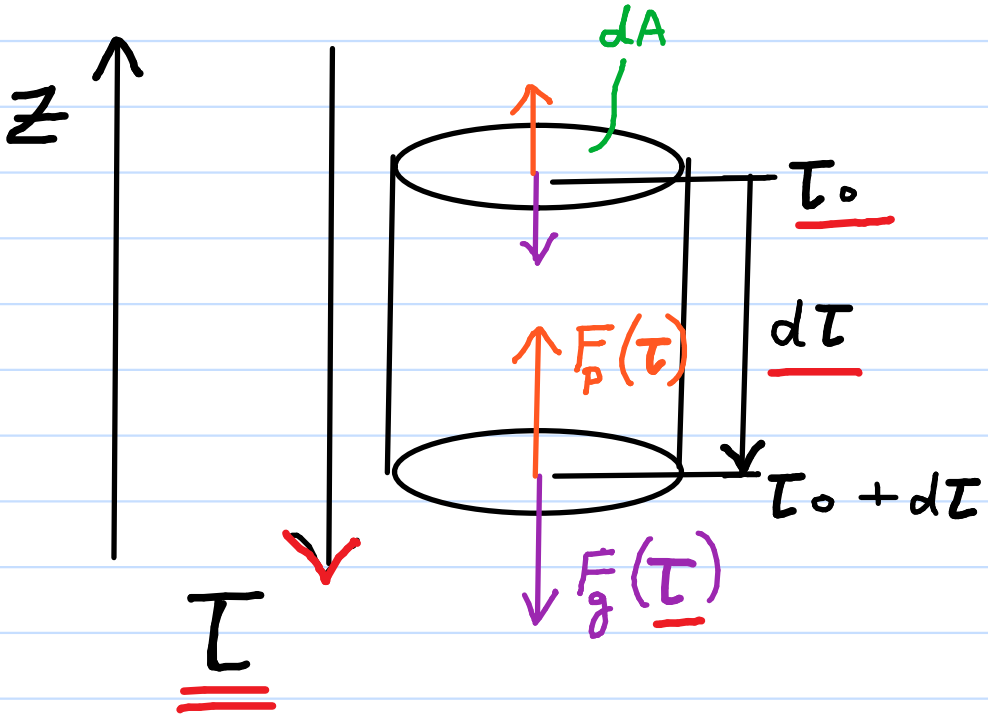
HSE on GRAY OPTICAL DEPTH SCALE IN -ve z DIRECTION,

$$d\tau(z) = -\bar{K}(z)\rho(z)\,dz :$$

$$\frac{dP(\tau)}{d\tau} = -\frac{(-g\rho(z))}{K(\tau)\rho(z)}$$

$$\frac{dP(\tau)}{d\tau} = \frac{g}{K(\tau)} > 0$$

HSE



HSE FORMAL SOLUTION:

- GRAY, ch. 9

$$P^{1/2}\,dP = P^{1/2}\,\frac{g}{K(\tau)}\,d\tau$$

$$\therefore P(\tau) = \left(\frac{3g}{2}\int_{t=0}^{\tau}\frac{P^{1/2}(t)}{K(t)}\,dt\right)^{2/3}$$

INTEGRATE INWARDS

ITERATIVE PROCEDURE:

1) INITIAL GUESS $P^{(1)}(\tau)$ FOR RHS

2) IMPROVED ESTIMATE $P^{(n)}(\tau)$ FROM $P^{(n-1)}(\tau)$ - ALL τ_i s

3) IMPROVED ESTIMATE $K^{(n)} = K(P^{(n)})$

ITERATE

4) CONVERGENCE CRITERION:

$$\frac{\Delta P}{P} = \frac{P^{(n)} - P^{(n-1)}}{P^{(n)}} < \varepsilon << 1$$

- ALL T_i

$$P(T) = P_{gas}(T) + P_{RAD}(T) + P_{MAG}(T) + P_{TURB}(T) + P_{CENTRIFUGE}(T)$$

MOST STARS, MOST T VALUES:

$$P_{gas}(T) = P(T) - P_{RAD}(T)$$