

ATOMIC TRANSITIONS:

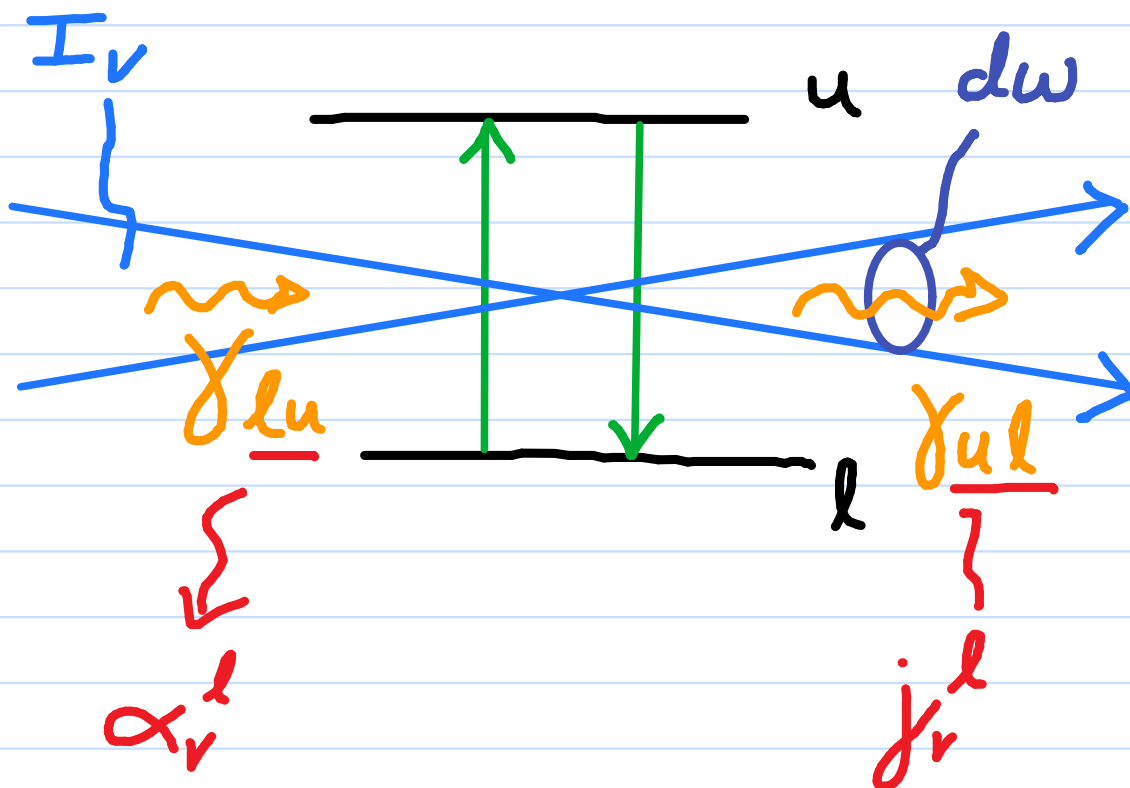
- ORIGIN OF α_ν & j_ν

BOUND-BOUND (b-b) TRANSITIONS
(LINE TRANSITIONS: α_ν^l & j_ν^l)

RADIATIVE b-b TRANSITIONS:

SPECTRAL LINES

SPECIES k :



EINSTEIN COEFFICIENTS FOR

RADIATIVE b-b TRANSITION $l \rightleftharpoons u$:

$$A_{ul}, B_{lu}, B_{ul}$$

- QUANTUM MECH. ATOMIC PARAMETERS FOR TRANSITION PROBABILITY

PERMITTED TRANSITIONS:

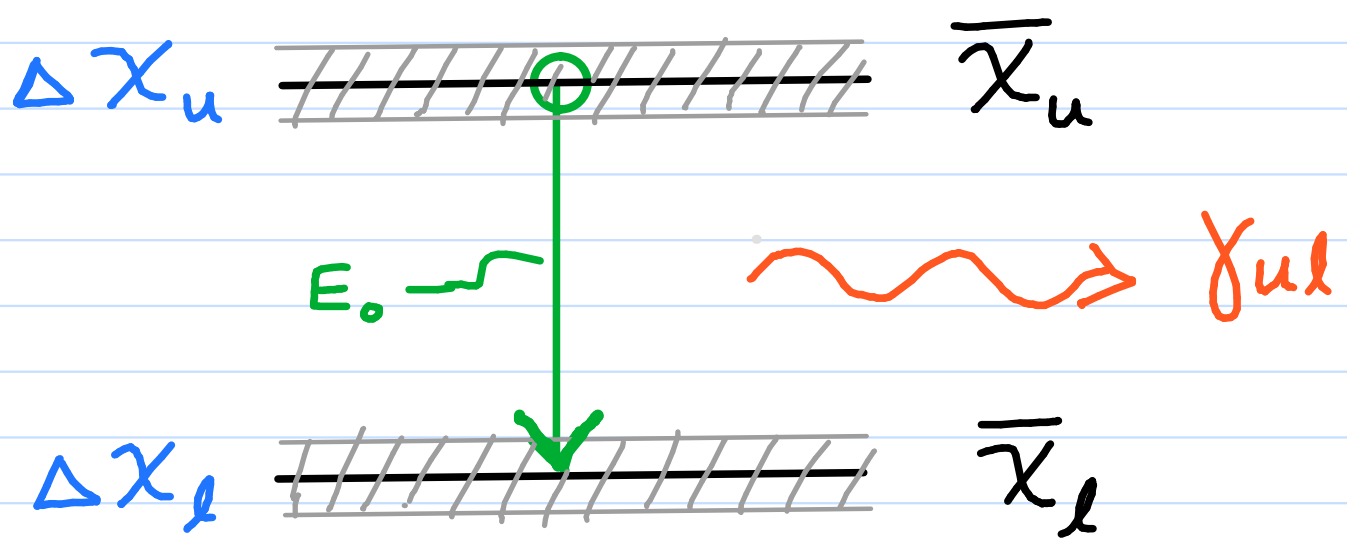
- ALLOWED BY Q.M. SELECTION RULES
- RELATIVELY LARGE A_{ul}, B_{ul}, B_{lu}

FORBIDDEN TRANSITIONS:

- VIOLATE Q.M. SELECTION RULES
- RELATIVELY SMALL A_{ul}, B_{ul}, B_{lu}

SPONTANEOUS RADIATIVE

DE-EXCITATION: $u \rightarrow l$
(SPONTANEOUS EMISSION)



PHOTON $E_0 = h\nu_0 = \bar{\chi}_u - \bar{\chi}_l$

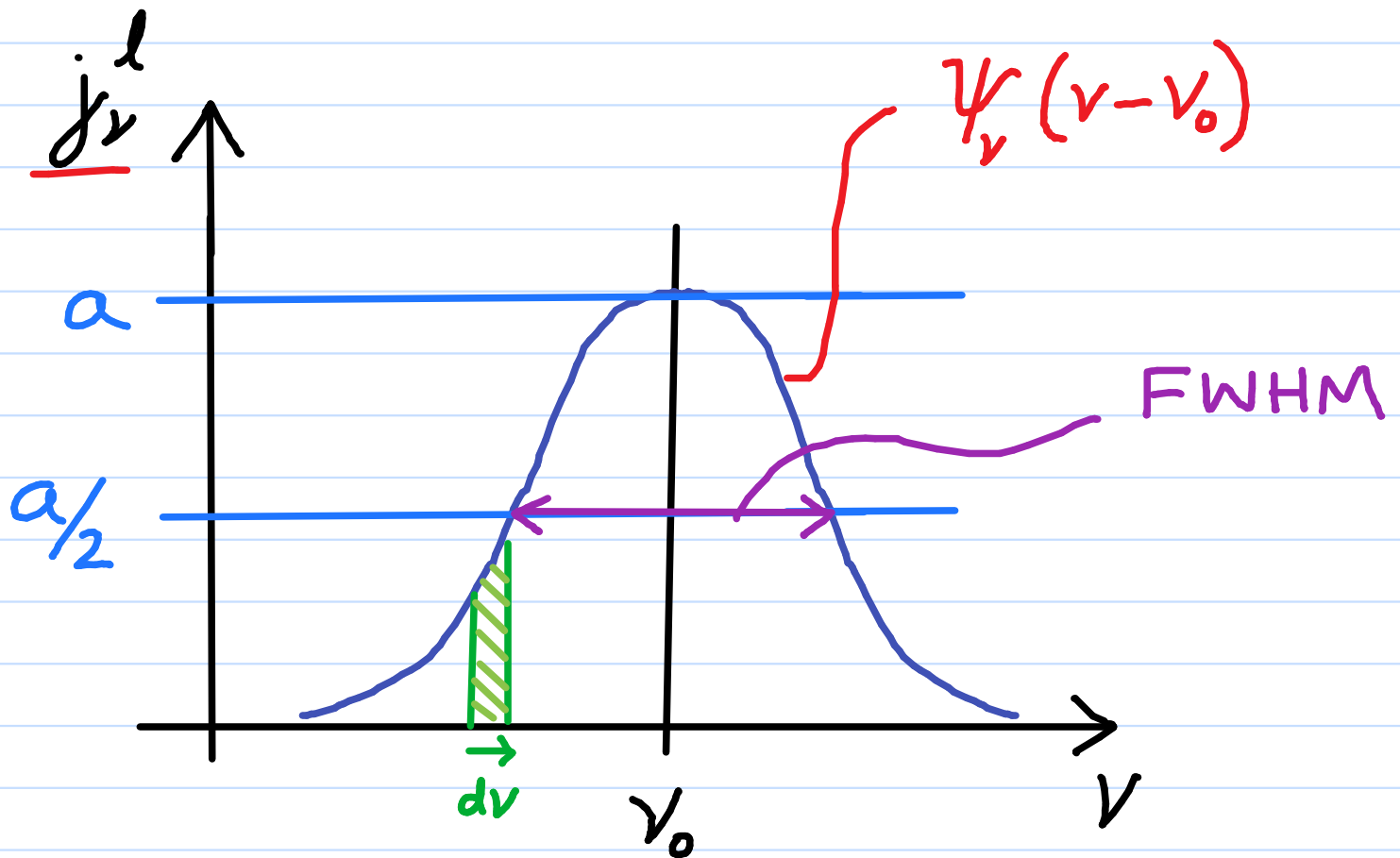
A_{ul} = NO. OF SPONTANEOUS $u \rightarrow l$ s
PER SEC. PER PARTICLE IN
LEVEL u (s^{-1}) (RUTTEN)

$\therefore \underline{n_u} A_{ul} =$ SPONT. $u \rightarrow l$ RATE PER
VOLUME ($s^{-1} \text{ cm}^{-3}$)

SPONT. EMISSION LINE PROFILE,

$$\underline{\Psi_\nu} = \Psi_\nu (\underline{\nu - \nu_0})$$

$$\rightarrow \nu_0 = \frac{\bar{\chi}_u - \bar{\chi}_l}{h}$$



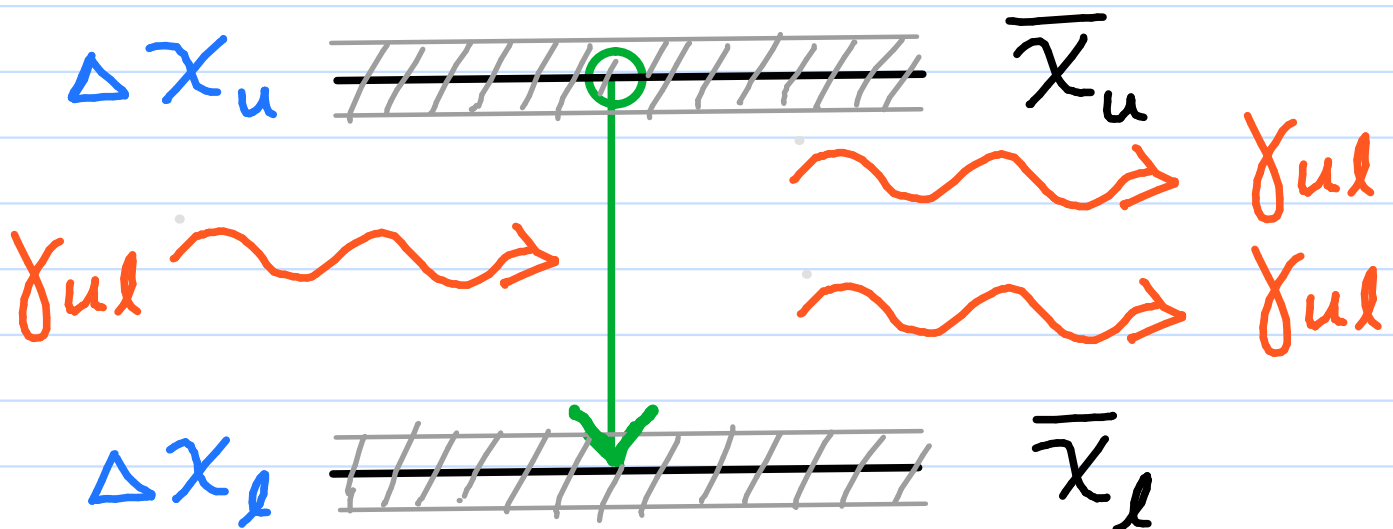
NORMALIZATION: $\int_{\nu=0}^{\infty} \Psi_\nu d\nu = \underline{1}$

$$\therefore A_{ul} = \int_0^{\infty} A_{ul} \psi_r \, dv$$

$$\underline{j_r^l} = n_u A_{ul} \psi_r$$

STIMULATED RADIATIVE

DE-EXCITATION: $u \rightarrow l$
(STIMULATED EMISSION)



Q.M. BOSON CLONING:

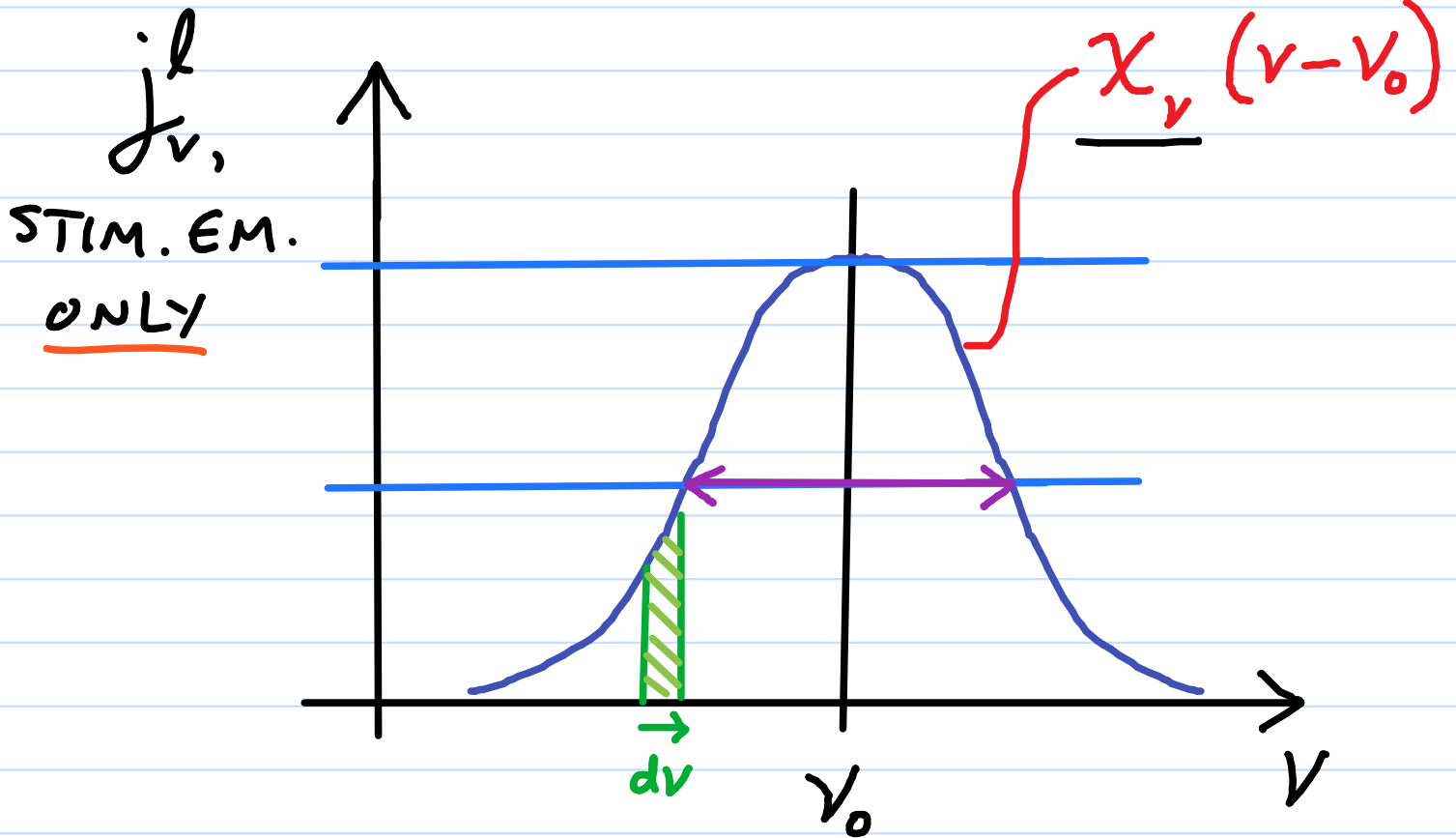
→ STIMULATED γ IDENTICAL TO ORIGINAL γ

- eg. ASTRONOMICAL MASERS

B_{ul} = NO. OF STIMULATED $u \rightarrow l$ s
PER SEC PER PARTICLE IN
LEVEL u PER $\text{erg/s/cm}^2/\text{STER/Hz}$

STIM. EMISSION LINE PROFILE,

$$\underline{x_\nu} = x_\nu (\underline{\nu - \nu_0})$$



NORMALIZATION: $\int_{\nu=0}^{\infty} x_\nu d\nu = \underline{1}$

\therefore NO. OF STIM. $u \rightarrow l$ s PER SEC. PER PARTICLE IN LEVEL u

$$= \underline{B_{ul}} \int_0^{\infty} \underline{J_{\nu}} \chi_{\nu}(\nu - \nu_0) d\nu = B_{ul} \overline{J_{\nu_0}}^x$$

(s⁻¹)

WHERE, $\overline{J_{\nu_0}}^x \equiv \int_0^{\infty} J_{\nu} \chi_{\nu}(\nu - \nu_0) d\nu$

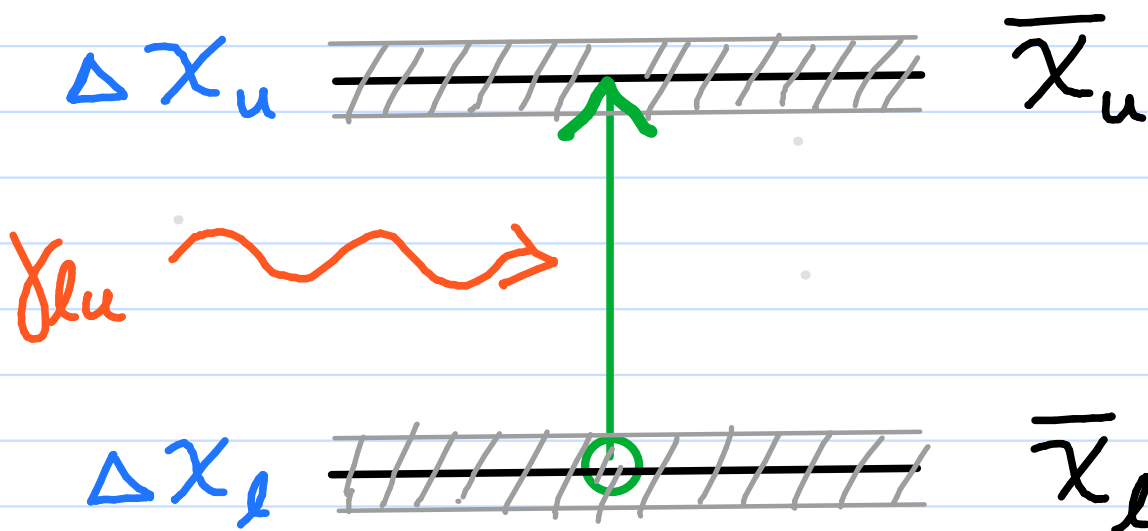
$$\therefore \underline{n_u} B_{ul} \overline{J_{\nu_0}}^x = \frac{\text{STIM. } u \rightarrow l}{\text{RATE}} \underline{\text{VOLUME}} \text{ (s}^{-1} \text{ cm}^{-3}\text{)}$$

∴ TOTAL RADIATIVE $u \rightarrow l$ RATE PER PARTICLE, R_{ul} :

$$R_{ul} = A_{ul} + B_{ul} \bar{J}_\nu^x \quad (\text{s}^{-1})$$

∴ TOTAL RADIATIVE $u \rightarrow l$ VOLUME RATE
 $= \underline{n_u} R_{ul} \quad (\text{s}^{-1} \text{ cm}^{-3})$

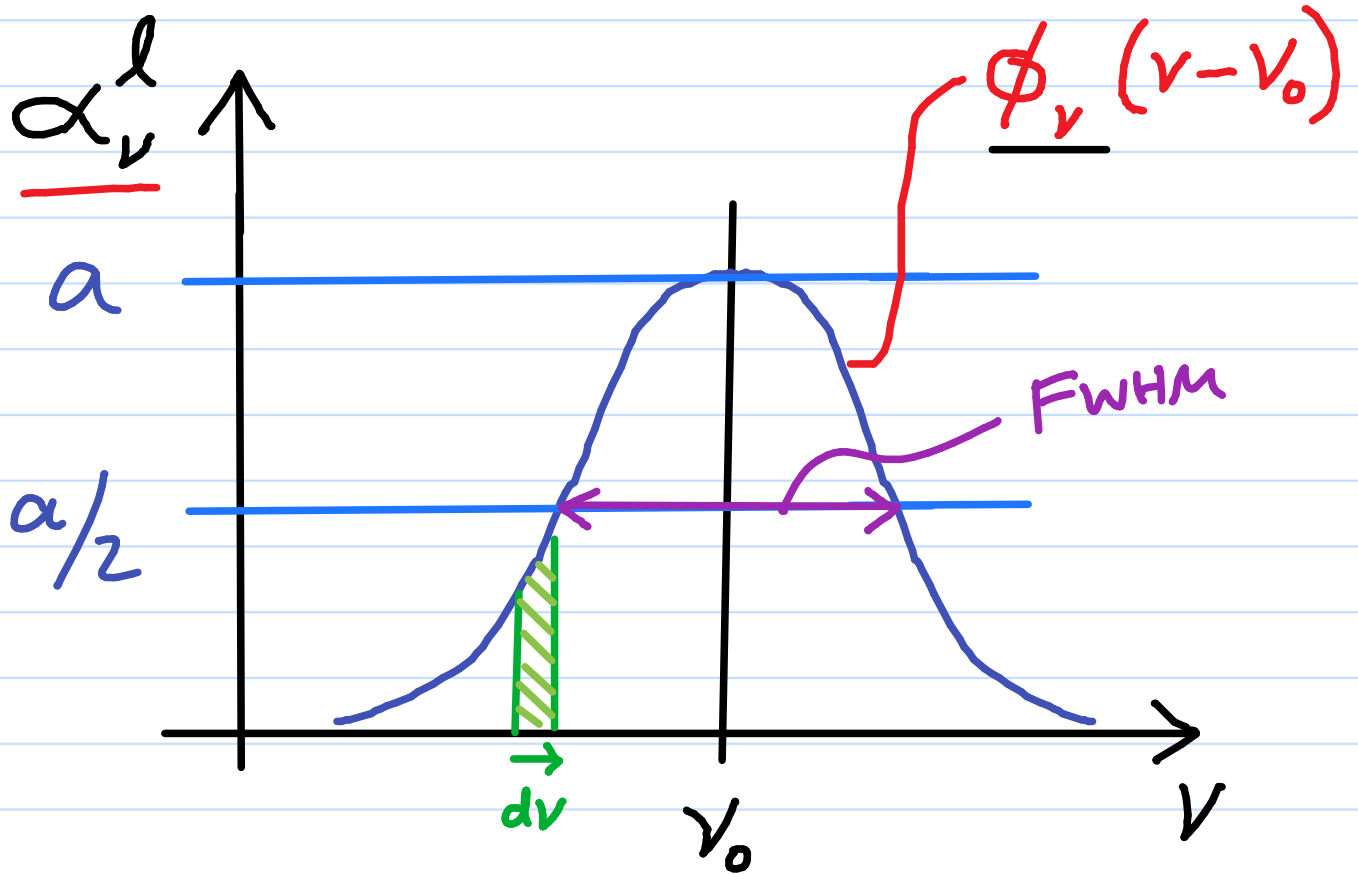
RADIATIVE EXCITATION: $l \rightarrow u$
(EXTINCTION)



B_{lu} = NO. OF STIMULATED $l \rightarrow u$ s
PER SEC, PER PARTICLE IN
LEVEL l PER $\text{erg/s/cm}^2/\text{STER/Hz}$

EXTINCTION LINE PROFILE,

$$\underline{\phi_\nu} = \phi_\nu (\underline{\nu - \nu_0})$$



NORMALIZATION: $\int_{\nu=0}^{\infty} \phi_\nu d\nu = \underline{1}$

\therefore NO. OF $l \rightarrow u$ s PER SEC. PER PARTICLE IN LEVEL l , R_{lu}

$$= \underline{B_{lu}} \int_0^{\infty} J_{\nu} \underline{\phi_{\nu}} d\nu = \underline{B_{lu}} \underline{\overline{J_{\nu_0}^{\phi}}} \quad (\text{s}^{-1})$$

WHERE, $\underline{\overline{J_{\nu_0}^{\phi}}} \equiv \int_0^{\infty} J_{\nu} \phi_{\nu} (\nu - \nu_0) d\nu$

$$\therefore n_l B_{lu} \overline{J_{\nu_0}^{\phi}} = l \rightarrow u \text{ VOLUME RATE} \quad (\text{s}^{-1} \text{ cm}^{-3})$$

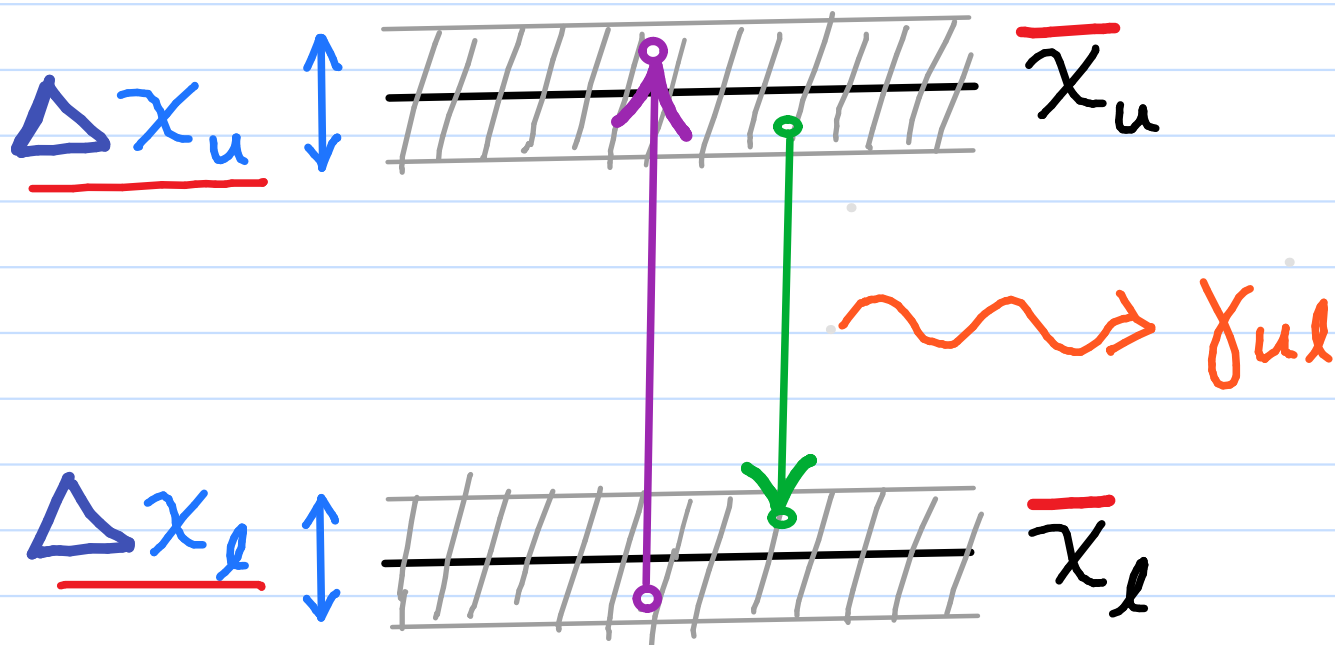
$$\begin{aligned} \therefore \text{TOTAL RADIATIVE } \underline{l \rightarrow u} \text{ VOLUME RATE} \\ = \underline{n_l} \underline{R_{lu}} = n_l (B_{lu} \overline{J_{\nu_0}^{\phi}}) \\ (\text{s}^{-1} \text{ cm}^{-3}) \end{aligned}$$

LINE PROFILES:

$\phi_\nu(\nu - \nu_0) = \text{EXTINCTION PROFILE}$

$\psi_\nu(\nu - \nu_0) = \text{SPONT. EMISSION PROFILE}$

$\chi_\nu(\nu - \nu_0) = \text{STIM. EMISSION PROFILE}$

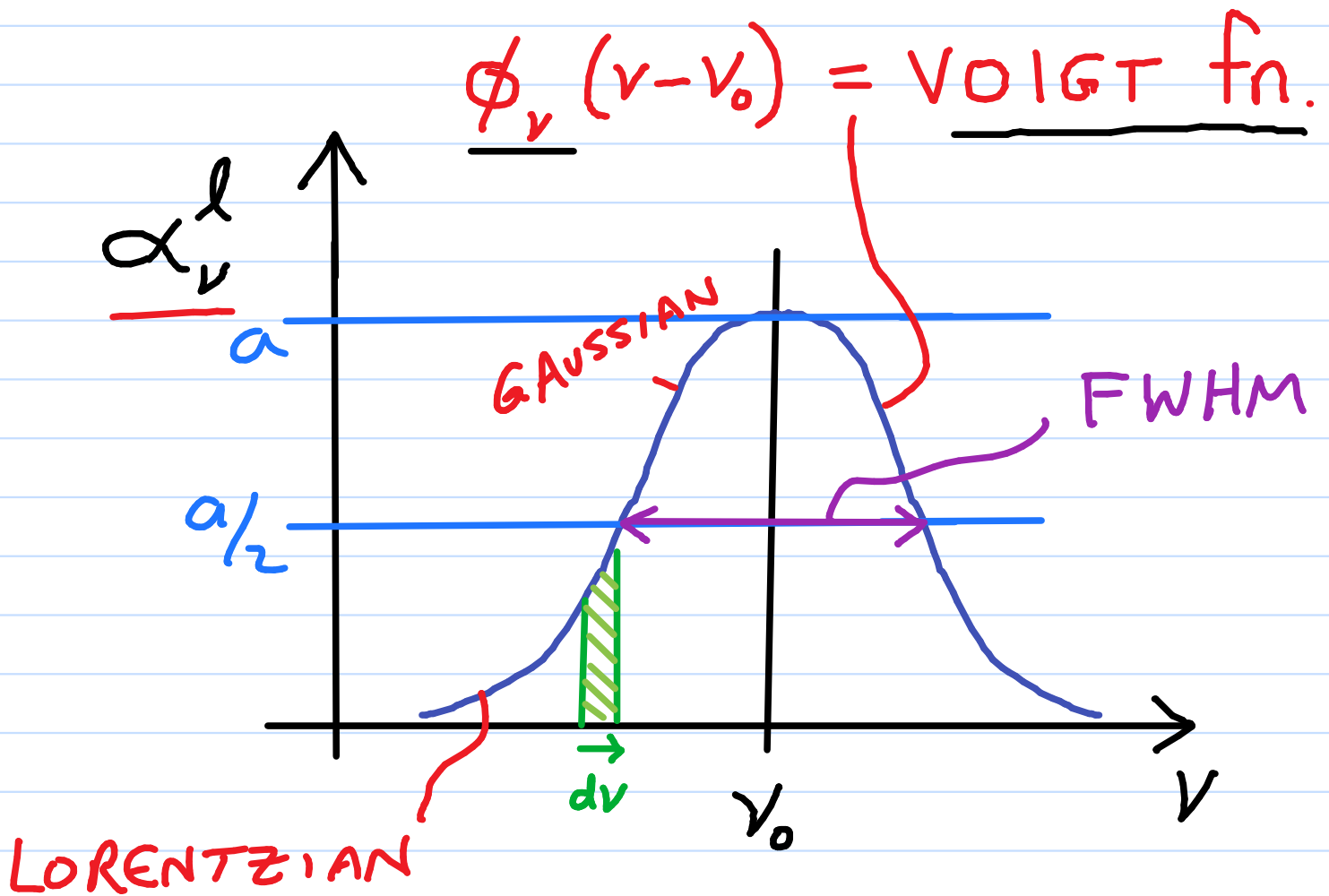


LINE CENTRE, $\nu_0 = \frac{\bar{\chi}_u - \bar{\chi}_l}{h}$

$\phi_\nu(\nu - \nu_0) \neq \delta(\nu - \nu_0)$

Ex. EXTINCTION ($l \rightarrow u$):

$$\phi_\nu(\nu - \nu_0) = \text{VOIGT } f_n \neq \delta(\nu - \nu_0)$$



NORMALIZATION: $\int_{\nu=0}^{\infty} \phi_\nu d\nu = \underline{1}$

GENERALLY, FOR TRANSITION $l \rightleftharpoons u$:

$$\phi_\nu \neq \psi_\nu \neq \chi_\nu$$

COMPLETE FREQ. REDISTRIBUTION (CRD)

APPROXIMATION:

$\phi_\nu, \psi_\nu, \chi_\nu$ UNCORRELATED

\therefore ASSUME $\psi_\nu = \chi_\nu = \underline{\underline{\phi_\nu}}$

\therefore ONLY NEED ϕ_ν ($\nu - \nu_0$)

CRD A GOOD APPROX. FOR MOST
SPECTRAL LINES IN STELLAR
ATMOSPHERES

IN ISM (LOW ρ):

- IN HIGH A_{ul} TRANSITIONS

- COHERENT SCATTERING APPROX:

$$\Rightarrow \psi_\nu \neq \chi_\nu \neq \phi_\nu$$

PARTIAL FREQ. REDISTRIBUTION (PRD):

- MORE REALISTIC

- DIFFICULT

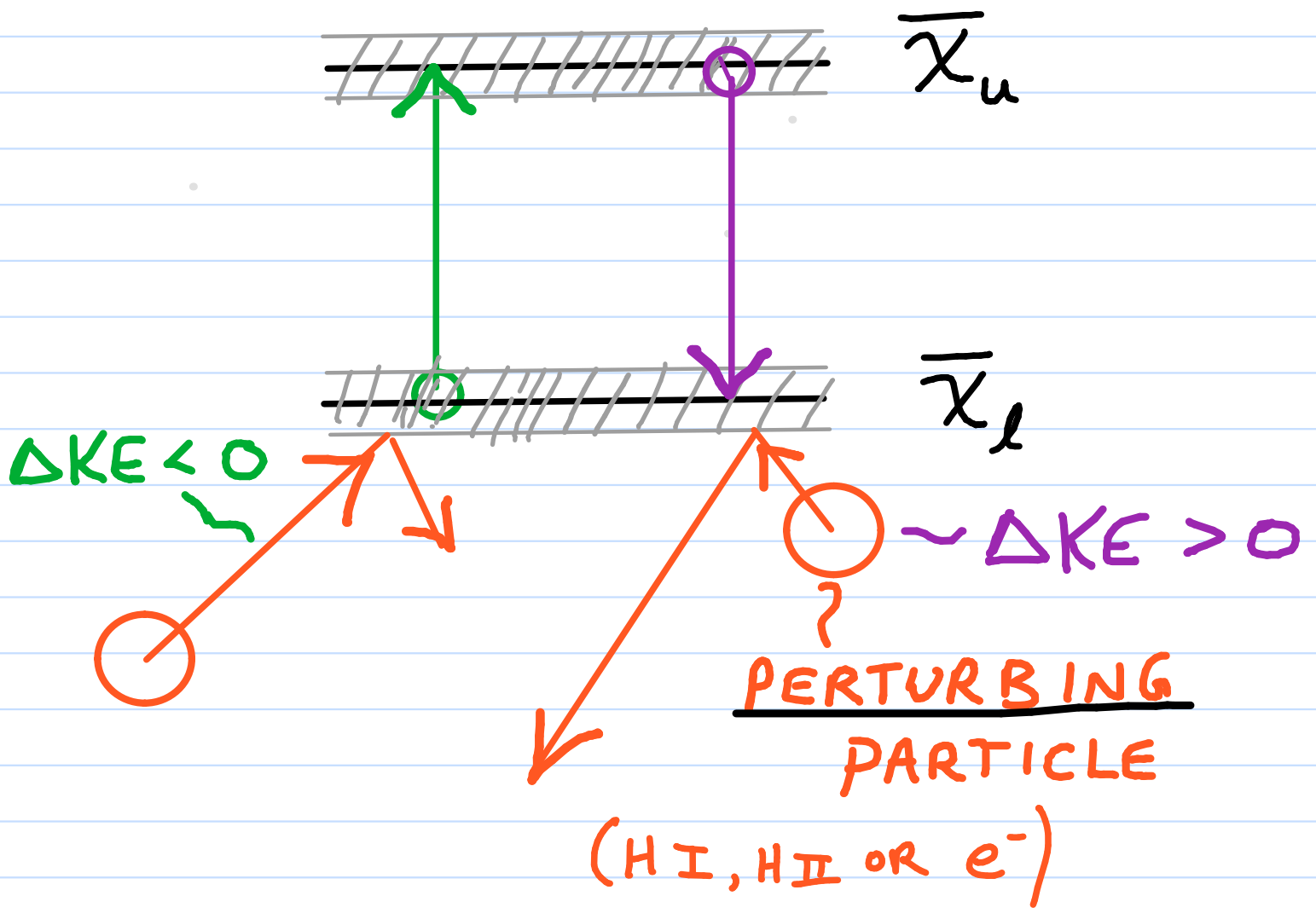
- $\psi_\nu \neq \chi_\nu \neq \phi_\nu$

- eg.

NEEDED FOR SOLAR Ca II HK
& Mg II hk LINE CORES

COLLISIONAL b-b TRANSITIONS $l \rightleftharpoons u$:

- AFFECTS n_l & n_u



"COLLISION": EM INTERACTION

COLLISIONAL EINSTEIN COEFFICIENTS

C_{lu} \equiv NO. OF COLLISIONAL $l \rightarrow u$ s PER SEC. PER PARTICLE IN LEVEL l (s^{-1})

C_{ul} \equiv NO. OF COLLISIONAL $u \rightarrow l$ s PER SEC. PER PARTICLE IN LEVEL u (s^{-1})

TOTAL VOLUME TRANSITION RATES:
($s^{-1} \text{ cm}^{-3}$)

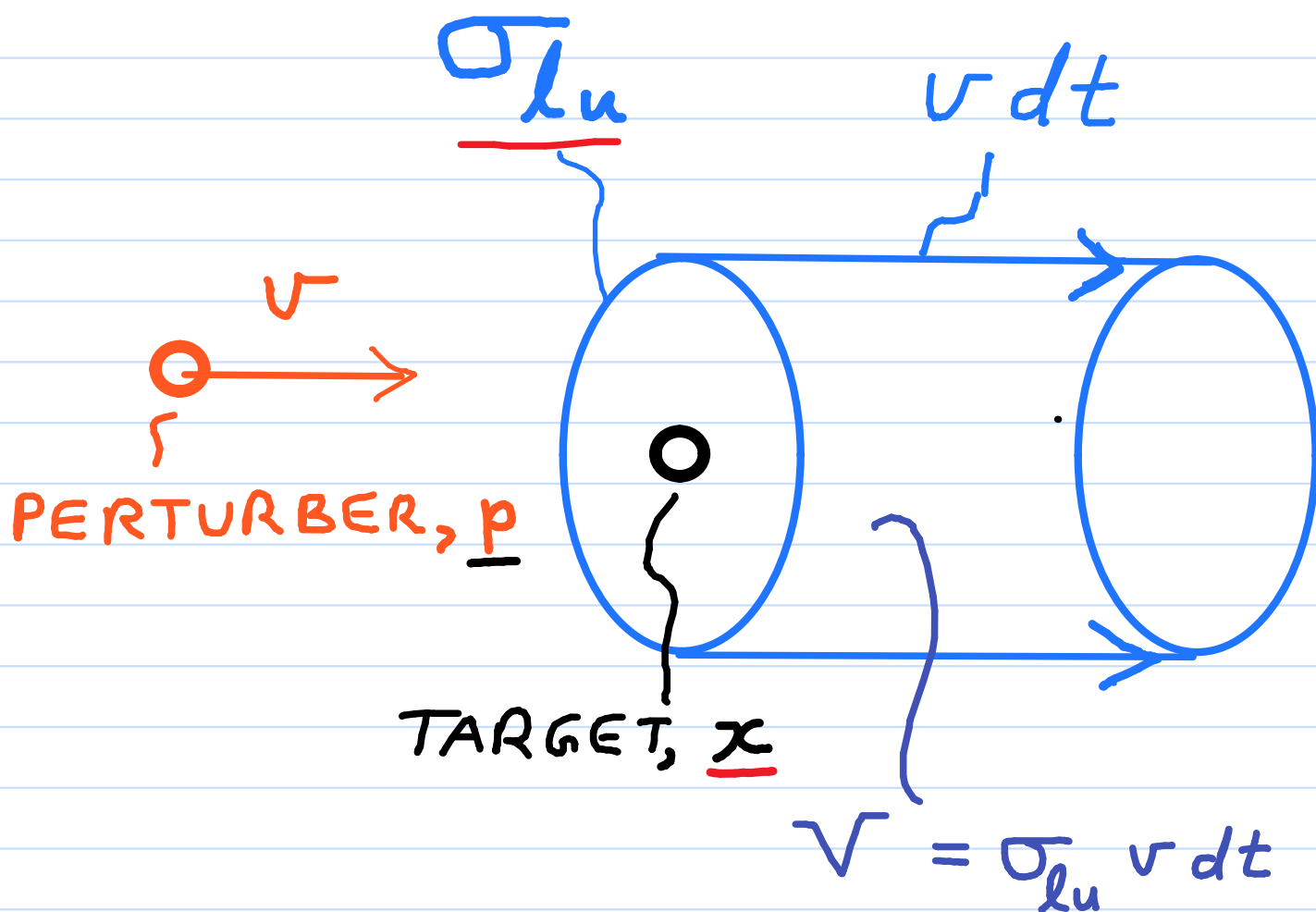
$u \rightarrow l$:

$$\underline{n_u} (R_{ul} + C_{ul}) = n_u (A_{ul} + B_{ul} \bar{J}_\nu^{\lambda} + C_{ul})$$

$l \rightarrow u$:

$$\underline{n_l} (R_{lu} + C_{lu}) = n_l (B_{lu} \bar{J}_\nu^{\phi} + C_{lu})$$

FOR COLLISIONAL EXCITATION
 $l \rightarrow u$:



FOR PERTURBER, p , OF SPEED v OF
 NO. DENSITY $\underline{n_{p,v}}$:

$l \rightarrow u$ RATE PER PARTICLE OF x :

$$= \sigma_{lu} v (n_{p,v} \underline{dv}) \quad (s^{-1})$$

$$\therefore \underline{C_{lu}} = \int_{\underline{v=v_0}}^{\infty} \underline{\sigma_{lu}(v)} v n_{p,v}(v) dv$$

THRESHOLD, $\underline{v_0}$:

$$\frac{1}{2} m_p \underline{v_0}^2 = \bar{\chi}_u - \bar{\chi}_l$$

$n_{p,v}(v) = \underline{M-B}$ DISTRIBUTION

IMPORTANT PERTURBERS (p):

FGKM STARS: HI (He I)

OBA STARS: e^- , HII (He II)

EINSTEIN RELATIONS:

FOR TRANSITION $l \leftrightarrow u$:

$$\frac{B_{lu}}{B_{ul}} = \frac{g_u}{g_l}$$

$$\frac{A_{ul}}{B_{ul}} = \frac{2h\nu_0^3}{c^2}$$

$$\therefore A_{ul} = B_{lu} \frac{g_l}{g_u} \cdot \frac{2h\nu_0^3}{c^2}$$

AND FOR PERTURBER p:

$$\begin{aligned}\frac{C_{lu}}{C_{ul}} &= \frac{g_u}{g_l} e^{-(\bar{\nu}_u - \bar{\nu}_l)/kT} \\ &= \frac{g_u}{g_l} e^{-\underline{h\nu_0}/kT}\end{aligned}$$

IN PRACTICE:

ONLY A_{ul} & C_{lu} PROVIDED