ATOMIC TRANSITIONS:

- ORIGIN OF α_ν & j_ν

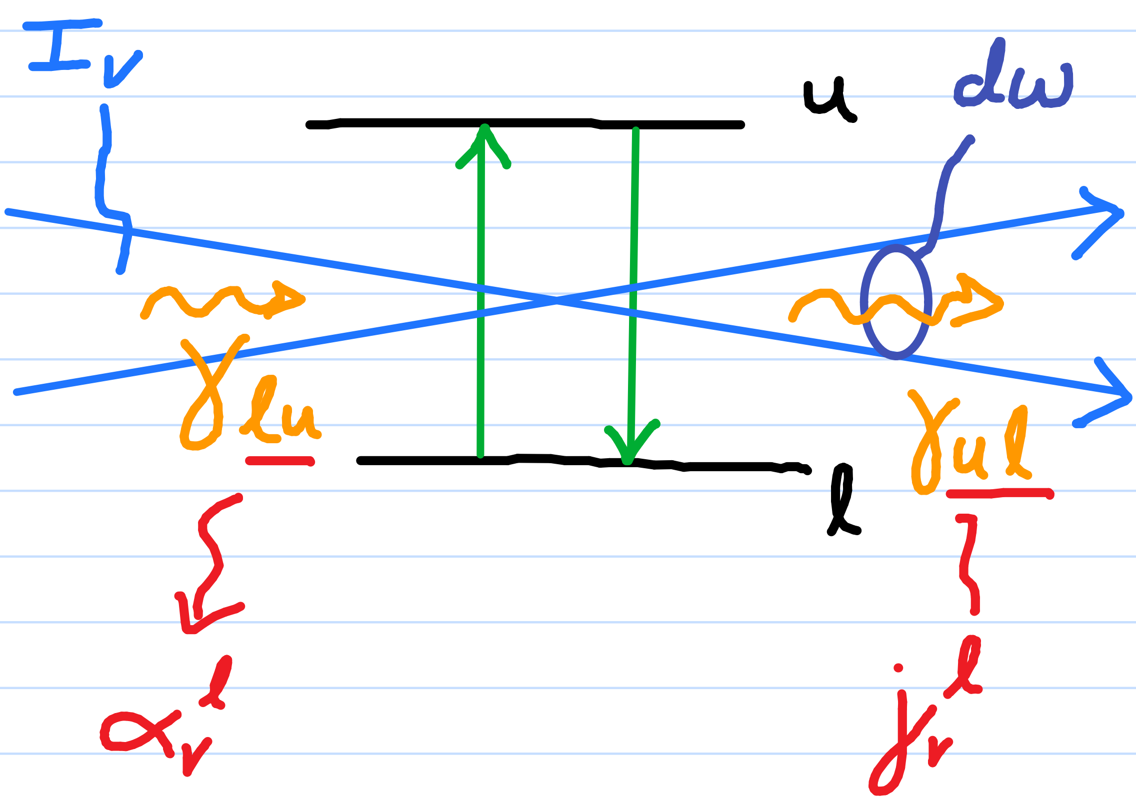
BOUND-BOUND (b-b) TRANSITIONS

(LINE TRANSITIONS: α_ν^{ℓ} & j_ν^{ℓ})

RADIATIVE b-b TRANSITIONS:

SPECTRAL LINES

SPECIES k:



EINSTEIN COEFFICIENTS FOR RADIATIVE b-b TRANSITION $l \rightleftarrows u$:

A_{ul}, B_{lu}, B_{ul}

- QUANTUM MECH. ATOMIC PARAMETERS FOR TRANSITION PROBABILITY

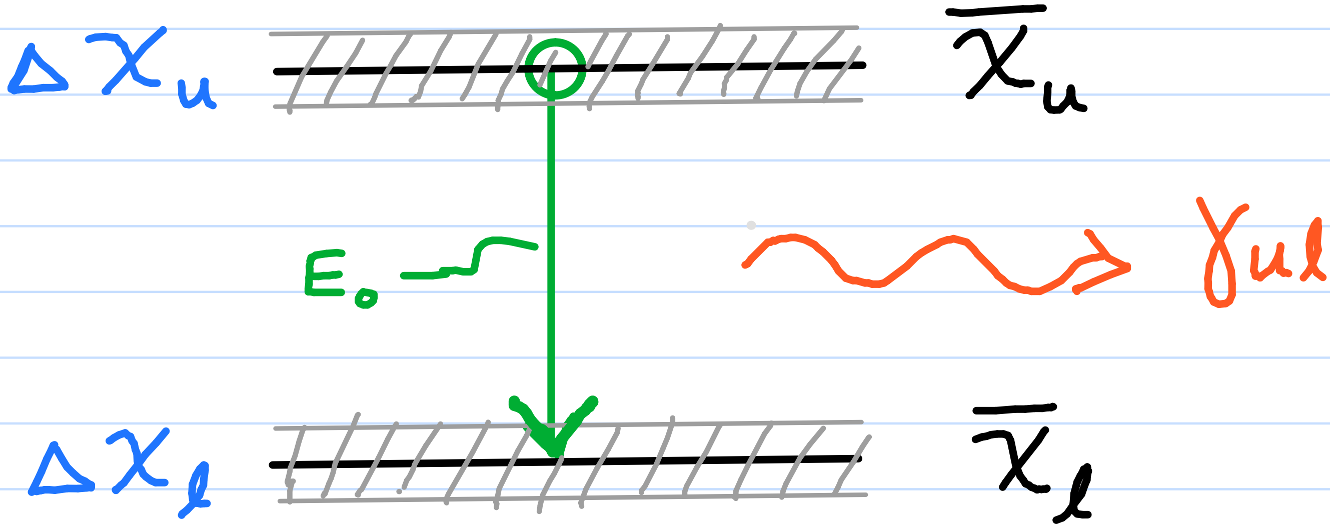
PERMITTED TRANSITIONS:

- ALLOWED BY Q.M. SELECTION RULES
- RELATIVELY LARGE A_{ul}, B_{ul}, B_{lu}

FORBIDDEN TRANSITIONS:

- VIOLATE Q.M. SELECTION RULES
- RELATIVELY SMALL A_{ul}, B_{ul}, B_{lu}

SPONTANEOUS RADIATIVE DE-EXCITATION: $u \rightarrow \ell$
(SPONTANEOUS EMISSION)



PHOTON $E_0 = h\nu_0 = \bar{\chi}_u - \bar{\chi}_\ell$

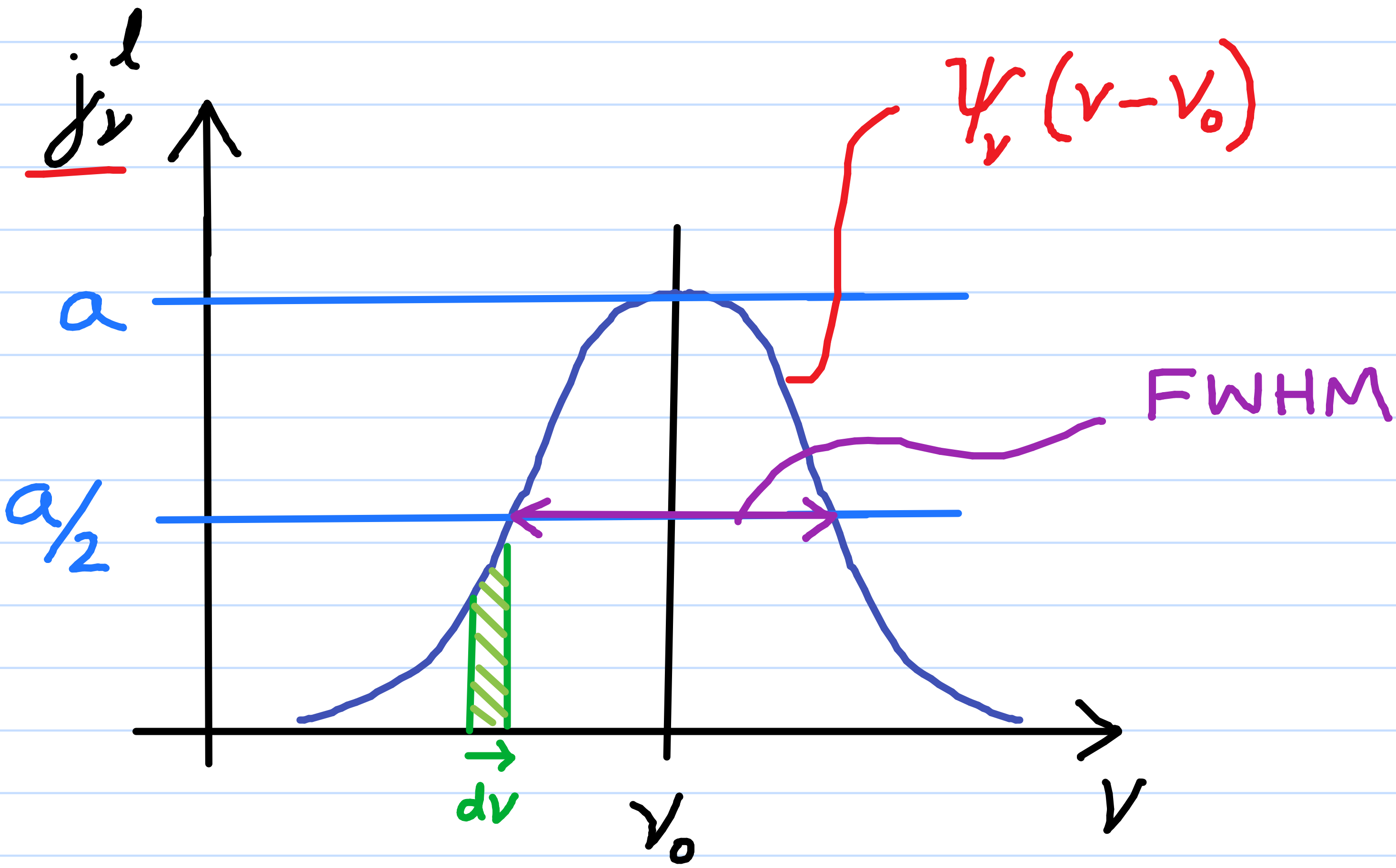
$A_{u\ell}$ = NO. OF SPONTANEOUS $u \rightarrow \ell$s PER SEC. PER PARTICLE IN LEVEL u (s^{-1}) (RUTTEN)

$\therefore n_u A_{u\ell}$ = SPONT. $u \rightarrow \ell$ RATE PER VOLUME $(s^{-1}\ cm^{-3})$

SPONT. EMISSION LINE PROFILE,

$$\psi_\nu = \psi_\nu(\nu - \nu_0)$$

$$\rightarrow \nu_0 = \frac{\chi_u - \chi_\ell}{h}$$



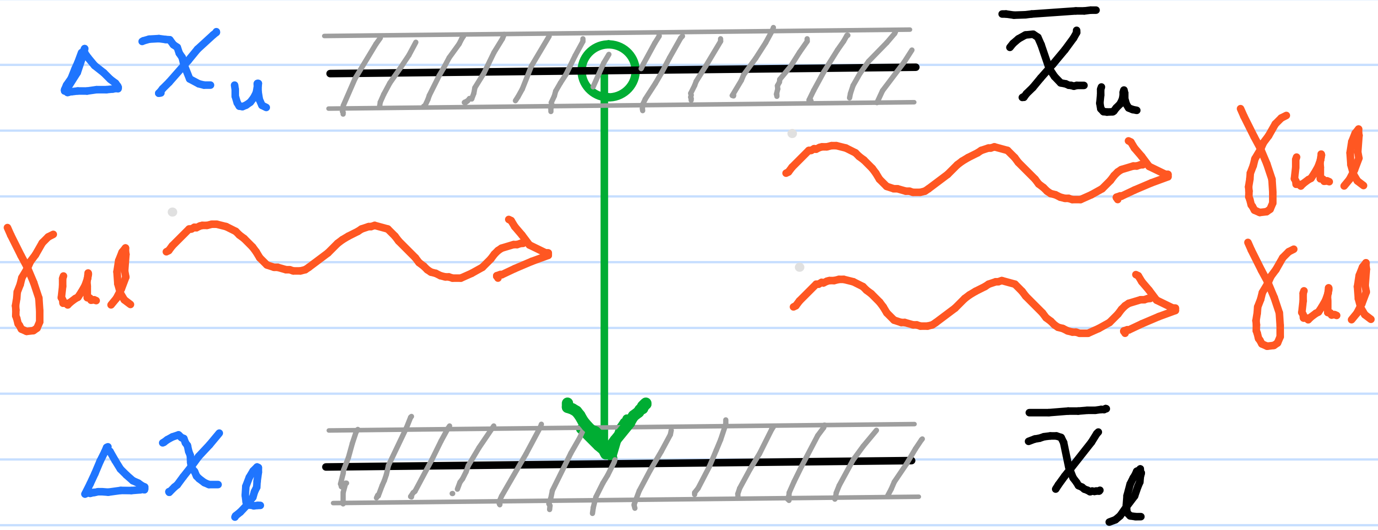
NORMALIZATION:

$$\int_{\nu=0}^{\infty} \psi_\nu \, d\nu = 1$$

$$\therefore A_{ul} = \int_0^\infty A_{ul} \psi_\nu \, d\nu$$

$$j_\nu^l = n_u A_{ul} \psi_\nu$$

STIMULATED RADIATIVE DE-EXCITATION: $u \rightarrow l$
(STIMULATED EMISSION)



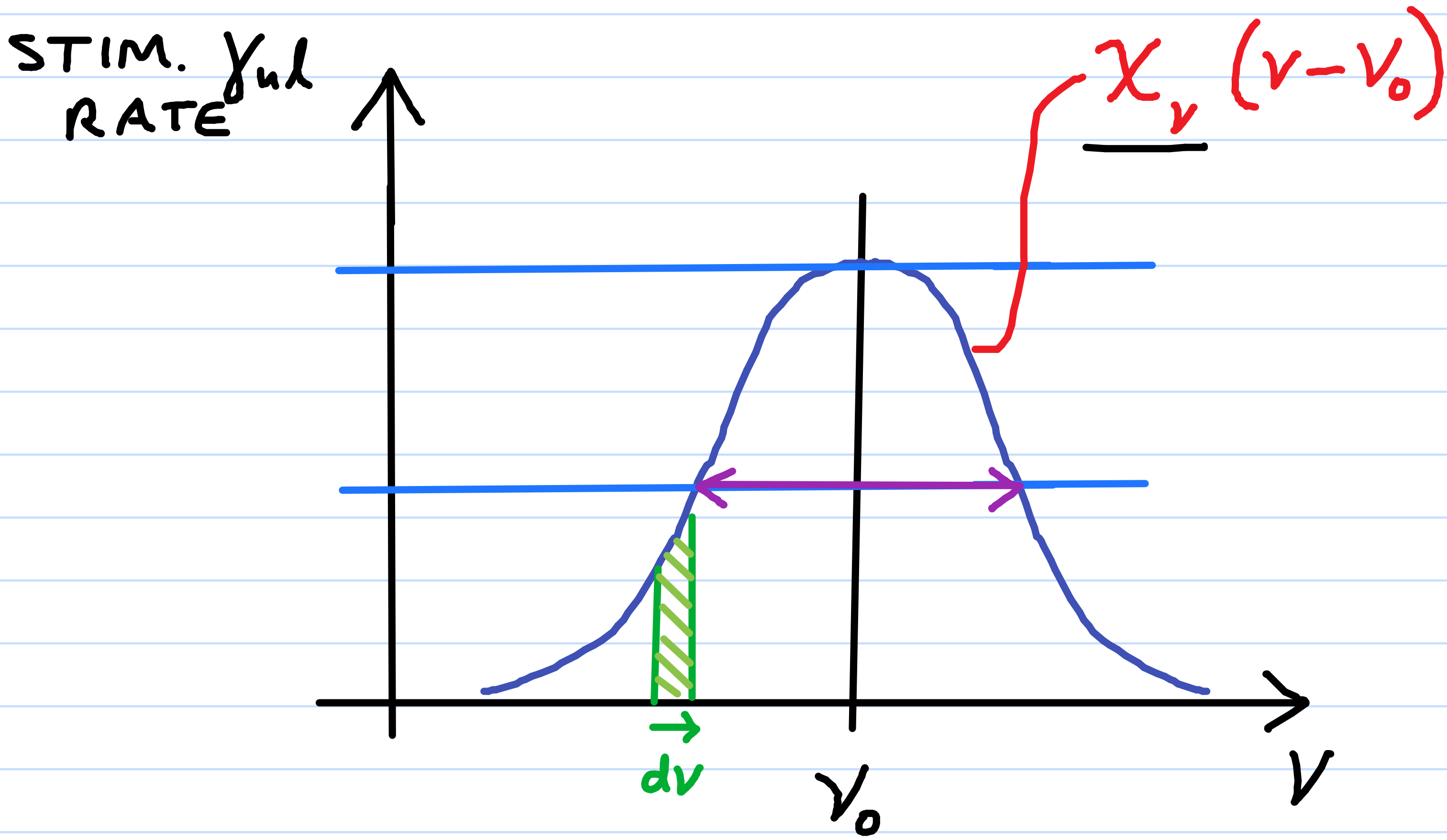
Q.M. BOSON CLONING:
→ STIMULATED γ IDENTICAL TO ORIGINAL γ

- eg. ASTRONOMICAL MASERS

B_{ul} = NO. OF STIMULATED $u \rightarrow l$ s
PER SEC PER PARTICLE IN
LEVEL u PER erg/s/cm^2/STER/Hz

STIM. EMISSION LINE PROFILE,

$$\chi_\nu = \chi_\nu(\nu - \nu_0)$$



NORMALIZATION: $\int_{\nu=0}^{\infty} \chi_\nu \, d\nu = 1$

$\therefore$ NO. OF STIM. $u \rightarrow l$s PER SEC. PER PARTICLE IN LEVEL u

$$= B_{ul} \int_0^\infty J_\nu \, \chi_\nu \, d\nu = B_{ul} \bar{J}_{\nu_0}^{\chi}$$

(s^{-1})

WHERE, $\bar{J}_{\nu_0}^{\chi} \equiv \int_0^\infty J_\nu \, \chi_\nu(\nu - \nu_0) \, d\nu$

$\therefore n_u B_{ul} \bar{J}_{\nu_0}^{\chi}$ = STIM. $u \rightarrow l$ VOLUME RATE $(s^{-1}\ cm^{-3})$

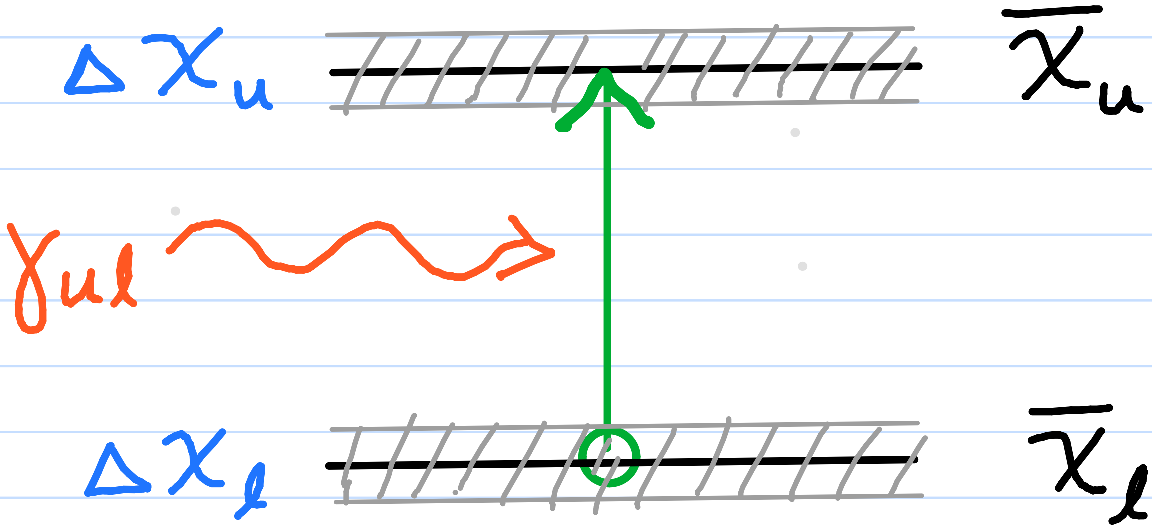
∴ TOTAL RADIATIVE $u \to \ell$ RATE PER PARTICLE, $R_{u\ell}$:

$$R_{u\ell} = A_{u\ell} + B_{u\ell} \bar{J}_{\nu_0}^{\chi} \quad (s^{-1})$$

∴ TOTAL RADIATIVE $u \to \ell$ VOLUME RATE

$$= n_u R_{u\ell} \quad (s^{-1}\ cm^{-3})$$

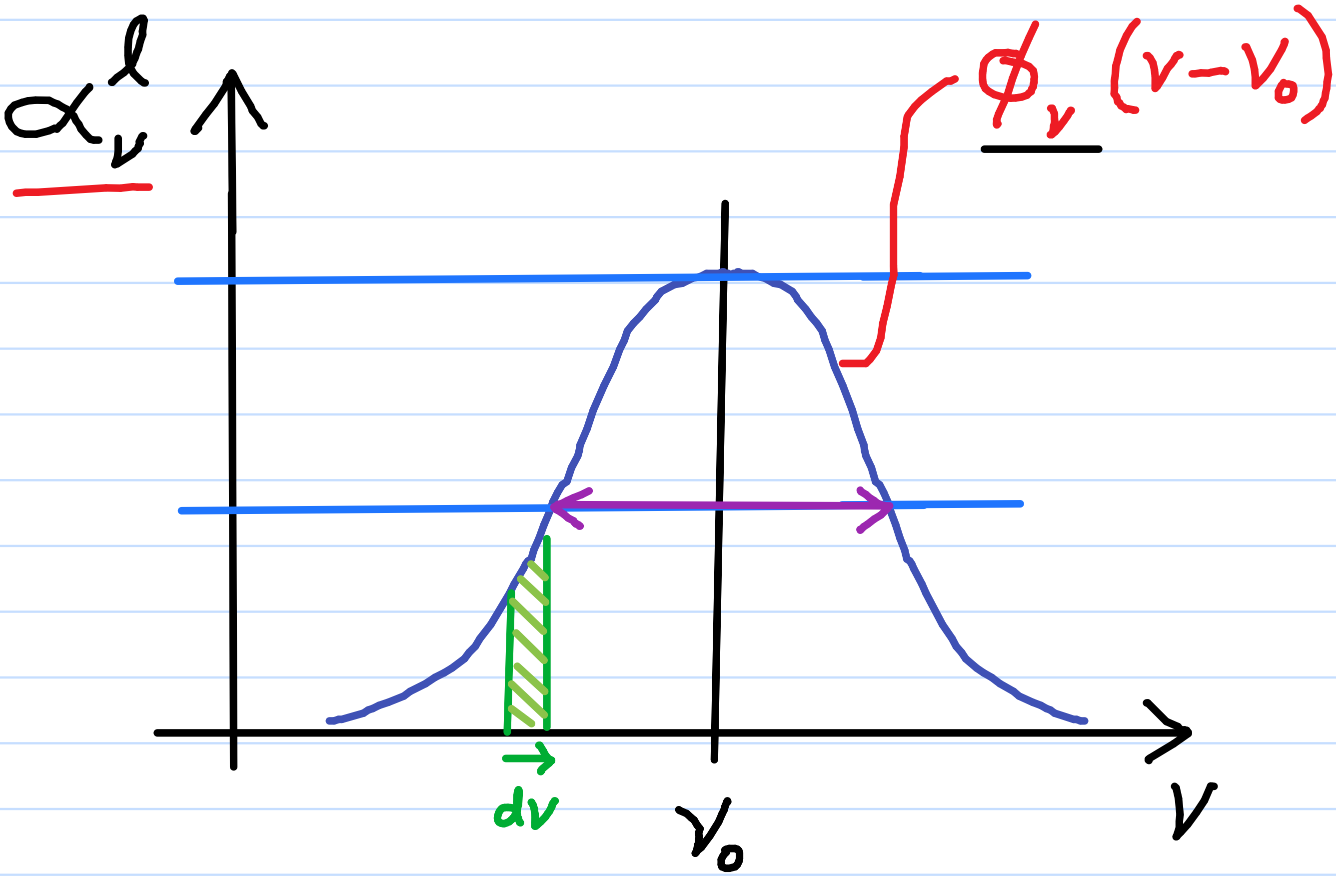
RADIATIVE EXCITATION: $\ell \to u$
(EXTINCTION)



$B_{\ell u}$ = NO. OF STIMULATED $\ell \to u$s PER SEC PER PARTICLE IN LEVEL u PER erg/s/cm^2/STER/Hz

EXTINCTION LINE PROFILE,

$$\phi_\nu = \phi_\nu\,(\nu - \nu_0)$$



NORMALIZATION: $\displaystyle\int_{\nu=0}^{\infty} \phi_\nu \, d\nu = 1$

$\therefore$ NO. OF $l \to u$s PER SEC. PER PARTICLE IN LEVEL l

$$= B_{lu} \int_0^\infty J_\nu \, \phi_\nu \, d\nu = B_{lu} \bar{J}_{\nu_0}^{\phi}$$

(s^{-1})

WHERE, $\bar{J}_{\nu_0}^{\phi} \equiv \int_0^\infty J_\nu \, \phi_\nu(\nu - \nu_0) \, d\nu$

$\therefore$ $= l \to u$ VOLUME RATE

$(s^{-1} \, cm^{-3})$

$\therefore$ TOTAL RADIATIVE $l \to u$ VOLUME RATE

$$= n_l R_{lu} = n_l \left(B_{lu} \bar{J}_{\nu_0}^{\phi} \right)$$

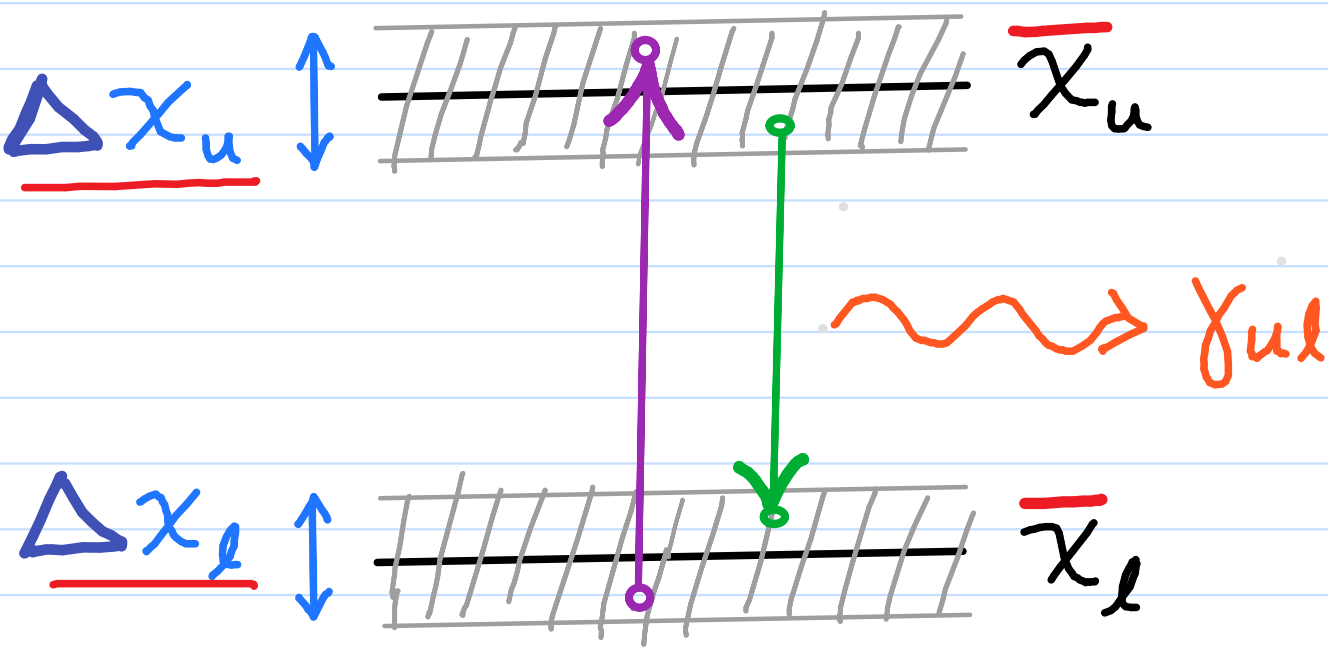
$(s^{-1} \, cm^{-3})$

LINE PROFILES:

$\phi_\nu(\nu-\nu_0)$ = EXTINCTION PROFILE

$\psi_\nu(\nu-\nu_0)$ = SPONT. EMISSION PROFILE

$\chi_\nu(\nu-\nu_0)$ = STIM. EMISSION PROFILE

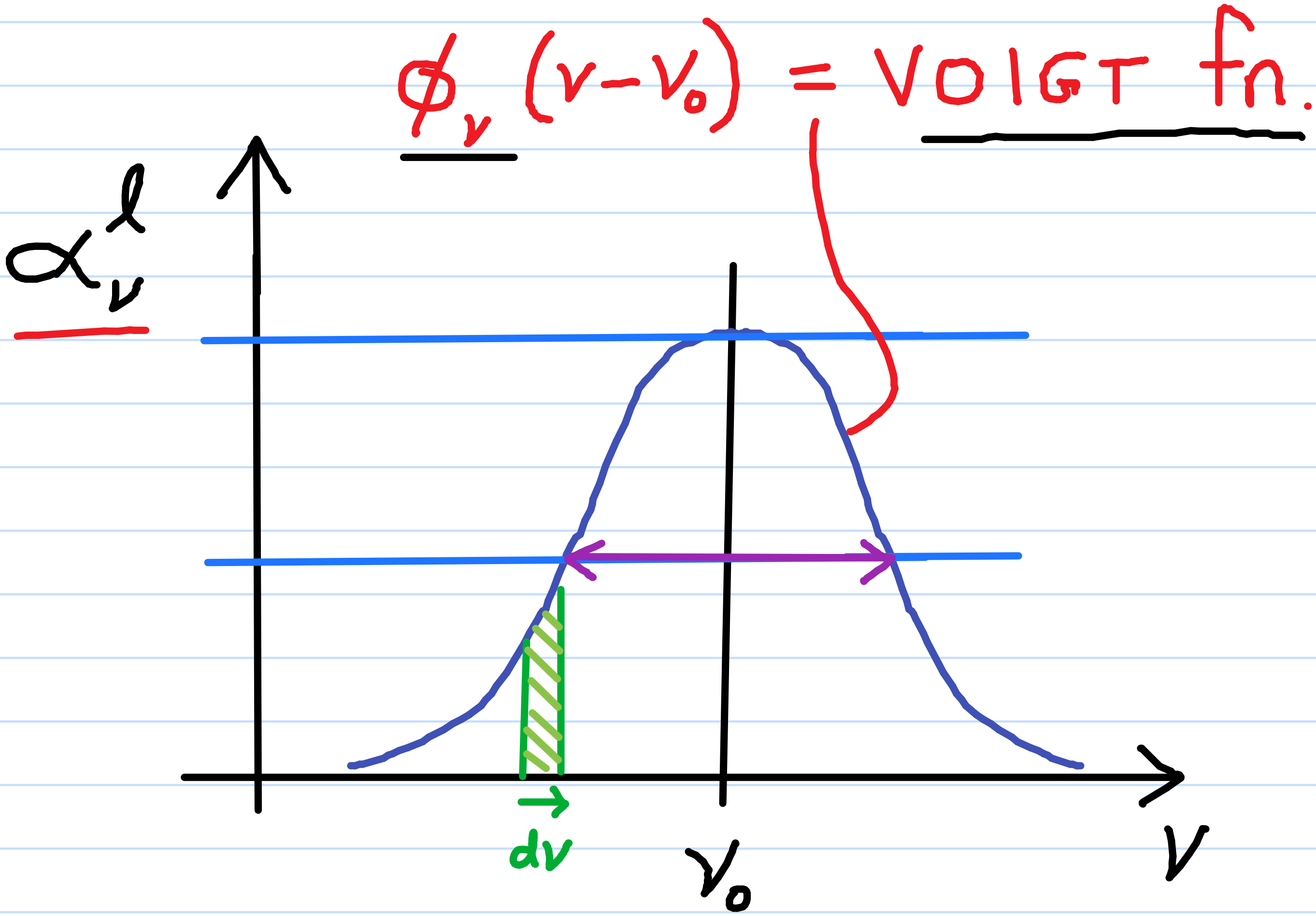


LINE CENTRE, $\nu_0 = \dfrac{\overline{\chi}_u - \overline{\chi}_l}{h}$

$$\phi_\nu(\nu-\nu_0) \neq \delta(\nu-\nu_0)$$

Eg. EXTINCTION $(l \to u)$:

$$\phi_\nu(\nu-\nu_0) = \text{VOIGT fn} \neq \delta(\nu-\nu_0)$$



NORMALIZATION: $\int_{\nu=0}^{\infty} \phi_\nu \, d\nu = 1$

GENERALLY, FOR TRANSITION $l \rightleftarrows u$:

$$\phi_\nu \neq \psi_\nu \neq \chi_\nu$$

COMPLETE FREQ. REDISTRIBUTION (CRD) APPROXIMATION:

$\phi_\nu, \psi_\nu, \chi_\nu$ UNCORRELATED

$\therefore$ ASSUME $\psi_\nu = \chi_\nu = \phi_\nu$

$\therefore$ ONLY NEED $\phi_\nu(\nu - \nu_0)$

CRD A GOOD APPROX. FOR MOST SPECTRAL LINES IN STELLAR ATMOSPHERES

IN ISM (LOW ρ):

- IN HIGH A_{ul} TRANSITIONS
- COHERENT SCATTERING APPROX:

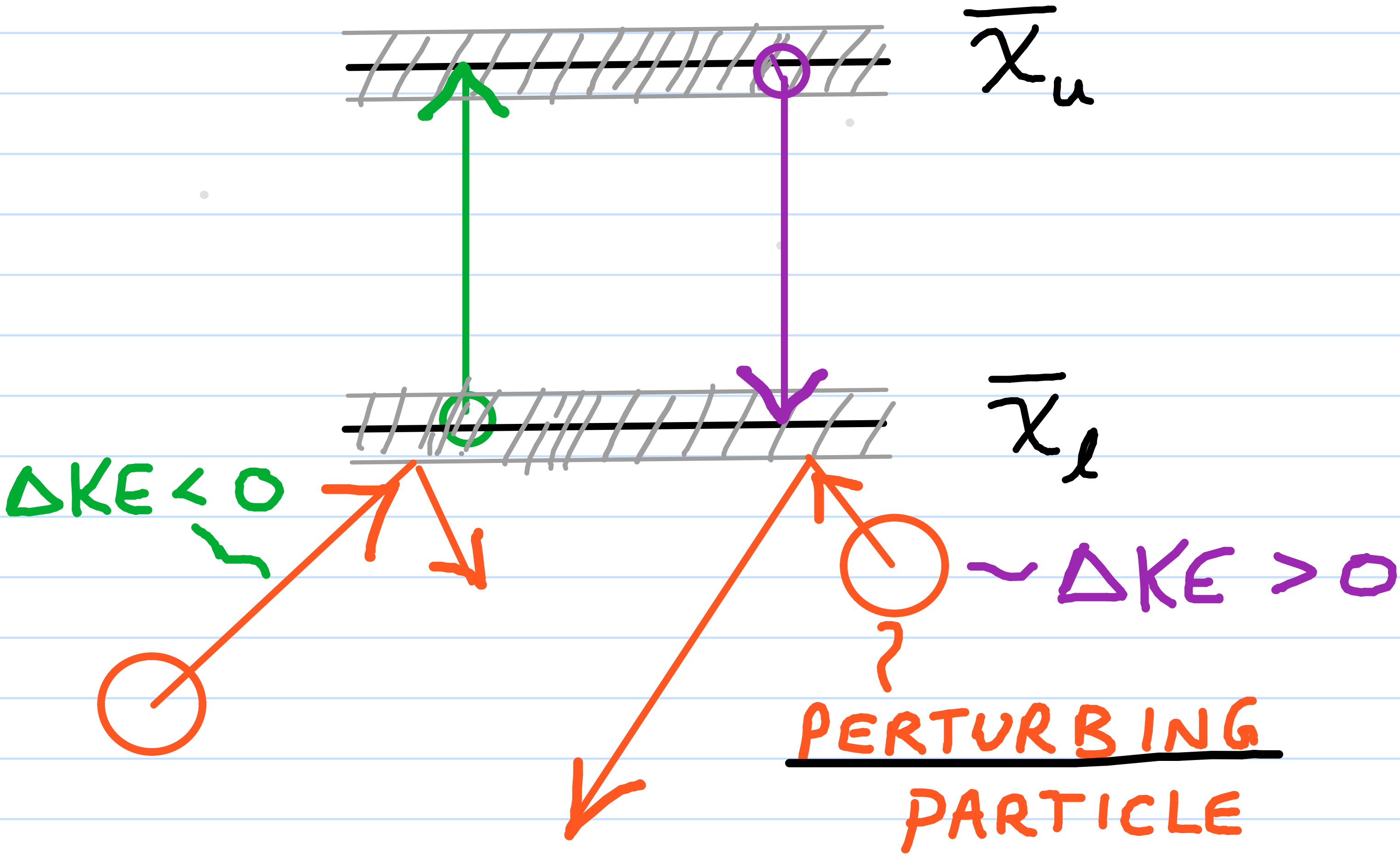
$\Rightarrow \psi_\nu \neq \chi_\nu \neq \phi_\nu$

PARTIAL FREQ. REDISTRIBUTION (PRD):

- MORE REALISTIC
- DIFFICULT
- $\psi_\nu \neq \chi_\nu \neq \phi_\nu$
- Eg. NEEDED FOR SOLAR Ca II HK & Mg II hk LINE CORES

COLLISIONAL b-b TRANSITIONS $l \rightleftarrows u$:

- AFFECTS n_l & n_u



"COLLISION": EM INTERACTION

COLLISIONAL EINSTEIN COEFFICIENTS

$C_{lu} \equiv$ No. OF COLLISIONAL $l \to u$s PER SEC. PER PARTICLE IN LEVEL l
(s^{-1})

$C_{ul} \equiv$ No. OF COLLISIONAL $u \to l$s PER SEC. PER PARTICLE IN LEVEL u
(s^{-1})

TOTAL VOLUME TRANSITION RATES:
($s^{-1}\ cm^{-3}$)

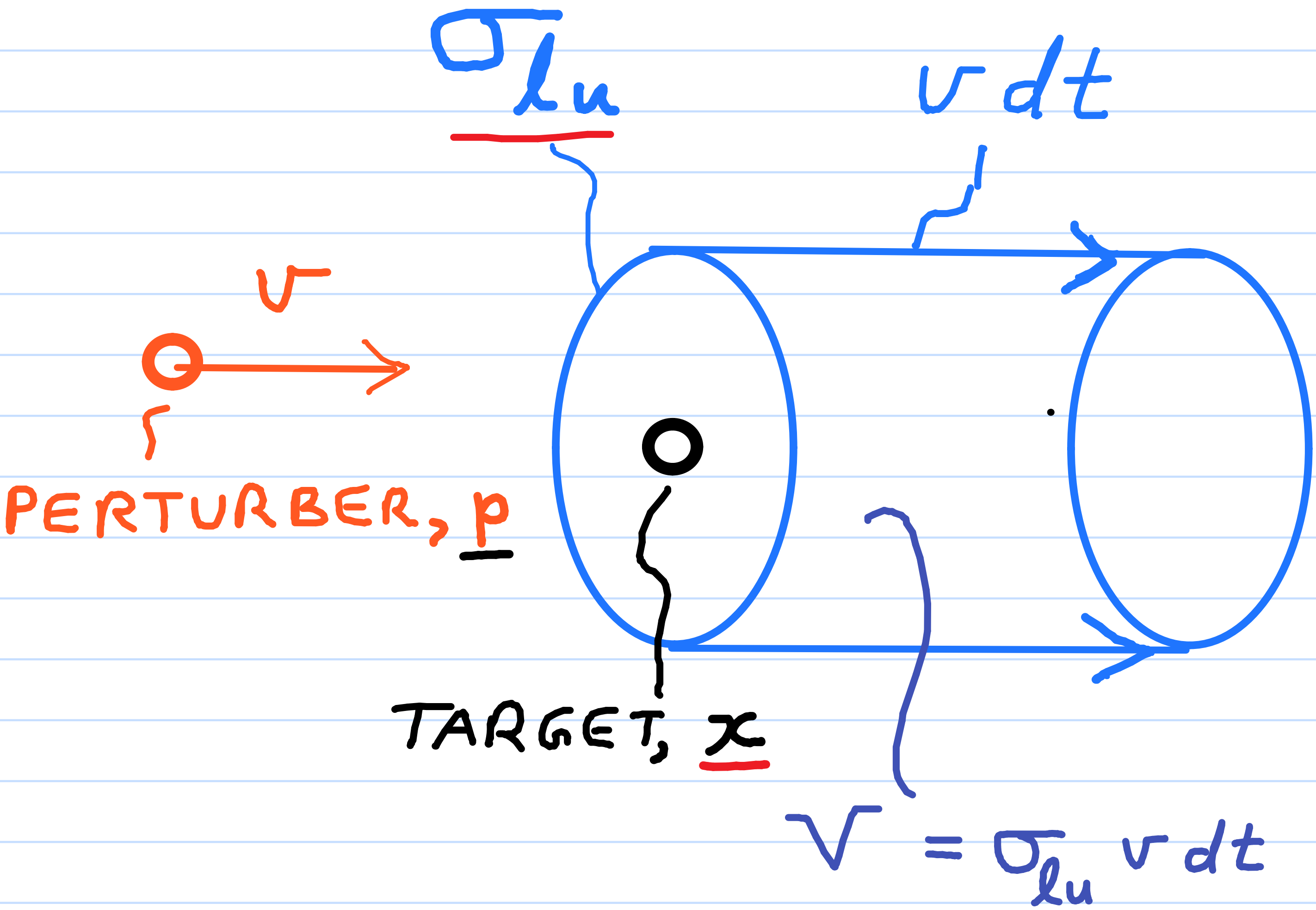
$u \to l$:

$$n_u (R_{ul} + C_{ul}) = n_u (A_{ul} + B_{ul} \bar{J}_{\nu_0}^{\chi} + C_{ul})$$

$l \to u$:

$$n_l (R_{lu} + C_{lu}) = n_l (B_{lu} \bar{J}_{\nu_0}^{\phi} + C_{lu})$$

FOR COLLISIONAL EXCITATION $l \to u$:



FOR PERTURBER, p, OF SPEED v OF NO. DENSITY $\underline{N_{p,v}}$:

$l \to u$ RATE PER PARTICLE OF x :

$$= \sigma_{lu}\, v \left(N_{p,v}\, dv\right) \quad (s^{-1})$$

$$\therefore \; C_{lu} = \int_{v=v_0}^{\infty} \sigma_{lu}(v)\, v\, N_{p,v}(v)\, dv$$

THRESHOLD, v_0 :

$$\frac{1}{2} m_p v_0^2 = \overline{\chi}_u - \overline{\chi}_l$$

$N_{p,v}(v)$ = M-B DISTRIBUTION

IMPORTANT PERTURBERS (p):

FGKM STARS: HI (HeI)

OBA STARS: e^-, HII (HeII)

EINSTEIN RELATIONS:

FOR TRANSITION $l \rightleftarrows u$:

$$\frac{B_{lu}}{B_{ul}} = \frac{g_u}{g_l}$$

$$\frac{A_{ul}}{B_{ul}} = \frac{2h\nu_0^3}{c^2}$$

$$\therefore A_{ul} = B_{lu} \frac{g_l}{g_u} \cdot \frac{2h\nu_0^3}{c^2}$$

AND FOR PERTURBER p:

$$\frac{C_{lu}}{C_{ul}} = \frac{g_u}{g_l} e^{-(\bar{\chi}_u - \bar{\chi}_l)/kT}$$

$$= \frac{g_u}{g_l} e^{-\underline{h\nu_0}/kT}$$

IN PRACTICE:

ONLY A_{ul} & C_{lu} PROVIDED