

GAS IN LTE:

RECALL: RADIATION IN LTE:

ASSUME:

$$S_\nu(z) = \frac{j_\nu(z)}{\alpha_\nu(z)} = B_\nu(T_{\text{KIN}}(z))$$

RECALL:

- FOR ELEMENT z:

- SPECIES, k = ION. STAGE, k

- ATOMIC E-LEVEL, i

$$\therefore N_z = \sum_k \overset{\text{STAGES}}{N_k} = \sum_k \sum_i \overset{\text{LEVELS}}{n_i} \quad (\text{cm}^{-3})$$

GAS PARTICLES DISTRIBUTED AMONG:

- 1) K.E. (v (OR p) VALUES)
- 2) EXCITATION E (ATOMIC E-LEVELS)
- 3) IONIZATION E (ION. STAGES)

1) KE DISTRIBUTION

→ v -DISTRIBUTION, n_v

$$N_k = \int_{\underline{v=0}}^{\infty} n_v(v) dv$$

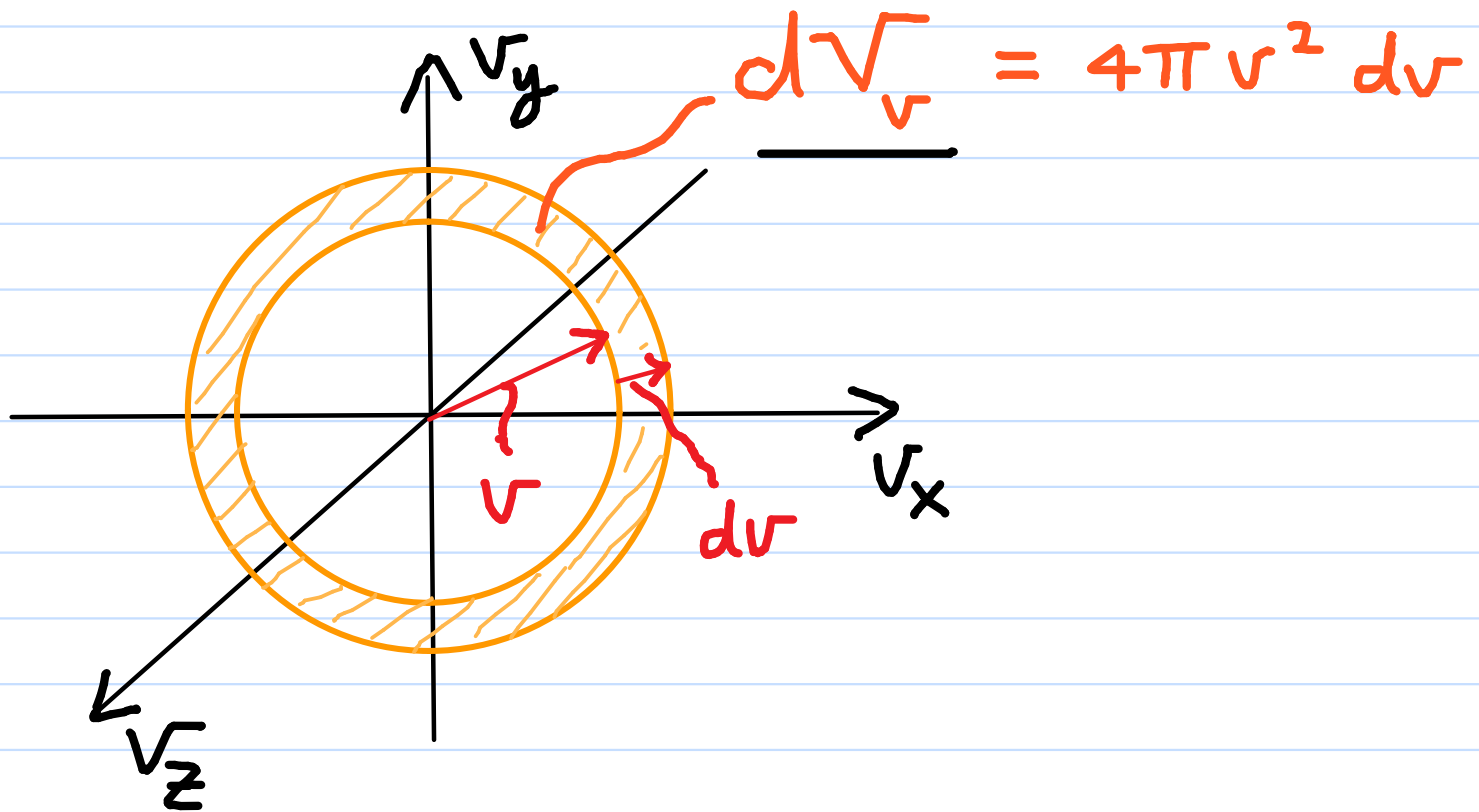
MAXWELL-BOLTZMANN (M-B) DISTRIBUTION

- \vec{v} DISTRIBUTION ISOTROPIC:

$$\bar{v}_x = \bar{v}_y = \bar{v}_z$$

FOR SPECIES k OF MASS m :

$$\frac{n_v(v) dv}{N_k} = \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-K.E.(v)/kT} dv_v$$



$$\text{K.E.}(v) = \frac{1}{2} m v^2 \quad (\text{erg})$$

$$T \equiv T_{\text{KIN}} \quad (\text{K})$$

$$\frac{n_v(v) dv}{N_A} =$$

$$\left(\frac{m}{2\pi R T_{\text{KIN}}} \right)^{3/2} e^{-\frac{mv^2}{2R T_{\text{KIN}}}} 4\pi v^2 dv$$

$$\frac{n_v(v) dv}{N_k} =$$

$$\left(\frac{m}{2\pi k T_{\text{KIN}}} \right)^{3/2} e^{-\frac{mv^2}{2k T_{\text{KIN}}}} 4\pi v^2 dv$$

$$\rightarrow v = (v_x^2 + v_y^2 + v_z^2)^{1/2}$$

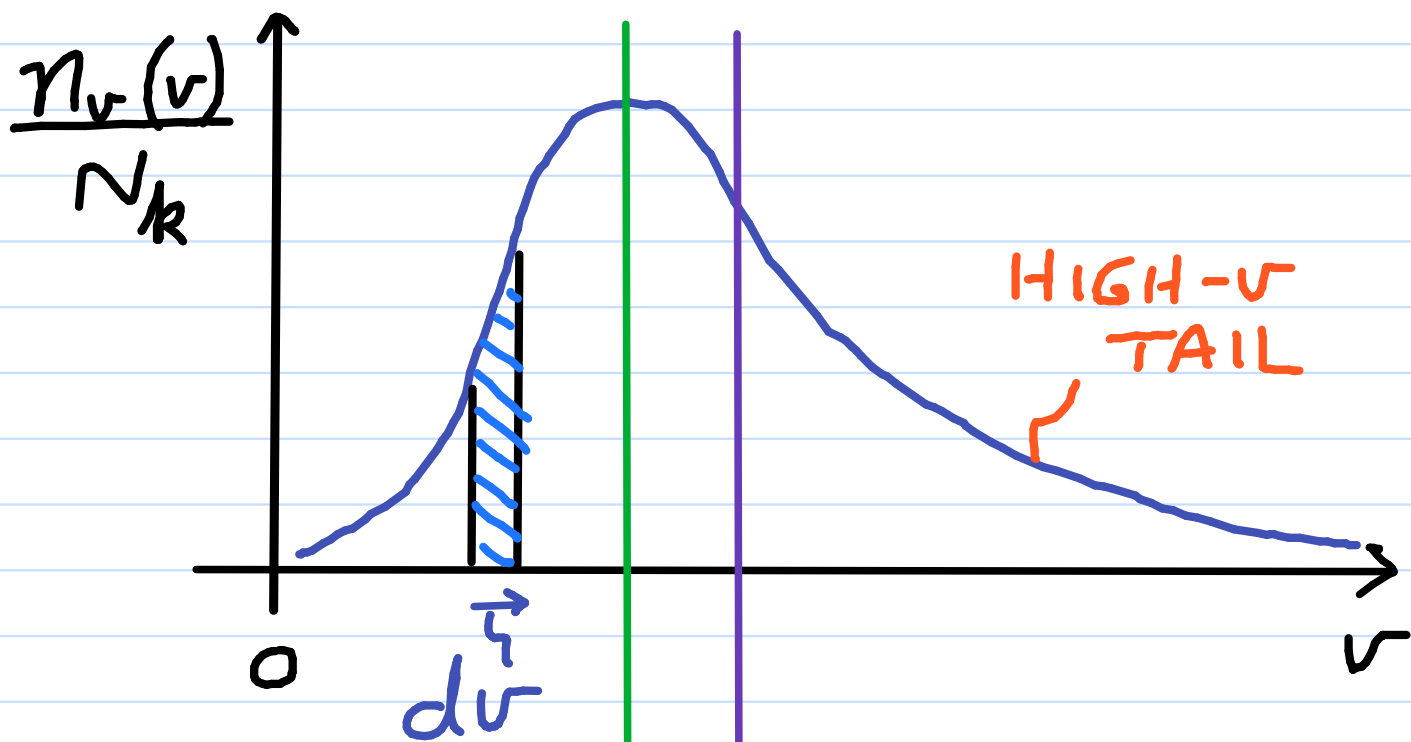
$\rightarrow e^{-\text{K.E.}/k T_{\text{KIN}}}$ IS A BOLTZMANN

FACTOR

$-k T_{\text{KIN}} \approx$ THERMAL E
PER PARTICLE

$$\frac{n_v(v) dv}{N_k} =$$

$$\left(\frac{m}{2\pi k T_{\text{KIN}}} \right)^{3/2} e^{-mv^2/2kT_{\text{KIN}}} 4\pi v^2 dv$$

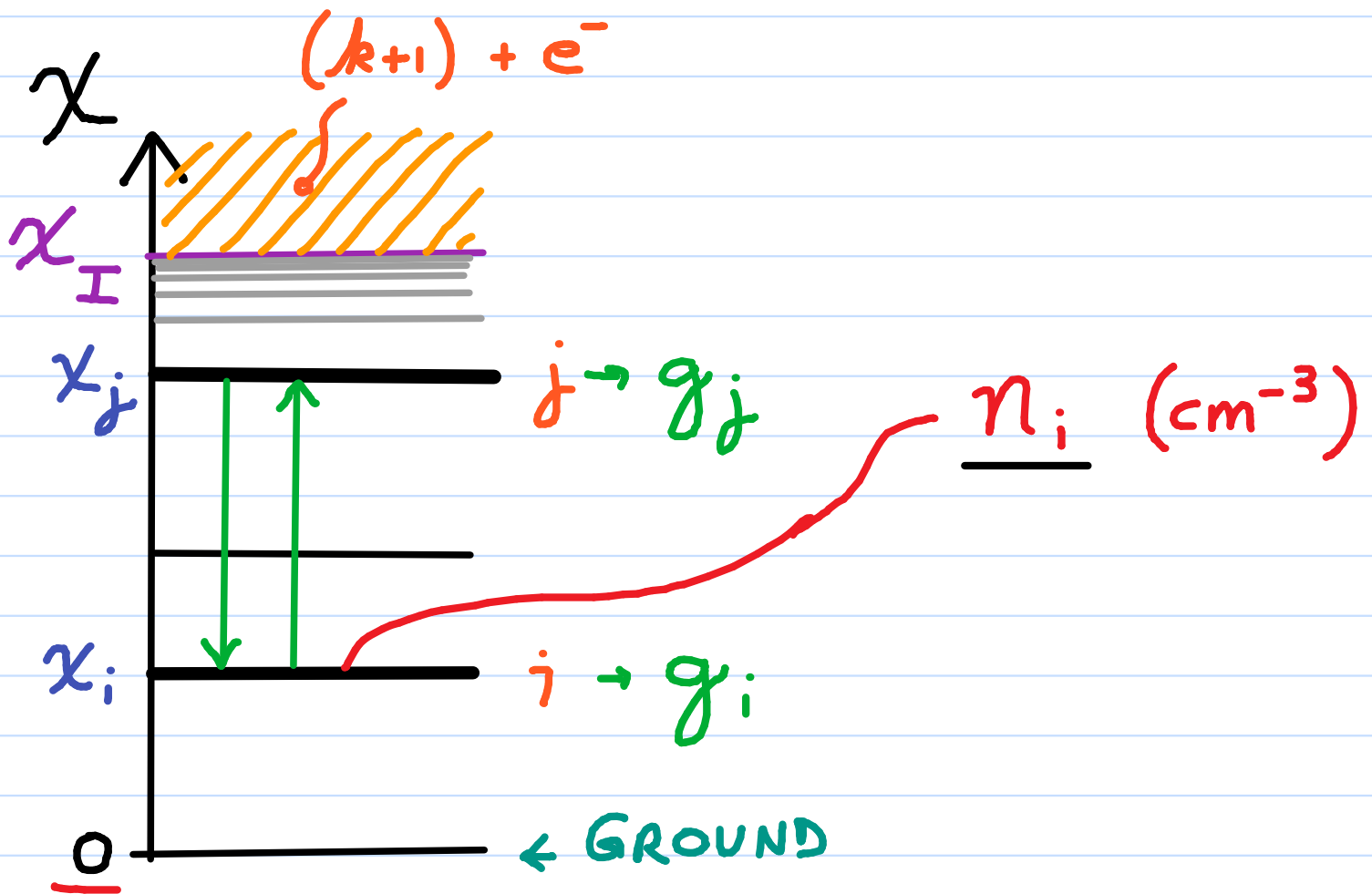


$$v_{\text{mode}} = \sqrt{\frac{2kT_{\text{KIN}}}{m}}$$

$$\bar{v} = \sqrt{\frac{3kT_{\text{KIN}}}{m}}$$

ATOMIC E-LEVEL & IONIZATION-STATE DISTRIBUTION:

SPECIES, k :



χ = EXCITATION E wrt GROUND ($\text{eV}, \text{cm}^{-1}$)

χ_I = ION. E wrt GROUND ($\text{eV}, \text{cm}^{-1}$)

g_i = STATISTICAL WEIGHT
(Q.M. DEGENERACY)

RADIATIVE TRANSITION $i \leftrightarrow j$:

$$\text{PHOTON } \lambda_{ij} = \frac{hc}{\underline{\chi_j} - \underline{\chi_i}} \quad (\text{cm})$$

$$\text{OR } \Delta E_{ij} = (\underline{\chi_j} - \underline{\chi_i}) = \frac{1}{\lambda_{ij}} \quad (\underline{\text{cm}^{-1}})$$

EXCITATION EQUILIBRIUM:

$$n_i, \text{ ALL } i \quad (\text{cm}^{-3})$$

BOLTZMANN DISTRIBUTION:

$$\frac{\underline{n_j}}{n_i} = \frac{\underline{g_j}}{g_i} e^{-\frac{(\underline{\chi_j} - \underline{\chi_i})}{kT_{\text{EXC}}}}$$

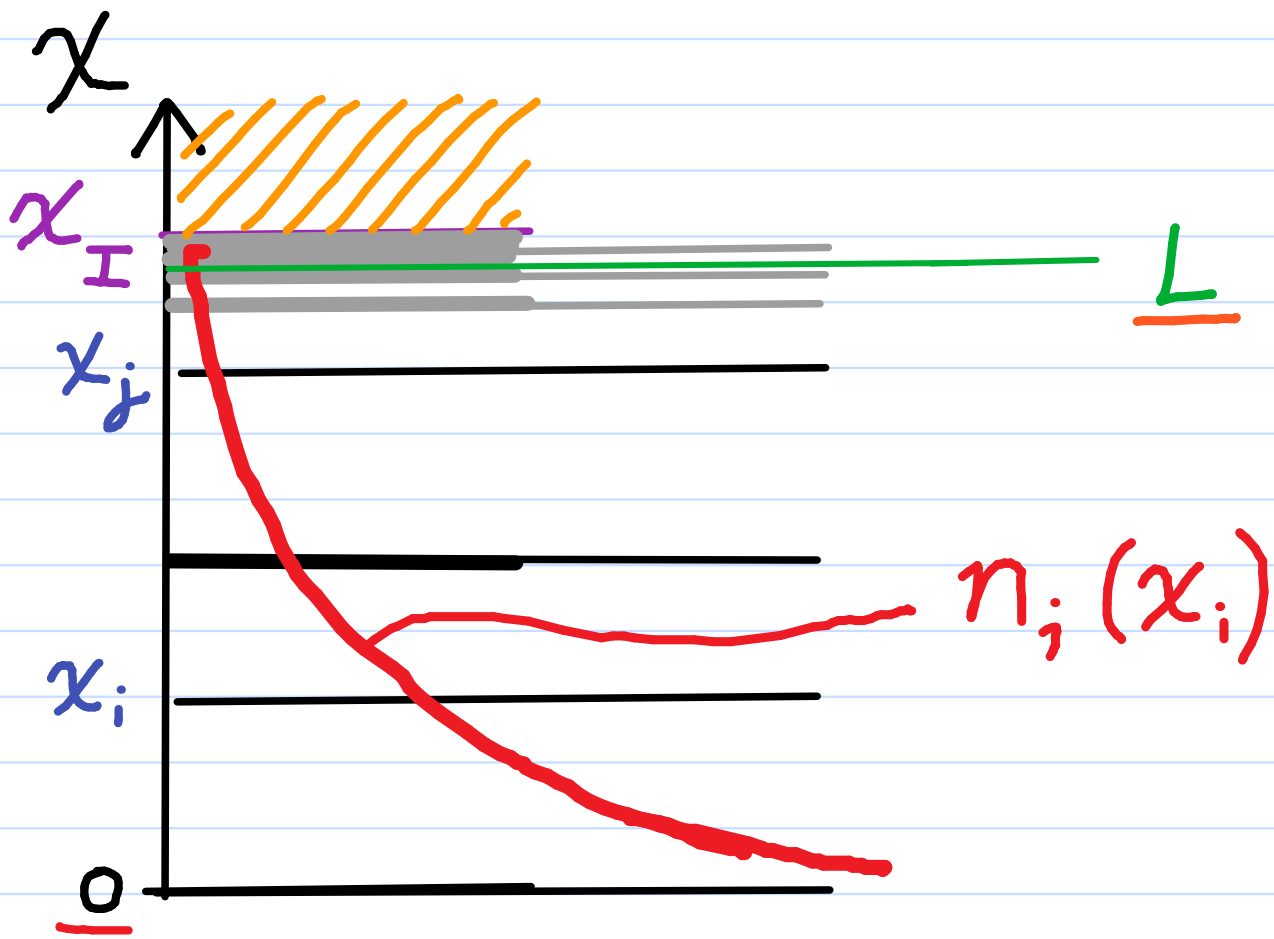
T \equiv EXCITATION TEMP., T_{EXC} (K)

$$\frac{n_j(T_R)}{n_i(T_R)} = \frac{g_j}{g_i} e^{-\frac{(\chi_j - \chi_i)}{kT_{\text{EXC}}(T_R)}}$$

$e^{-\frac{(\chi_j - \chi_i)}{kT_{\text{EXC}}(T_R)}}$ IS ANOTHER
BOLTZMANN FACTOR

$$\text{LTE: } T_{\text{EXC}}(T_R) = T_{\text{KIN}}(T_R)$$

$$\frac{n_j}{n_i} = \frac{g_j}{g_i} e^{-(x_j - x_i)/kT_{\text{Exc}}} :$$



THEN:

$$\underline{N_k} \approx \sum_{i=1}^L n_i, \quad \text{FOR } L \text{ SUCH THAT}$$

$$\underline{x_L} \approx x_I$$

USEFUL: $\frac{n_i}{N_k}$:

$$\begin{aligned}\frac{N_k}{n_i} &= \frac{\sum_j n_j}{n_i} = \sum_j \frac{n_j}{n_i} \\ &= \sum_j \frac{g_j}{g_i} e^{-(x_j - x_i)/kT} \\ &= \frac{\sum_j g_j e^{-x_j/kT}}{g_i e^{-x_i/kT}}\end{aligned}$$

$$\therefore \frac{n_i}{N_k} = \frac{g_i e^{-x_i/kT}}{\sum_j g_j e^{-x_j/kT}}$$

OR

$$\frac{n_i}{N_k} = \frac{g_i e^{-\chi_i/kT}}{U_k(T)}$$

WHERE $\underline{U_k(T)} \equiv \sum_j g_j e^{-\chi_j/kT}$

- PARTITION fn FOR SPECIES k

- IN LTE: $T = T_{KIN}$

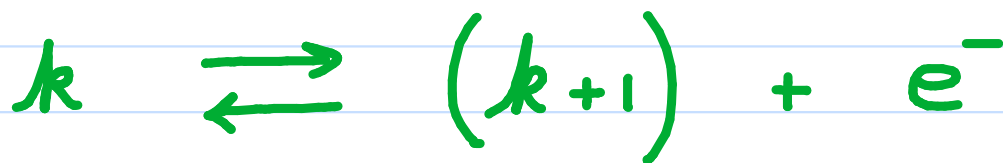
FOR $T \lesssim 5000$ K, MANY SPECIES:

$$U_k(T) \approx g_1 e^{-\chi_1/kT}$$

IONIZATION EQUILIBRIUM:

$$N_k, \text{ ALL } k \text{ (cm}^{-3}\text{)}$$

FOR SPECIES k & $k+1$ OF ELEMENT Z , SUCH THAT



SAHA DISTRIBUTION:

$$\frac{n_{k+1,1} n_e}{n_{k,1}} =$$

$$\frac{2 g_{k+1,1}}{g_{k,1}} \left(\frac{2 \pi m_e k T}{h^2} \right)^{3/2} e^{-\chi_{I,k}/kT}$$

- $T =$ IONIZATION TEMP, T_{ion} (K)

$$\frac{N_{k+1}(T_R)}{N_k(T_R)} =$$

$$\frac{1}{N_e(T_R)} \frac{2 U_{k+1}(T_R)}{U_k(T_R)} \left(\frac{2 \pi m_e k T_{ION}(T_R)}{h^2} \right)^{3/2} \times e^{-\chi_{I,k}/k T_{ION}(T_R)}$$

$e^{-\chi_{I,k}/k T_{ION}}$ IS A BOLTZMANN FACTOR

LTE: $T_{ION}(T_R) = T_{KIN}(T_R)$

NOTE: $\frac{N_{k+1}}{N_k} < 1$ OR $\frac{N_{k+1}}{N_k} > 1$

USE: RECALL IONIZATION

FRACTIONS, f_k

Σg. IF $N_Z = N_I + N_{II} + N_{III}$

$$f_{II} \equiv \frac{N_{II}}{N_k} = \frac{N_{II}}{N_I + N_{II} + N_{III}}$$

$$= \frac{N_{II}/N_I}{1 + \underbrace{N_{II}/N_I}_{\text{SAHA } \Sigma g.} + \underbrace{\left(\frac{N_{III}}{N_{II}}\right)\left(\frac{N_{II}}{N_I}\right)}_{\text{SAHA } \Sigma g.}}$$

SAHA $\Sigma g.$

TEMPERATURES FOR GAS & RADIATION:

$T_{\text{KIN}} : n_{\nu}(\nu);$ M-B DIST.

$T_{\text{EXC}} : n_i(x_i);$ BOLTZMANN DIST.

$T_{\text{ION}} : n_k(x_{I,k});$ SAHA DIST.

$T_{\text{RAD}} : S_{\nu}(\nu);$ PLANCK fn DIST.

IN LTE:

$$\begin{aligned} T_{\text{EXC}}(T_R) &= T_{\text{ION}}(T_R) = T_{\text{RAD}}(T_R) \\ &= \underline{\underline{T_{\text{KIN}}}}(T_R) \end{aligned}$$