SPECTRAL LINE BROADENING (DAMPING)

→ LINE PROFILE, $\phi_\nu(\nu-\nu_0)$:

RECALL: b-b TRANSITIONS:

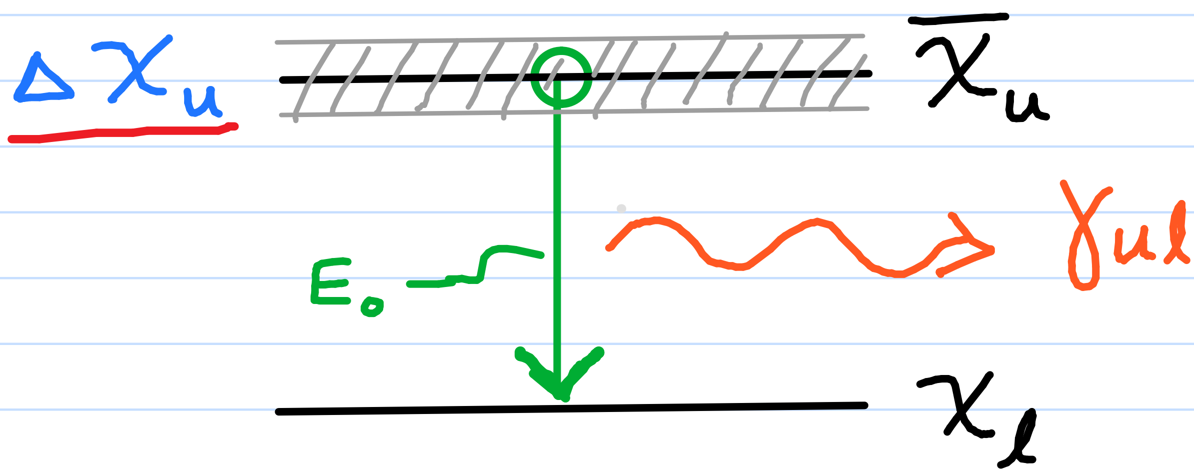
IN CRD: $\psi_\nu = \chi_\nu = \phi_\nu$

$$\alpha_\nu^l(\nu) = \alpha_{\nu_0}^l \, \phi_\nu(\nu-\nu_0) \neq \alpha_{\nu_0}\, \delta(\nu-\nu_0)$$

SPONTANEOUS DE-EXCITATION, $u \to l$

ASSUME $\Delta\chi_l = 0$:

2-LEVEL ATOM:



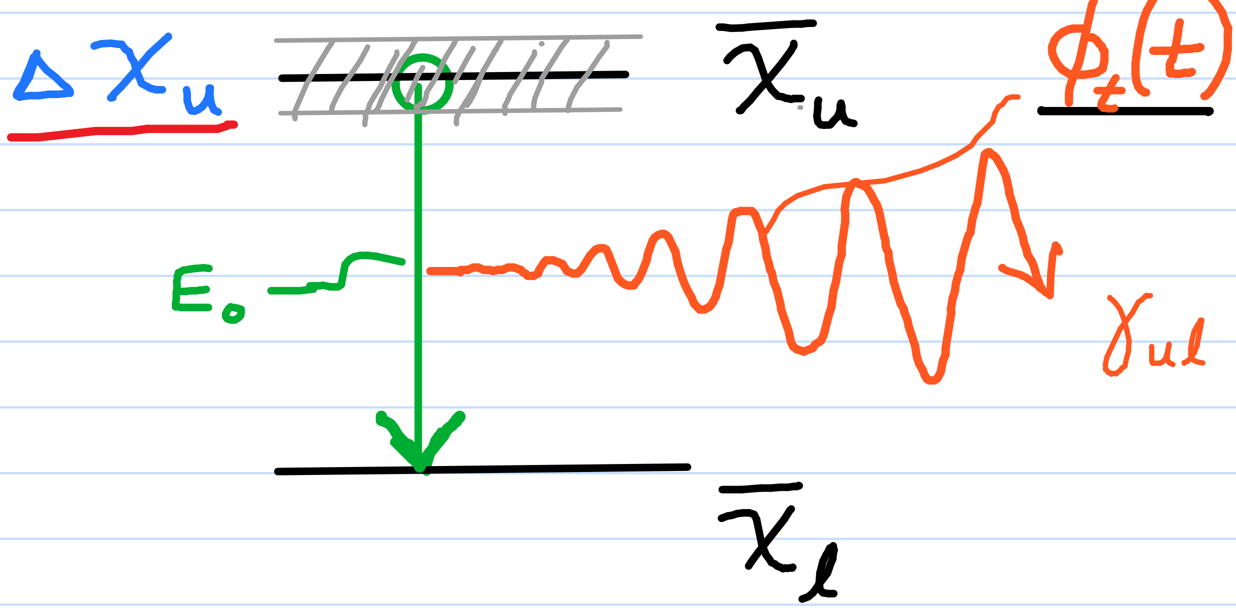
PHOTON $E_0 = h\nu_0 = \bar{\chi}_u - \chi_l$

RADIATION (NATURAL) BROADENING:

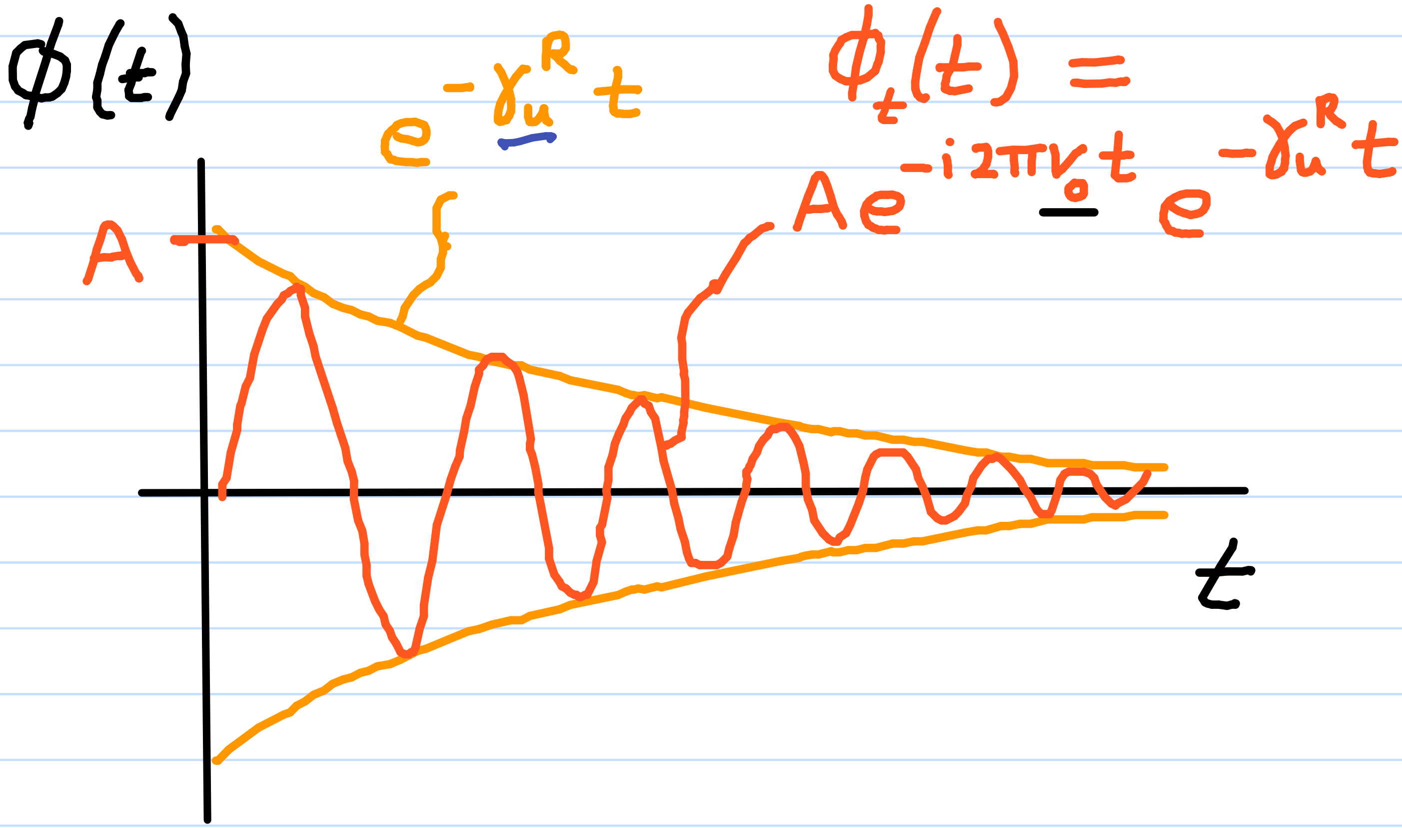
MODEL DE-EXCITING ATOM AS UNDERDAMPED, DRIVEN HARMONIC OSCILLATOR

- ν_0 = NATURAL (RESONANT) FREQ.
- ν = DRIVING FREQ.
- γ_u^R = DAMPING PARAMETER OF E-LEVEL u
- $\phi(t)$ = UNDER-DAMPED Sol'n FOR EMITTED CLASSICAL EM WAVE

TRANSIENT Sol'n:



EMITTED EM WAVE:



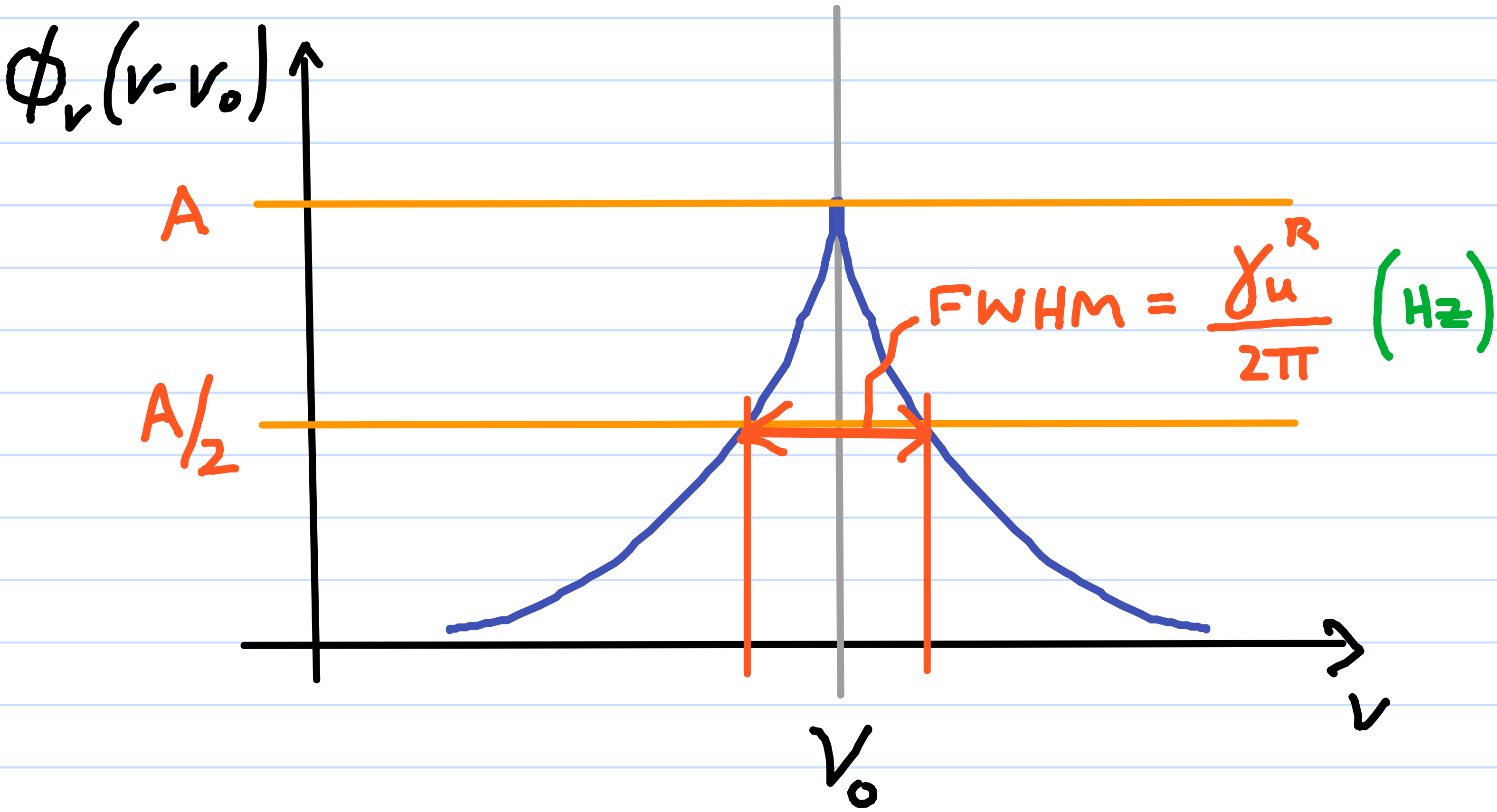
$$\phi_\nu(\nu-\nu_0) = F_T(\phi_t(t))$$

$$= A^* F_T(e^{-\gamma_u^R t}) * F_T(e^{-i2\pi\nu_0 t})$$

$$= A^* F_T(e^{-\gamma_u^R t}) * \delta(\nu-\nu_0)$$

$$\therefore \quad \phi_\nu(\nu-\nu_0) = \frac{\gamma_u^R/4\pi^2}{(\nu-\nu_0)^2 + \left(\gamma_u^R/4\pi\right)^2}$$

- LORENTZIAN DAMPING PROFILE



NORMALIZED: $\int_{\nu=0}^{\infty} \phi_\nu \, d\nu = 1$

QUANTUM MECH. INTERPRETATION (γ_u^R):

HEISENBERG $\Delta E - \Delta t$ UNCERTAINTY PRINCIPLE FOR E-LEVEL u:

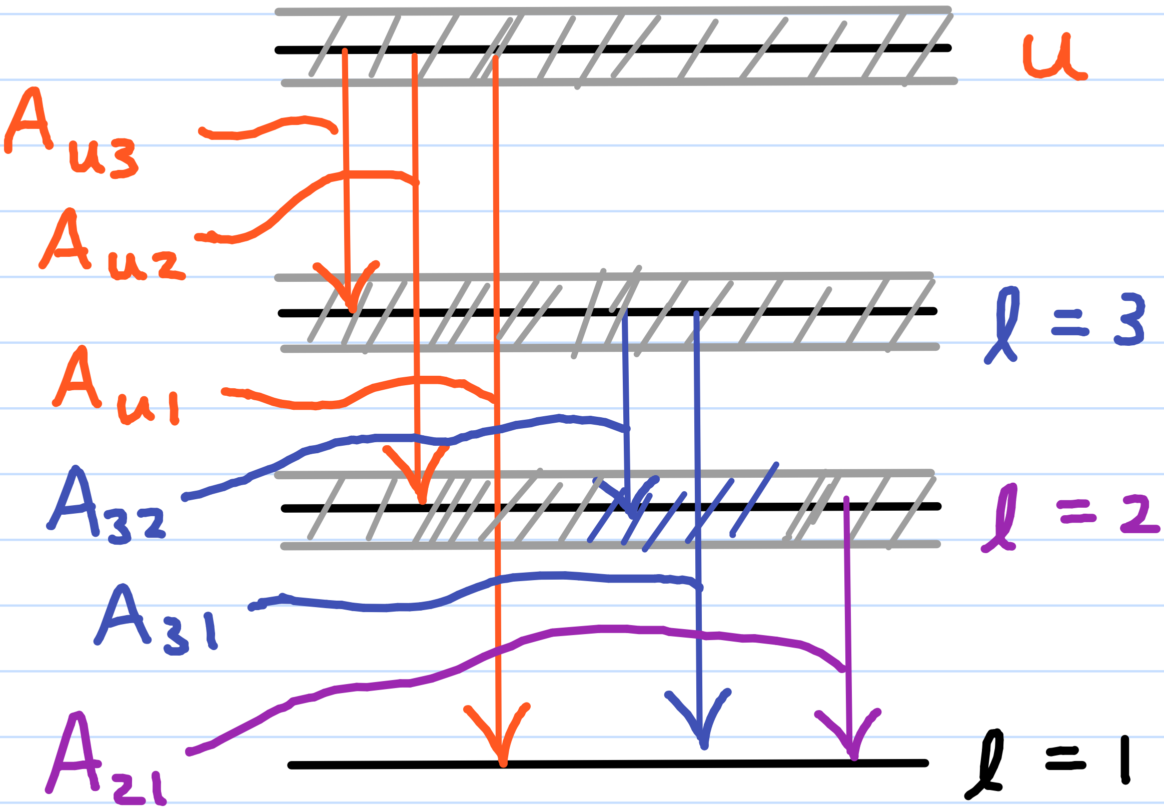
$$\underline{\Delta \chi_u} \, \Delta t = h/2\pi$$

→ Δt = MEAN LIFE-TIME IN LEVEL u

$= A_{ul}^{-1}$ (s)

$$\underline{\gamma_u^R} \equiv \frac{2\pi}{h} \Delta \chi_u = A_{ul} \quad \text{(Hz)}$$

MULTI-LEVEL ATOM:



$$\gamma_{\underline{u}}^{R} = \underset{\underline{\ell < u}}{\sum} A_{u\ell}$$

FOR $u \leftrightarrow \ell$: $\gamma^R = \gamma_u^R + \gamma_{\underline{\ell}}^R$

$\ell = 1$ (GROUND):

$$\Delta t = \infty \;\therefore\; \Delta x_1 = 0 \;\therefore\; \underline{\gamma_1^R = 0}$$

COLLISIONAL (PRESSURE) BROADENING:

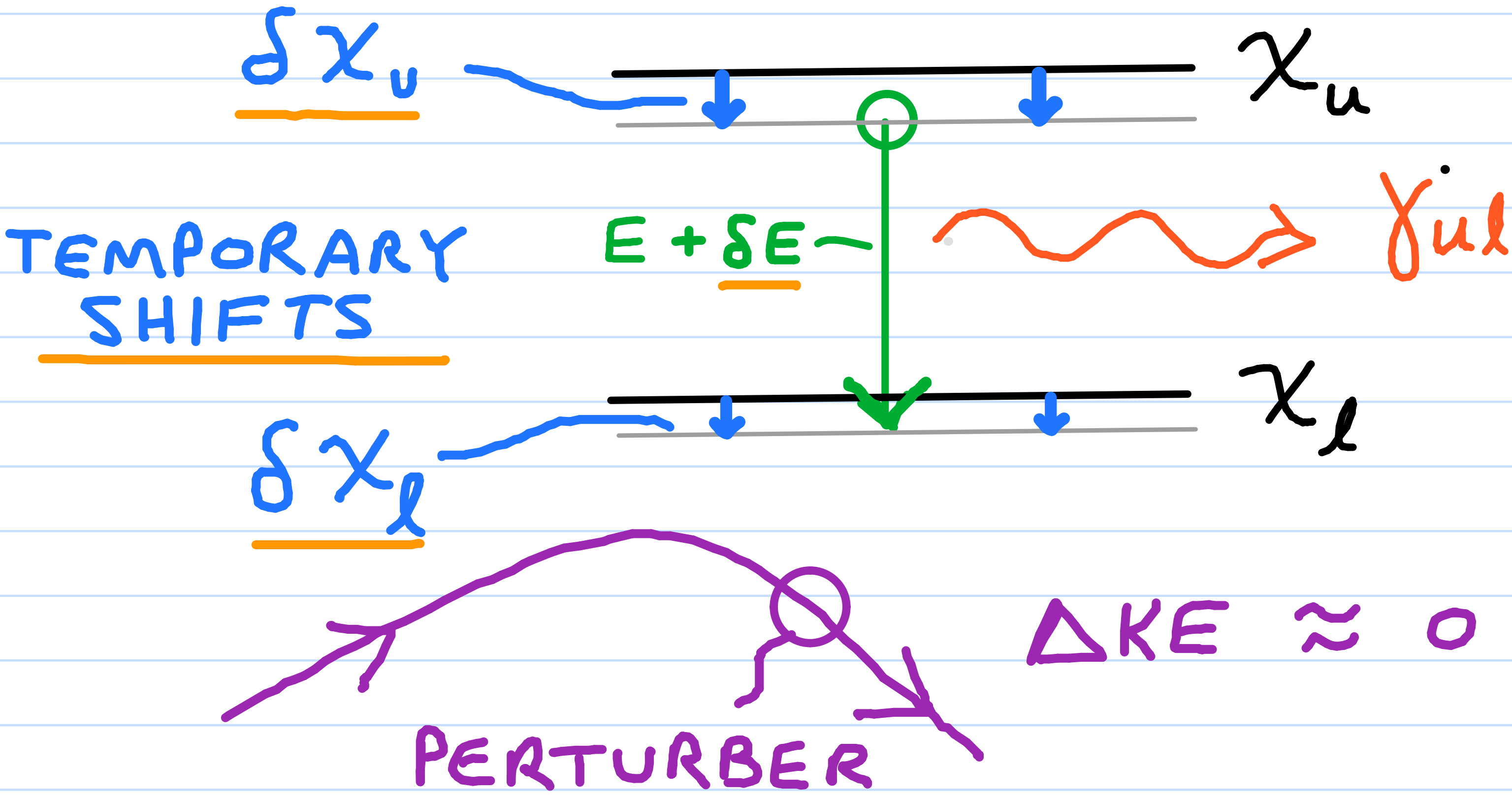
- ELASTIC EM INTERACTION WITH
  A PERTURBER
  → $\Delta KE \approx 0$

IMPORTANT PERTURBERS:

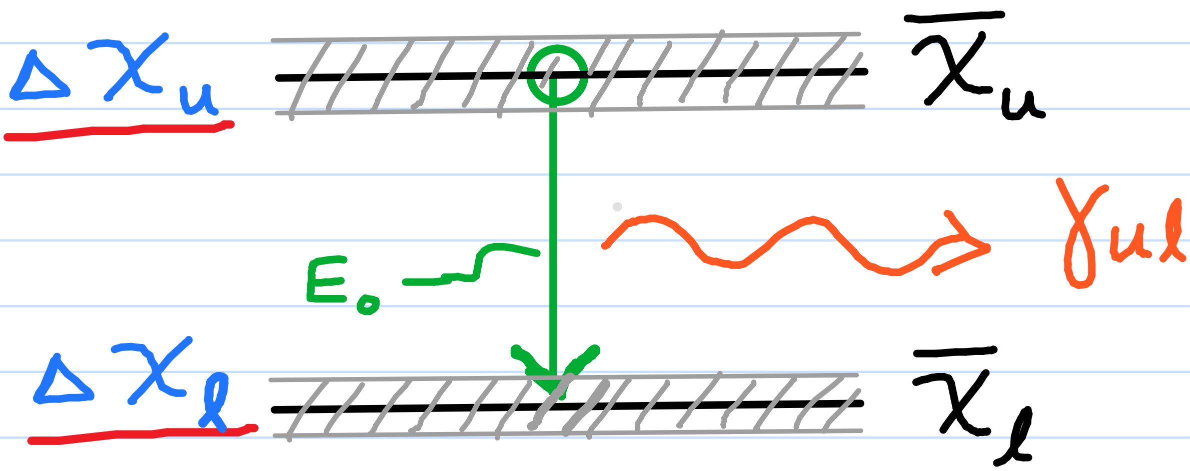
OBA STARS: e^-, H II

FGKM STARS: H I

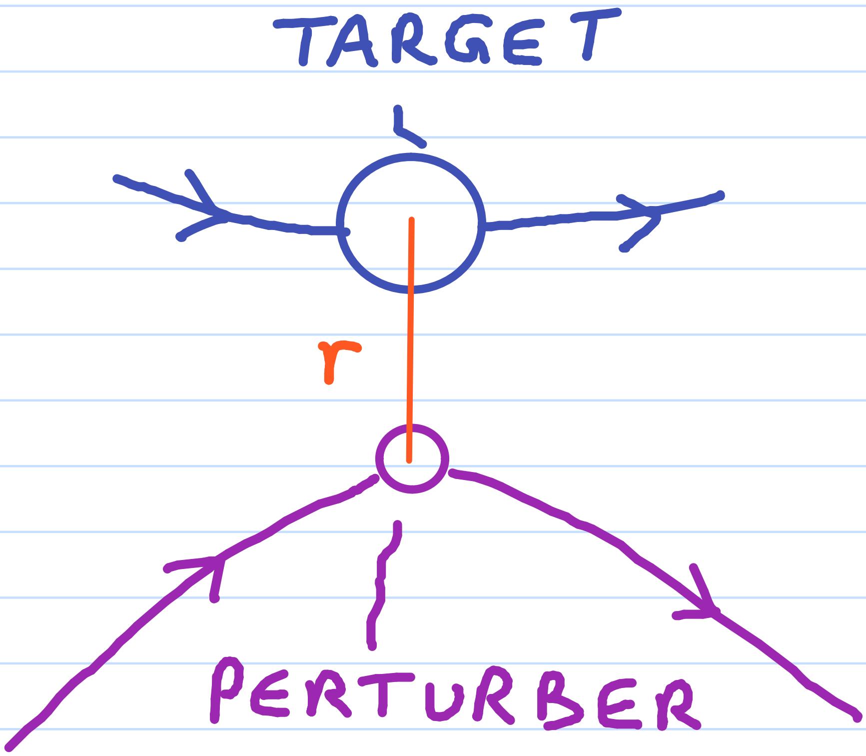
2-LEVEL ATOM, SINGLE COLLISION:



2-LEVEL ATOM: MANY COLLISION, ENSEMBLE REPRENTATIVE



COLLISION: EM INTERACTION



r = MINIMUM SEPARATION

$\phi(r)$ = EM POTENTIAL

n = POWER-LAW INDEX (MONOPOLE, DIPOLE, QUADRUPOLE...)

$$\phi(r) \propto r^{-n}$$

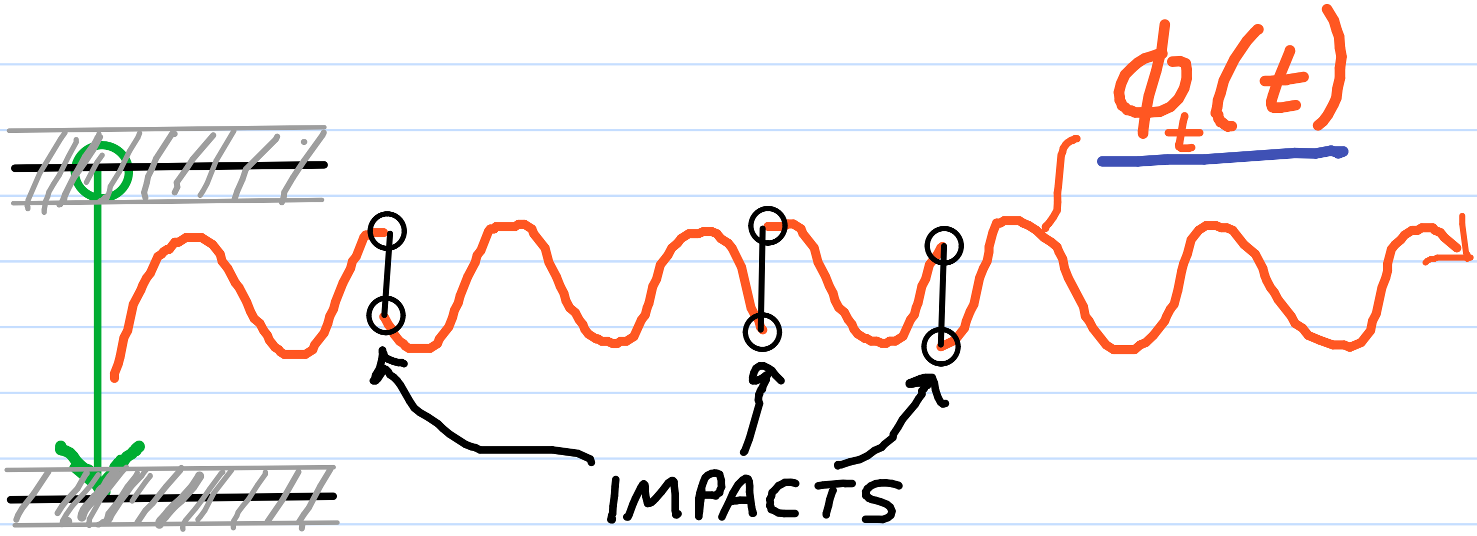
C_n = INTERACTION CONSTANT

$$\text{SHIFT } \Delta V = C_n / r^n$$

δt = MEAN IMPACT DURATION

Δt = MEAN INTERVAL BETWEEN COLLISIONS

IMPACT APPROXIMATION: $\delta t \ll \Delta t$



$$\phi_\nu(\nu-\nu_0) = F_T(\phi_t(t))$$

$$= \frac{\gamma_n/4\pi^2}{(\nu-\nu_0)^2 + (\gamma_n/4\pi)^2}$$ – LORENTZIAN AGAIN!

FWHM $= \gamma_n/2\pi$

COLLISIONAL BROADENING TYPES WELL-DESCRIBED BY IMPACT APPROX.:

- LARGER PERTURBER v
  - LOWER PERT. m
- LARGER n,

1) LINEAR STARK BROADENING:

PERTURBER: e^-

TARGET: HYDROGENIC SPECIES (INC. HI BALMER LINES)

$\therefore$ REGIME: B, A STARS

$n = 2$

$\therefore \Delta\nu = C_2 / r^2$

DAMP. PARAM., γ_2

2) RESONANCE (SELF) BROADENING:

PERTURBER: H I

TARGET: H I

∴ REGIME: GK STARS

$n = 3$

$\therefore \Delta\nu = C_3/r^3$

DAMP. PARAM., γ_3

3) QUADRATIC STARK BROADENING:

PERTURBER: e^-, H II

TARGET: NON-HYDROGENIC SPECIES

$\therefore$ REGIME: B, A STARS

$n = 4$

$\therefore \Delta\nu = C_4 / r^4$

DAMP. PARAM., γ_4

4) VAN DER WAALS BROADENING: (VdW)

PERTURBER: H I

TARGET: NON-HYDROGENIC SPECIES

$\therefore$ REGIME: GK STARS

$n = 6$

$\therefore \Delta\nu = C_6/r^6$

DAMP. PARAM., γ_6 (γ_{vw})

Eg. VdW DAMPING DOMINATES BROAD LORENTZIAN WINGS OF MANY FRAUNHOFER LINES IN SOLAR SPECTRUM

- Ca II HK, Mg II hk, Na I D LINES

TOTAL LORENTZIAN DAMPING :

$$\phi_\nu = \phi_\nu^R * \phi_\nu^{n=2} * \phi_\nu^{n=3} * \phi_\nu^{n=4} * \phi_\nu^{n=6}$$

$$= \frac{\gamma/4\pi^2}{(\nu-\nu_0)^2 + \left(\gamma/4\pi\right)^2}$$

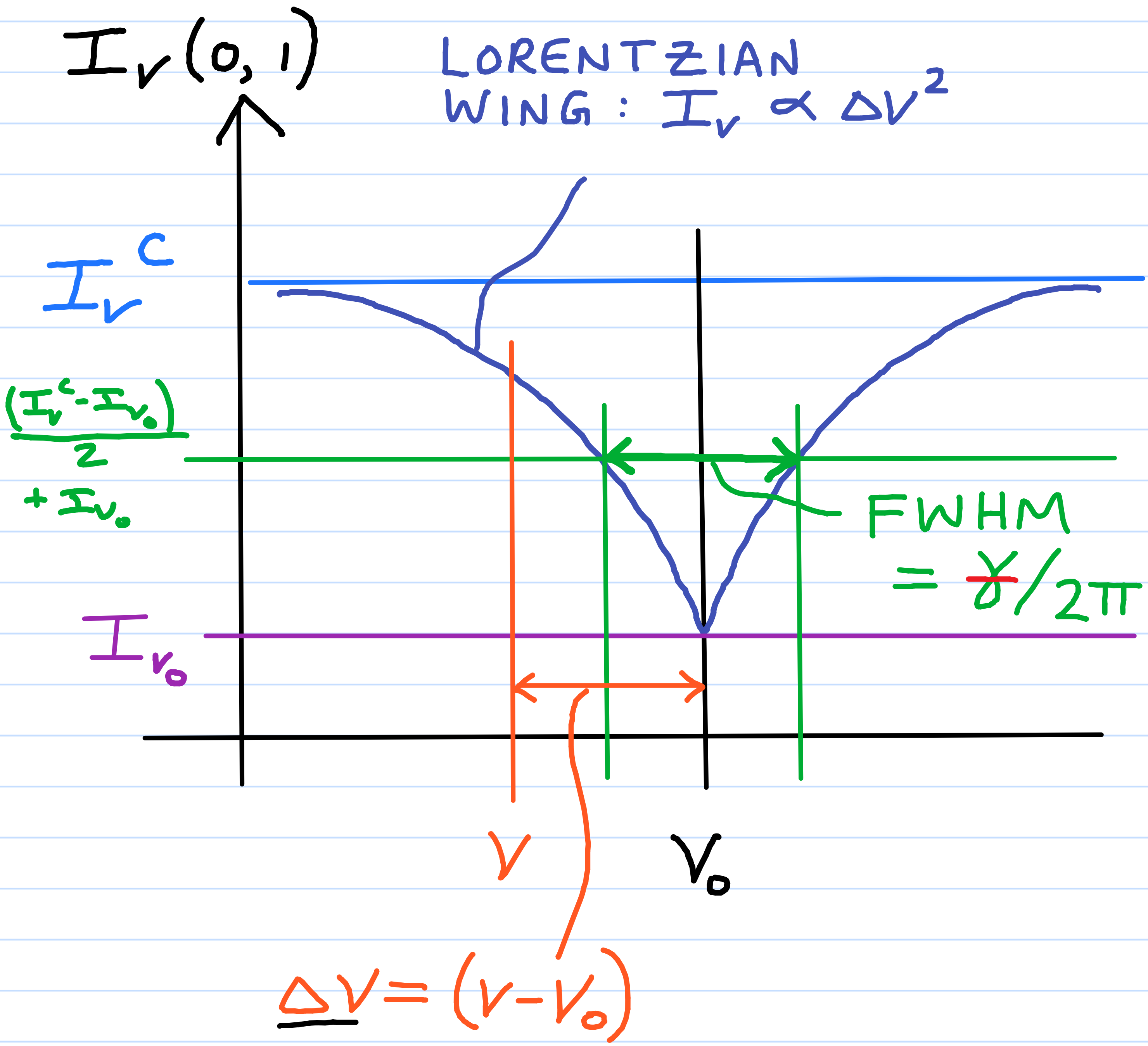
– STILL LORENTZIAN !

γ = TOTAL DAMPING PARAMETER

$$= \gamma^R + (\gamma_2 + \gamma_3 + \gamma_4 + \gamma_6)$$

$$= \gamma^R + \gamma^C$$

EMERGENT LINE PROFILE FROM SINGLE REPRENTATIVE ATOM AT REST :



Eg. Ca II HK LINES, SUN:

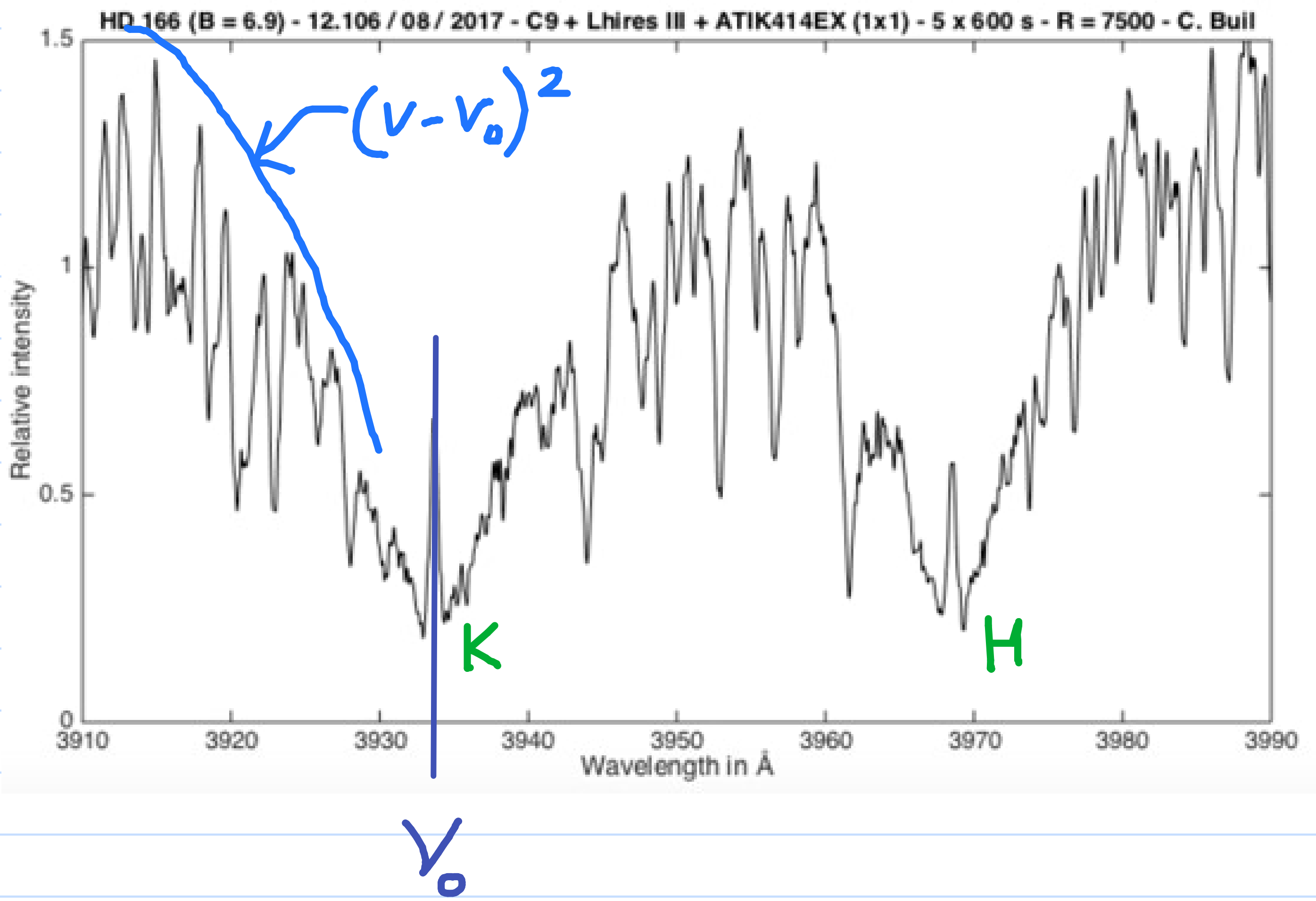


IMAGE CREDIT: ASTROSURF, CHRISTIAN BUIL

SPECIAL EXCEPTION:

LINEAR STARK BROADENING BY HII

PERTURBER: HII (p^+)

TARGET: HYDROGENIC SPECIES
(INC. HI BALMER LINES)

∴ REGIME: B, A STARS
BALMER LINES

$\delta t \gg \Delta t$

∴ NEED STATIC APPROXIMATION

→ $\phi_\nu \approx$ HOLTSMARK PROFILE

- NON-LORENTZIAN