

1D RADIATIVE EQUILIBRIUM (RE) MODELS:

RECALL:

BOLOMETRIC FLUX-CONSTANCY:

$$\frac{dF(\tau_v)}{d\tau_v} = 0, \quad \frac{dF(z)}{dz} = 0$$

$$\therefore F(\tau_v) = F(\underline{0}) = \sigma T_{\underline{eff}}^4 = \frac{L}{4\pi R^2}$$

RECALL:

ANGLE-ANG. OF 1D RAD. TRANS. Eq:

$$\frac{d\tilde{F}_\nu(\tau_\nu)}{d\tau_\nu} = 4\pi \left\{ \underline{J}_\nu(\tau_\nu) - \underline{S}_\nu(\tau_\nu) \right\}$$

RECALL: $\tau_\nu = -\alpha_\nu dz$, $S_\nu = j_\nu / \alpha_\nu$

$$\frac{d\tilde{F}_\nu(z)}{dz} = 4\pi \left\{ \underline{j}_\nu(z) - \underline{\alpha}_\nu(z) \underline{J}_\nu(z) \right\}$$

BOLOMETRIC:

$$\frac{d\tilde{F}(z)}{dz} = 4\pi \int_{\nu=0}^{\infty} \left\{ j_\nu(z) - \alpha_\nu(z) J_\nu(z) \right\} d\nu$$

$$= 0 \sim \text{R.E.}$$

∴ IN 1D R.E.:

$$\int_0^{\infty} j_v(z) dv = \int_0^{\infty} \alpha_v(z) J_v(z) dv$$

- STRÖMGREN Σ q.

OR:

$$\int_0^{\infty} \underbrace{\alpha_v(z)}_{\downarrow} S_v(z) dv = \int_0^{\infty} \underbrace{\alpha_v(z)}_{\downarrow} J_v(z) dv$$

E EMITTED

E ABSORBED

RAD. COOLING

RAD. HEATING

NOTE: IN GENERAL:

$$S_v(z) \neq J_v(z)$$

RECALL: 0th MOMENT OF FORMAL Soln:

$$J_v(\tau_v) = \frac{1}{2} \int_0^{\infty} S_v(t_v) E_1(|t_v - \tau_v|) dt_v$$

$$= \underline{\Lambda}_{\tau_v} [S_v(\tau_v)]$$

$$\text{OR } J_v(\underline{z}) = \underline{\Lambda}_{\underline{z}} [S_v(\underline{z})]$$

∴ STRÖMGREN Eq.:

$$\int_0^{\infty} \alpha_v(z) S_v(z) dv = \int_0^{\infty} \alpha_v(z) \underline{\Lambda}_{\underline{z}} [S_v(z)] dv$$

$$\underline{\text{LTE}}: S_v(z) = B_v(T_{\underline{\text{KIN}}}(z))$$

\therefore LTE STRÖMGREN Σq :

$$\int_0^{\infty} \alpha_v(z) B_v(T_{\text{KIN}}(z)) dv$$
$$= \int_0^{\infty} \alpha_v(z) \Lambda_z [B_v(\underline{T_{\text{KIN}}}(z))] dv$$

- SOLVE FOR $T_{\underline{\text{KIN}}}(z) \rightarrow T_{\underline{\text{KIN}}}(\underline{T_r})$

SIMPLE COMPUTATIONAL PROCEDURE:

INITIAL GUESS: $T_{KIN}^{(0)}(z)$

$$\begin{aligned} \delta T^{(n+1)}(z) = & \int_0^{\infty} \alpha_v^{(n)}(z) \Lambda_z [B_v(T_{KIN}^{(n)}(z))] dv \\ & - \int_0^{\infty} \alpha_v^{(n)}(z) B_v(T_{KIN}^{(n)}(z)) dv \end{aligned}$$

UNTIL:

$$\frac{\delta T}{T} = \frac{T^{(n+1)} - T^{(n)}}{T^{(n)}} < \epsilon \ll 1, \quad \text{ALL } z$$

THE GRAY APPROXIMATION: $T_{KIN}(\tau)$

USEFUL:

- INITIAL GUESS $T_{KIN}^{(0)}(\tau)$
- TEST CASE

ASSUME GRAY EXTINCTION:

$$\alpha_{\nu}(z, \nu) = \alpha(z)$$

$$\left(\text{AND } K(z) = \alpha(z) / \rho(z) \right)$$

VERY CRUDE!

- ONLY THOMSON SCATTERING IS GRAY

⇒ LEAST BAD FOR O STARS

THEN: GRAY RADIAL τ -SCALE:

$$d\tau(z) = -\alpha(z) dz$$

BOLOMETRIC GRAY RAD. TRANS. Eq.:

$$\mu \frac{d}{dz} \int_0^{\infty} \underline{I}_\nu(z, \mu) d\nu = \underline{\alpha}(z) \int_0^{\infty} \underline{S}_\nu(z) d\nu - \underline{\alpha}(z) \int_0^{\infty} \underline{I}_\nu(z) d\nu$$

$$\therefore \mu \frac{dI(z)}{dz} = \alpha(z) (S(z) - I(z))$$

RECALL: $d\tau = -\alpha dz$

$$\therefore \mu \frac{dI(\tau)}{d\tau} = (I(\tau) - S(\tau)) \quad \text{--- } \textcircled{1}$$

- BUT: $I_\nu = I_\nu(\underline{\nu})$ & $S_\nu = S_\nu(\underline{\nu})$

GRAY STRÖMGREN Eq. (R.E.):

$$\underline{\alpha} \int_0^{\infty} S_{\nu}(z) d\nu = \underline{\alpha} \int_0^{\infty} J_{\nu}(z) d\nu$$

$$\therefore S(z) = J(z)$$

$$\text{OR } S(\tau) = J(\tau) \quad \text{--- } \textcircled{2}$$

$$\text{--- } \underline{\text{BUT}}: S_{\nu}(\tau, \nu) \neq J_{\nu}(\tau, \nu)$$

RECALL: 1ST ANGLE-AVG. OF 1D RAD.
TRANS. Eq.

- ON GRAY T-SCALE

$$\frac{dK_v(\tau)}{d\tau} = \frac{F_v(\tau)}{4\pi}$$

∴ BOLOMETRIC:

$$\frac{d}{d\tau} \int_0^{\infty} K_v(\tau) dv = \frac{1}{4\pi} \int_0^{\infty} F_v(\tau) dv$$

$$\therefore \frac{dK(\tau)}{d\tau} = \frac{F(\tau)}{4\pi}$$

BOLOMETRIC FLUX-CONSTANCY (R.E.):

$$F(\tau) = F :$$

$$\frac{dK(\tau)}{d\tau} = \frac{F}{4\pi}$$

Sol'n: $K(\tau) = \frac{F\tau}{4\pi} + \underline{C} - \textcircled{3}$

AND $C = K(0)$

BOLOMETRIC, ON GRAY T-SCALE:

RECALL: 1ST EDD. APPROX:

$$K(\tau) = \frac{1}{3} J(\tau)$$

$$\therefore \underline{K(0)} = \frac{1}{3} J(0)$$

RECALL: 2ND EDD. APPROX:

$$J(0) = \frac{\mathcal{F}(0)}{2\pi} = \frac{\mathcal{F}}{2\pi}$$

$$\therefore Q = \frac{\mathcal{F}}{6\pi}$$

\therefore Eq. (3) BECOMES:

$$\underline{\frac{1}{3} J(\tau)} = \frac{F\tau}{4\pi} + \underline{\frac{F}{6\pi}}$$

$$\text{OR } J(\tau) = \frac{3}{4\pi} F\left(\tau + \frac{2}{3}\right)$$

Eq. (2) : GRAY R.E.: $J(\tau) = S(\tau)$

$$\underline{\text{LTE}} : S(\tau) = \int_0^{\infty} B_{\nu}(\tau) d\nu \equiv B(\tau)$$

\therefore IN GRAY R.E.:

$$\underline{B(\tau)} = \frac{3}{4\pi} F\left(\tau + \frac{2}{3}\right)$$

$$\underline{B(\tau)} = \frac{3}{4\pi} \underline{F\left(\tau + \frac{2}{3}\right)}$$

STEFAN-BOLTZMANN LAW:

$$\underline{B(\tau)} = \frac{\sigma}{\pi} \underline{T_{KIN}^4(\tau)}$$

Def'n of T_{eff} : $\underline{F} = F(0) \equiv \sigma \underline{T_{eff}^4}$

∴ IN GRAY R.E.:

$$\frac{\sigma}{\pi} \underline{T_{KIN}^4(\tau)} = \frac{3}{4\pi} \sigma \underline{T_{eff}^4\left(\tau + \frac{2}{3}\right)}$$

$$\underline{\therefore T_{KIN}(\tau)} = \underline{T_{eff}} \left(\frac{3}{4} \underline{\tau} + \frac{1}{2} \right)^{1/4}$$

GRAY $T_{KIN}(\tau)$ STRUCTURE

GRAY $T_{KIN}(\tau)$ MODEL:

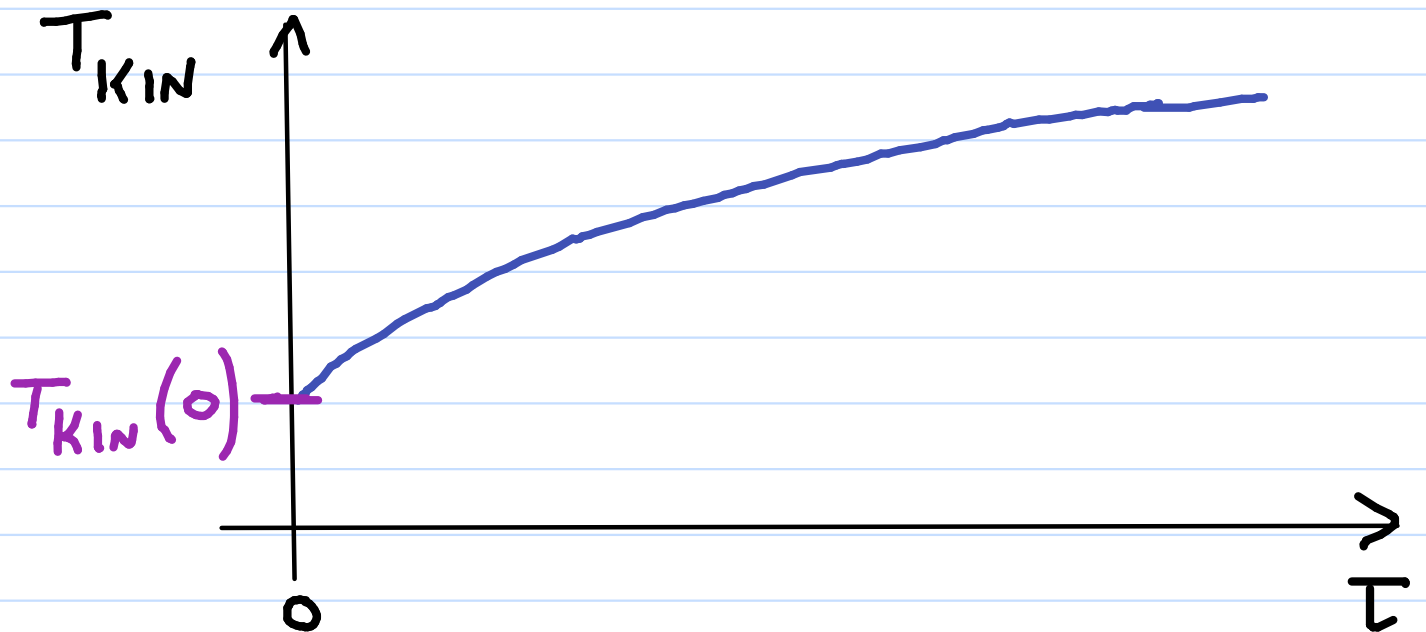
RECALL ROSSELAND τ -SCALE:

$$d\tau_R = -\alpha_R(z) dz \text{ IS } \underline{\text{GRAY}}$$

$$T_{KIN}(\tau) = T_{eff} \left(\frac{3}{4} \tau_R + \frac{1}{2} \right)^{1/4}$$

- T_{eff} = INPUT PARAMETER

$$\therefore T_{KIN}(\underline{0}) = \frac{T_{eff}}{16} ; T_{KIN}(\underline{2/3}) = T_{eff}$$



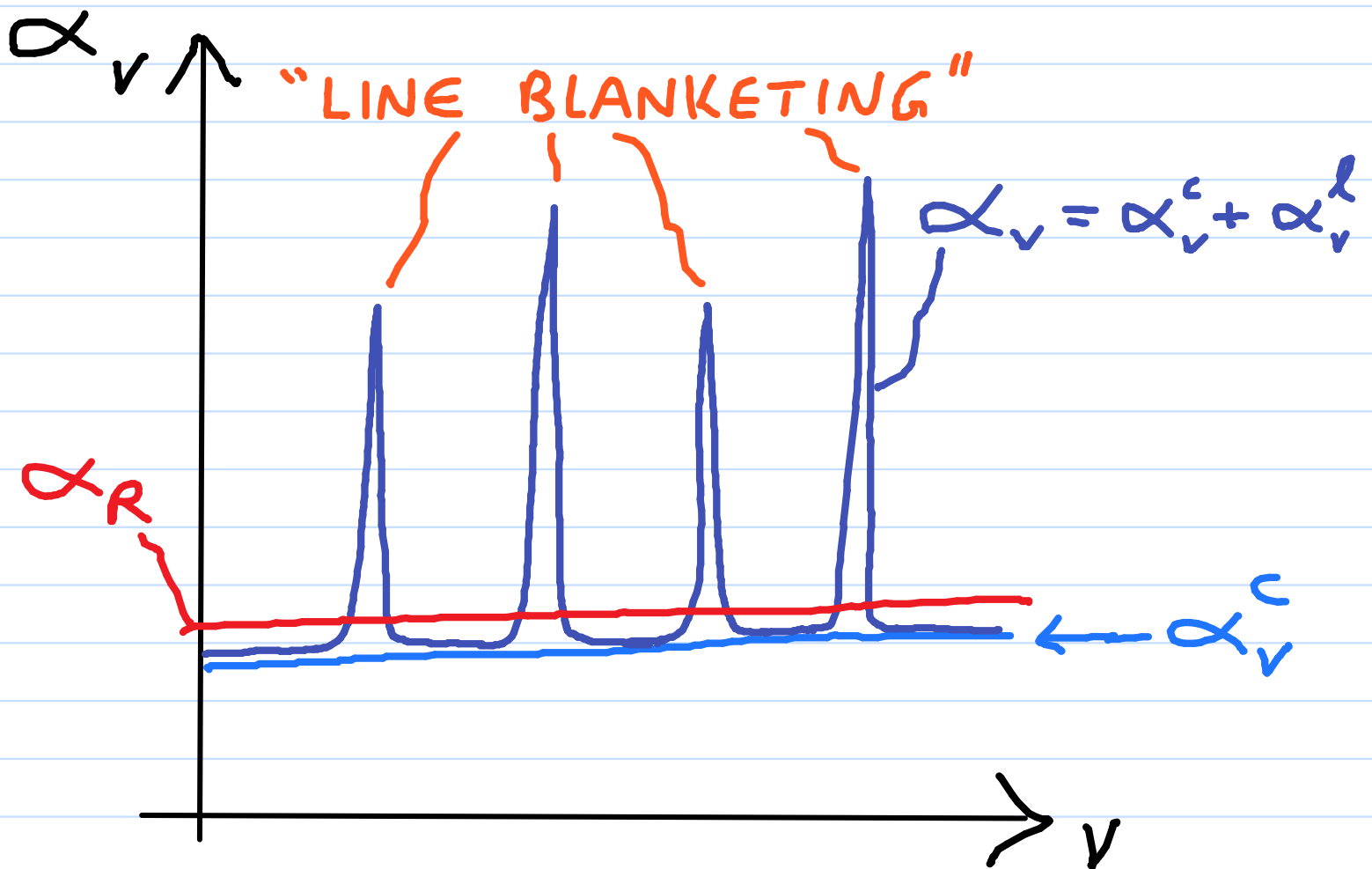
LINE-BLANKETED R.E. MODELS

- NON-GRAY: $\alpha_\nu(\nu) = \alpha_\nu^c + \alpha_\nu^l$

α_ν^c IS APPROXIMATELY GRAY

RECALL:

$\Delta\nu \approx \text{FEW} \times 10^{15} \text{ Hz}$



$\alpha_R > \alpha_\nu^c$

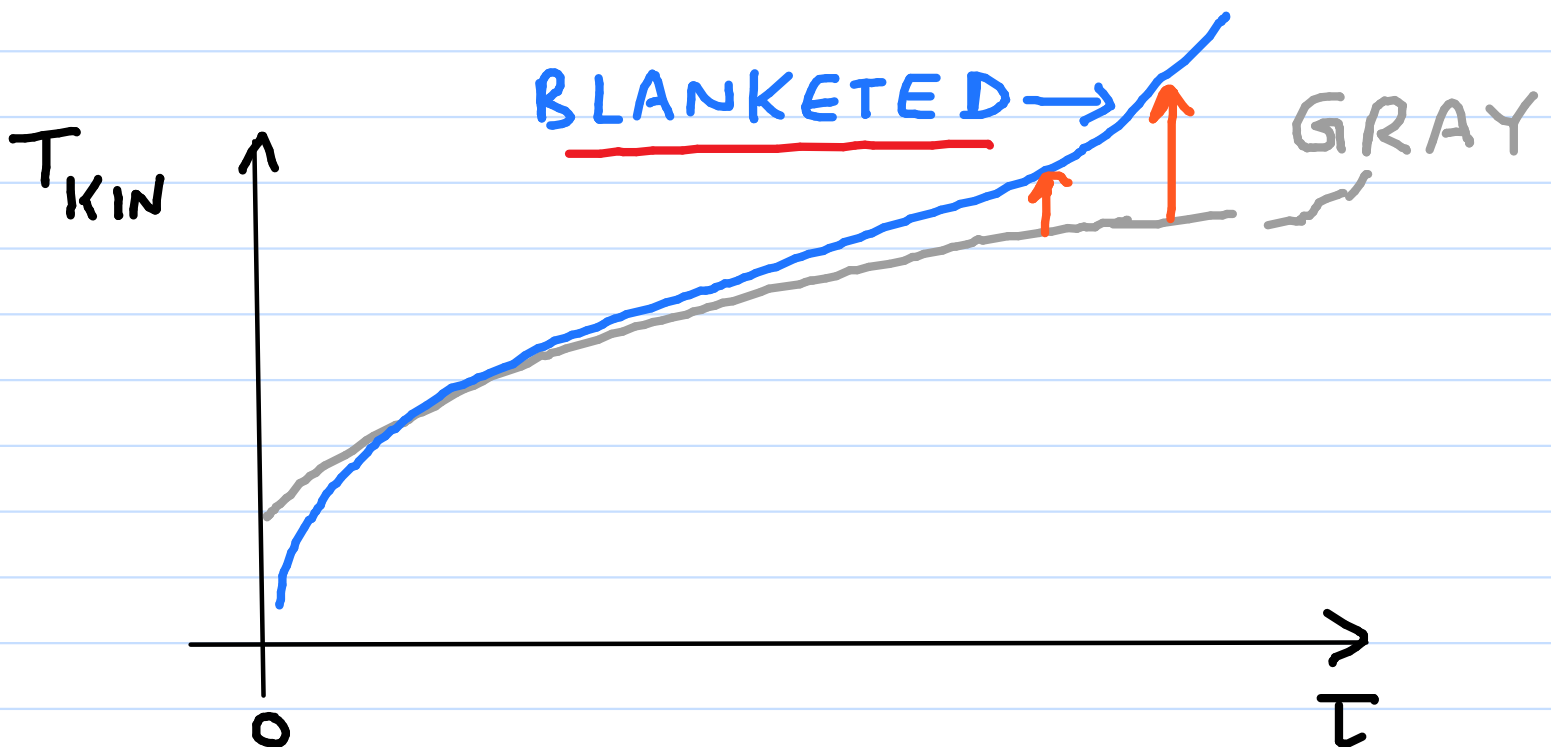
RECALL: DIFFUSION APPROX:

$$\kappa(z) \approx -\frac{16\sigma}{3} \frac{1}{\alpha(z)} T_{\text{KIN}}^3(z) \frac{dT_{\text{KIN}}(z)}{dz}$$

AND $\alpha_R > \alpha_V^s$

$$\therefore \left| \frac{dT}{dz} \right|_{\text{BLANKETED}} > \left| \frac{dT}{dz} \right|_{\text{GRAY}}$$

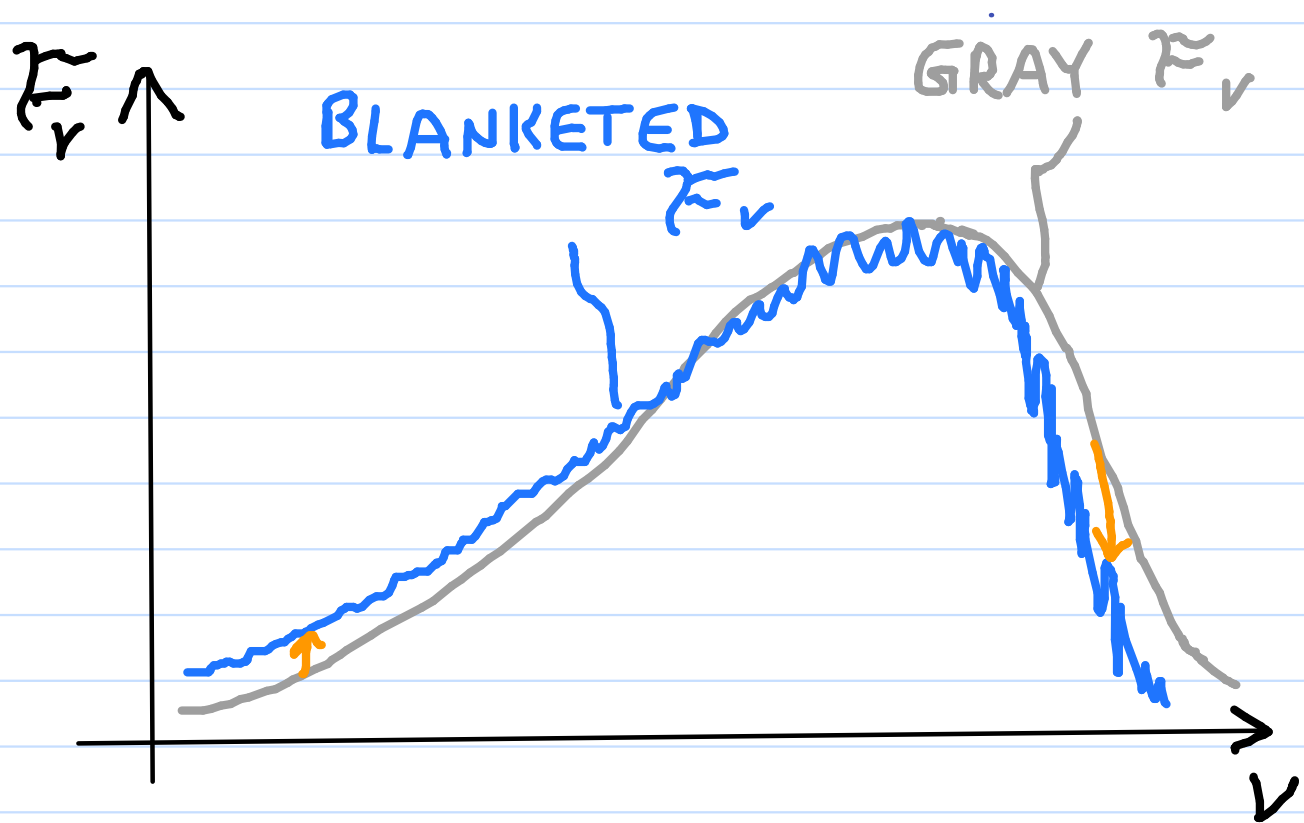
- BACK WARMING



FLUX RE-DISTRIBUTION:

SPECTRAL LINE DENSITY, $\frac{(\text{NO. LINES})}{\Delta \nu}$

INCREASES W. INCREASING ν



$$F_{\text{BLANKETED}} = F_{\text{GRAY}} = \sigma T_{\text{eff}}^4$$