GAS IN LTE:

RECALL: RADIATION IN LTE:

ASSUME:

$$S_\nu(z) = \frac{j_\nu(z)}{\alpha_\nu(z)} = B_\nu(T_{\underline{KIN}}(z))$$

RECALL:

- FOR ELEMENT $\underline{Z}$:
  - SPECIES, $\underline{k}$ = ION. STAGE, $\underline{k}$
  - ATOMIC E-LEVEL, $\underline{i}$

$$\therefore N_Z = \overset{\text{STAGES}}{\sum_k} \underline{N}_k = \sum_k \overset{\text{LEVELS}}{\sum_i} \underline{n}_i \qquad (\text{cm}^{-3})$$

GAS PARTICLES DISTRIBUTED AMONG:

1) K.E. (v (OR p) VALUES)
2) EXCITATION E (ATOMIC E-LEVELS)
3) IONIZATION E (ION. STAGES)

1) KE DISTRIBUTION

→ v- DISTRIBUTION, n_v

$$N_k = \int_{v=0}^{\infty} n_v(v)\, dv$$

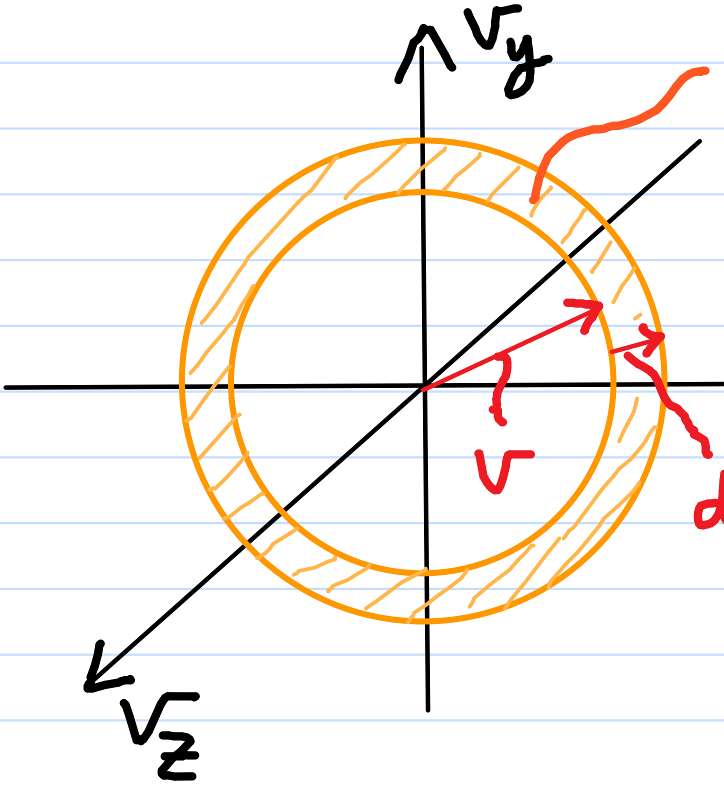
MAXWELL-BOLTZMANN (M-B) DISTRIBUTION

- $\vec{v}$ DISTRIBUTION ISOTROPIC:

$$\bar{v}_x = \bar{v}_y = \bar{v}_z$$

FOR SPECIES k OF MASS m:

$$\frac{n_v(v)\, dv}{N_k} = \left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{K.E.(v)/k_B T}\, dV_v$$



$$\underline{dV_v} = 4\pi v^2 dv$$

$$K.E.(v) = \frac{1}{2}mv^2 \qquad (erg)$$

$$T \equiv T_{\underline{KIN}} \qquad (K)$$

$$\frac{n_v(v)\,dv}{N_k} = \left(\frac{m}{2\underline{\pi k}_B \underline{T_{KIN}}}\right)^{3/2} e^{-\underline{m}v^2/2k_B\underline{T_{KIN}}}\, 4\pi v^2 dv$$

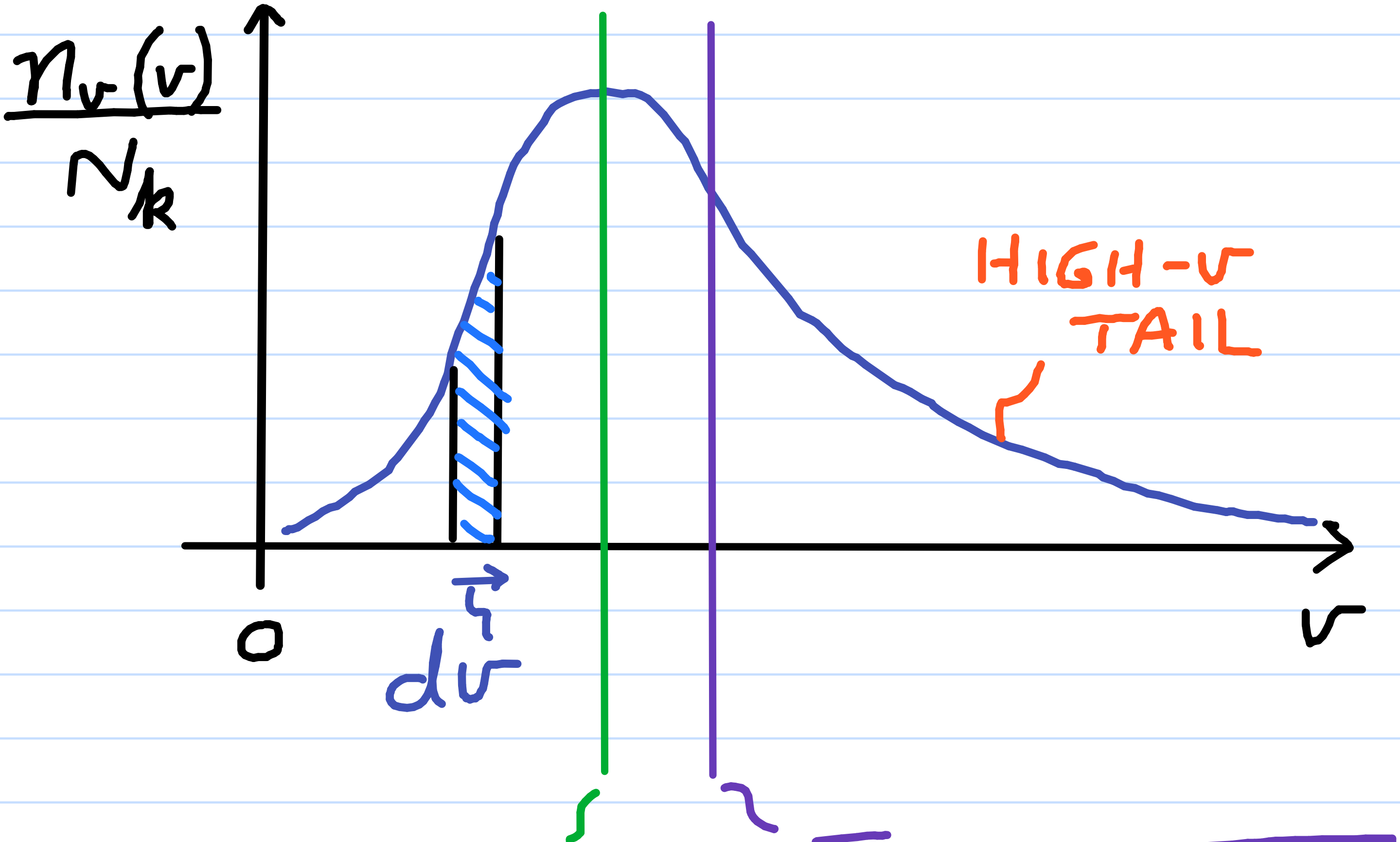
$$\frac{n_v(v)\,dv}{N_k} = \left(\frac{\underline{m}}{2\pi k_B \underline{T_{KIN}}}\right)^{3/2} e^{-\underline{m}v^2/2k_B \underline{T_{KIN}}}\, 4\pi v^2\, dv$$

$\rightarrow v = \left(v_x^2 + v_y^2 + v_z^2\right)^{1/2}$

$\rightarrow e^{-K.E./kT_{KIN}}$ IS A BOLTZMANN FACTOR

$-k_B T_{KIN} \approx$ THERMAL E PER PARTICLE

$$\frac{n_v(v)\,dv}{N_k} = \left(\frac{m}{2\pi k_B T_{KIN}}\right)^{3/2} e^{-mv^2/2k_BT_{KIN}}\, 4\pi v^2\, dv$$

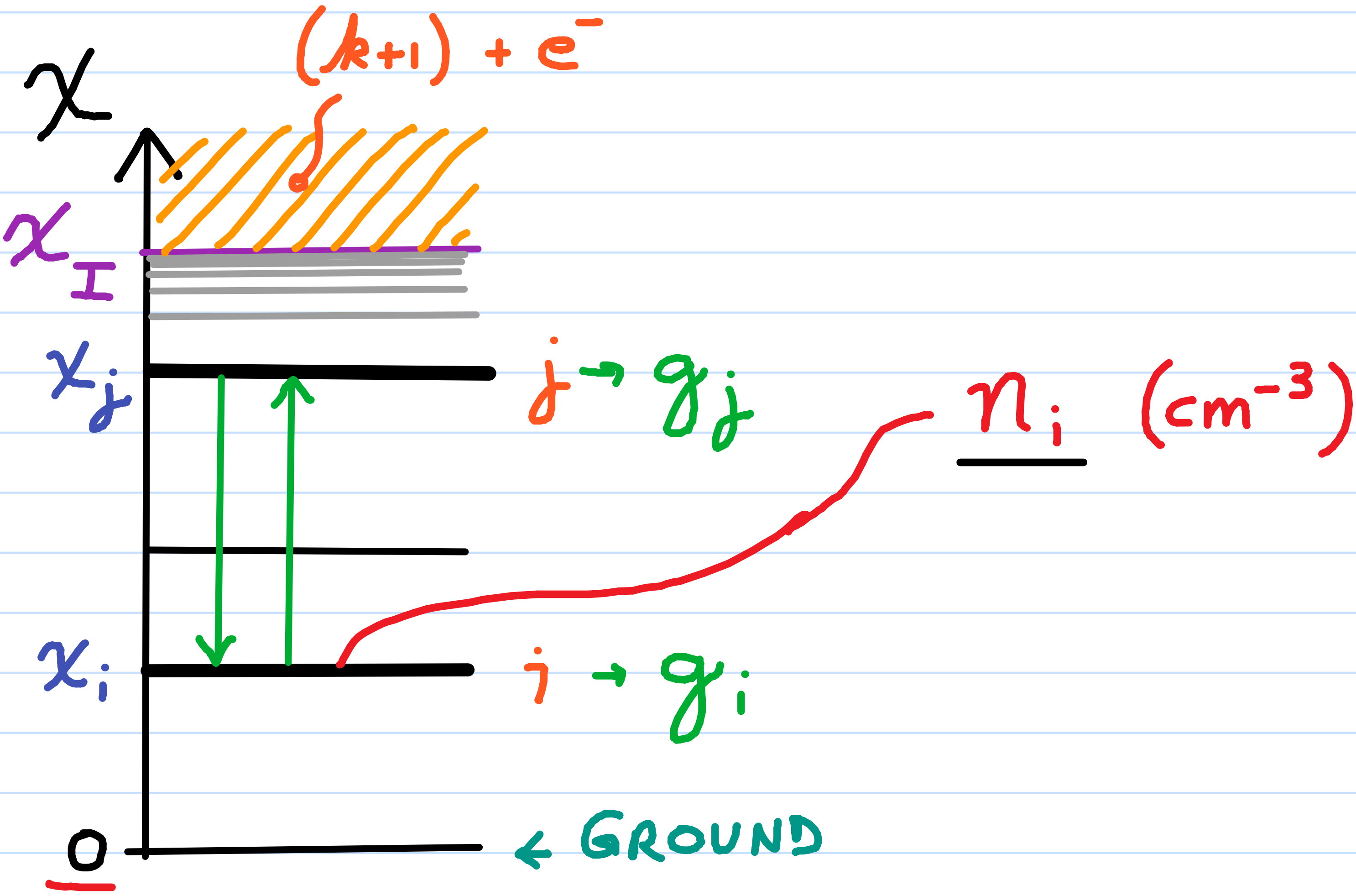


$$v_{mode} = \sqrt{\frac{\underline{2}\,k_B T_{KIN}}{m}}$$

$$\bar{v} = \sqrt{\frac{\underline{3}\,k_B T_{KIN}}{m}}$$

ATOMIC E-LEVEL & IONIZATION-STATE DISTRIBUTION:

SPECIES, k:



χ = EXCITATION E wrt GROUND (eV, cm^{-1})

χ_I = ION. E wrt GROUND (eV, cm^{-1})

g_i = STASTICAL WEIGHT (Q.M. DEGENERACY)

RADIATIVE TRANSITION $i \longleftrightarrow j$:

PHOTON $\lambda_{ij} = \dfrac{hc}{\underline{\chi_j} - \underline{\chi_i}}$ (cm)

OR $\Delta E_{ij} = (\chi_j - \chi_i) = \dfrac{1}{\lambda_{ij}}$ $\underline{(\text{cm}^{-1})}$

EXCITATION EQUILIBRIUM:

n_i, ALL i (cm^{-3})

BOLTZMANN DISTRIBUTION:

$$\frac{n_{\underline{j}}}{n_i} = \frac{g_{\underline{j}}}{g_i} e^{-(\chi_{\underline{j}} - \chi_i)/k_B \underline{T}_{\text{EXC}}}$$

$T \equiv$ EXCITATION TEMP., T_{EXC} (K)

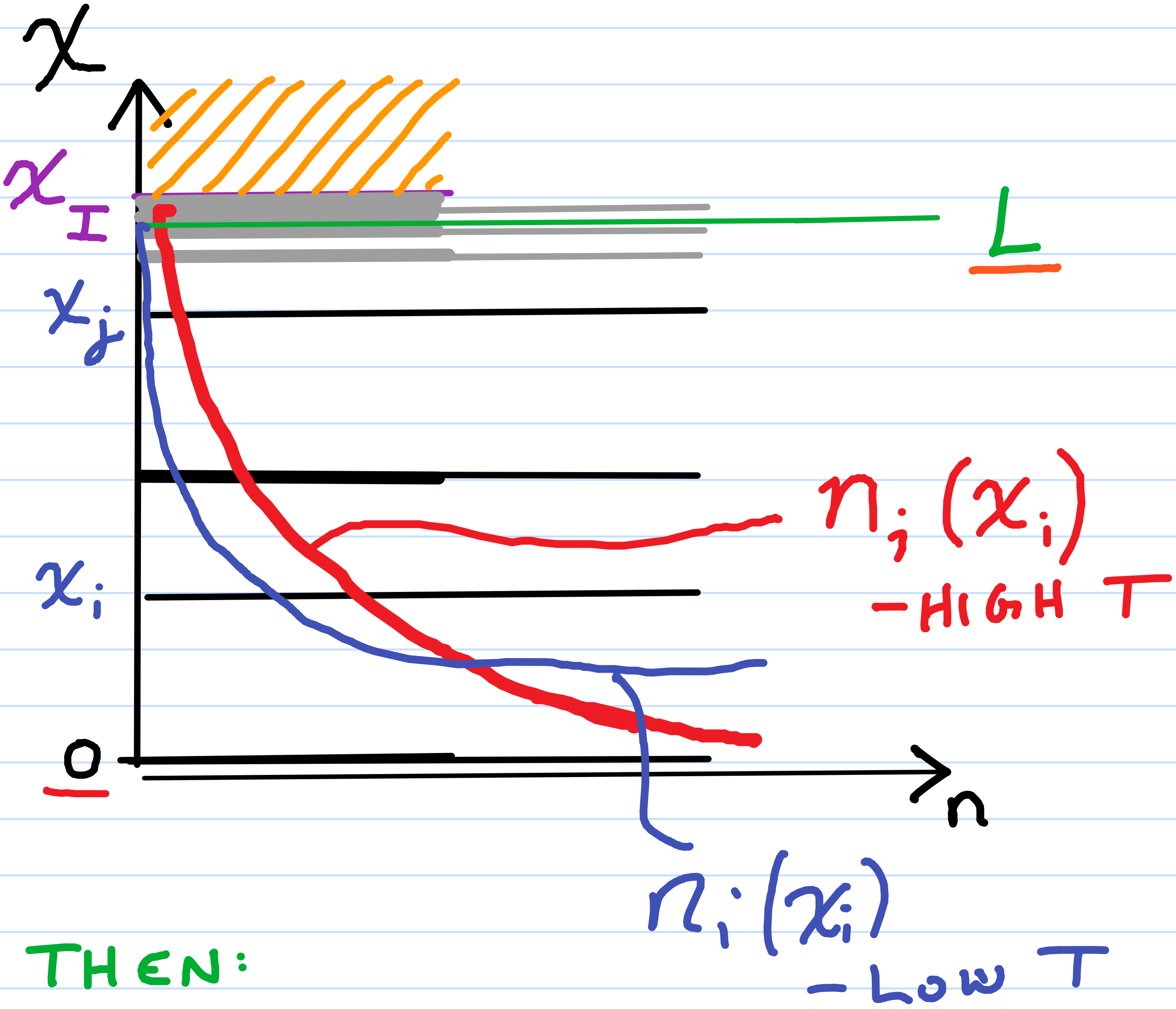
$$\frac{n_j(\tau_R)}{n_i(\tau_R)} = \frac{g_j}{g_i}\, e^{-(\chi_j-\chi_i)/k_B T_{\underline{EXC}}\underline{(\tau_R)}}$$

$e^{-(\chi_j-\chi_i)/k_B T_{EXC}(\tau_R)}$ IS ANOTHER

BOLTZMANN FACTOR

LTE : $T_{EXC}(\tau_R) = T_{\underline{KIN}}(\tau_R)$

$$\frac{n_j}{n_i} = \frac{g_j}{g_i} e^{-(\chi_j - \chi_i)/k_B T_{EXC}} :$$



THEN:

$$\underline{N_k} \gtrsim \sum_{i=1}^{\underline{L}} n_i \ , \quad \text{FOR L SUCH THAT} \quad \chi_{\underline{L}} \lesssim \chi_I$$

$\underline{\text{USEFUL}}$: $\frac{n_i}{N_k}$:

$$\underline{\frac{N_k}{n_i}} = \frac{\sum_{\underline{j}} n_{\underline{j}}}{n_i} = \sum_j \frac{n_j}{n_i}$$

$$= \sum_j \frac{g_j}{g_i} e^{-(\chi_j - \chi_i)/k_B T}$$

$$= \frac{\sum_{\underline{j}} g_{\underline{j}} e^{-\chi_{\underline{j}}/k_B T}}{g_{\underline{i}} e^{-\chi_{\underline{i}}/k_B T}}$$

$$\therefore \underline{\frac{n_{\underline{i}}}{N_k}} = \frac{g_{\underline{i}} e^{-\chi_{\underline{i}}/k_B T}}{\sum_j g_j e^{-\chi_j/k_B T}}$$

OR $$\frac{n_i}{N_k} = \frac{g_i e^{-\chi_i/k_B \underline{T}}}{U_k(\underline{T})}$$

WHERE $$\underline{U_k(\underline{T})} \equiv \sum_j^L g_j e^{-\chi_j/k_B T}$$

- PARTITION fn FOR SPECIES $\underline{k}$

- IN LTE: $T = T_{KIN}$

FOR $T \lesssim 5000$ K, MANY SPECIES:

$$U_k(T) \gtrsim g_{\underline{1}} e^{-\chi_{\underline{1}}/k_B T}$$

IONIZATION EQUILIBRIUM:

N_k, ALL k (cm^{-3})

FOR SPECIES k & $k+1$ OF ELEMENT Z, SUCH THAT

$$k \rightleftarrows (k+1) + e^-$$

SAHA DISTRIBUTION:

$$\frac{n_{k+1,1} N_e}{n_{k,1}} = \frac{2 g_{k+1,1}}{g_{k,1}} \left(\frac{2 \pi m_e k_B T}{h^2} \right)^{3/2} e^{-\chi_{I,k}/k_B T}$$

- T = IONIZATION TEMP, T_{ION} (K)

$$\frac{\underline{N_{k+1}}(\tau_R)}{N_{\underline{k}}(\tau_R)} =$$

$$\frac{1}{N_e(\tau_R)} \frac{2\, U_{\underline{k+1}}(\tau_R)}{\underline{U_k}(\tau_R)} \left(\frac{2\pi m_e k_B T_{\underline{ION}}(\tau_R)}{h^2} \right)^{3/2} \times$$

$$e^{-\chi_{I,k}/k_B T_{\underline{ION}}(\tau_R)}$$

$e^{-\chi_{I,k}/k_B T_{ION}}$ IS A BOLTZMANN FACTOR

LTE: $T_{ION}(\tau_R) = T_{KIN}(\tau_R)$

NOTE: $\frac{N_{k+1}}{N_k} < 1$ OR $\frac{N_{k+1}}{N_k} > 1$

<u>USE</u>: RECALL IONIZATION FRACTIONS, f_k

<u>Eg.</u> IF $N_Z = N_I + N_{II} + N_{III}$

$$f_{II} \equiv \frac{N_{II}}{N_Z} = \frac{N_{II}}{N_I + N_{II} + N_{III}}$$

$$= \frac{N_{II}/N_I}{1 + N_{II}/N_I + \left(\frac{N_{III}}{N_{II}}\right)\left(\frac{N_{II}}{N_I}\right)}$$

SAHA Eq.

TEMPERATURES FOR GAS & RADIATION:

T_{KIN} : $n_v(v)$; M-B DIST.

T_{EXC} : $n_i(\chi_i)$; BOLTZMANN DIST.

T_{ION} : $n_k(\chi_{I,k})$; SAHA DIST.

T_{RAD} : $S_\nu(\nu)$; PLANCK fn.

BUT IN LTE:

$$T_{RAD}(\tau) = T_{EXC}(\tau) = T_{ION}(\tau) = T_{KIN}(\tau)$$