1D RADIATIVE EQUILIBRIUM (RE) MODELS:

Eg. B, A STARS

- NO CONVECTION

$$\therefore \quad F(z) = \mathcal{F}(z)$$

RECALL: THERMAL EQUILIBRIUM:

BOLOMETRIC FLUX-CONSTANCY:

$$\frac{d\mathcal{F}(\tau_\nu)}{d\tau_\nu} = 0, \quad \frac{d\mathcal{F}(z)}{dz} = 0$$

$$\therefore \mathcal{F}(\tau_\nu) = \mathcal{F}(0) = \sigma T_{eff}^4 = \frac{L}{4\pi R^2}$$

RECALL:

ANGLE-AVG. OF 1D RAD. TRANS. Eq:

$$\frac{dF_\nu(\tau_\nu)}{d\tau_\nu} = 4\pi\left\{J_\nu(\tau_\nu) - S_\nu(\tau_\nu)\right\}$$

RECALL: $d\tau_\nu = -\alpha_\nu dz$, $S_\nu = j_\nu/\alpha_\nu$

$$\frac{dF_\nu(z)}{dz} = 4\pi\left\{j_\nu(z) - \alpha_\nu(z) J_\nu(z)\right\}$$

BOLOMETRIC:

$$\frac{dF(z)}{dz} = 4\pi \int_{\nu=0}^{\infty}\left\{j_\nu(z) - \alpha_\nu(z) J_\nu(z)\right\} d\nu$$

$$= 0 \sim \text{R.E.}$$

∴ IN 1D R.E.:

$$\int_0^\infty j_\nu(z)\, d\nu = \int_0^\infty \alpha_\nu(z)\, J_\nu(z)\, d\nu$$

- STRÖMGREN Eq.

OR:

$$\int_0^\infty \underbrace{\alpha_\nu(z)}_{\text{E EMITTED}} S_\nu(z)\, d\nu = \int_0^\infty \underbrace{\alpha_\nu(z)}_{\text{E ABSORBED}} J_\nu(z)\, d\nu$$

E EMITTED → RAD. COOLING

E ABSORBED → RAD. HEATING

NOTE: IN GENERAL:

$$S_\nu(z) \neq J_\nu(z)$$

RECALL: 0^{th} MOMENT OF FORMAL Soln:

$$J_\nu(\tau_\nu) = \frac{1}{2}\int_0^\infty S_\nu(t_\nu)\,E_1(|t_\nu - \tau_\nu|)\,dt_\nu$$

$$= \Lambda_{\tau_\nu}[S_\nu(\tau_\nu)]$$

OR $J_\nu(z) = \Lambda_z[S_\nu(z)]$

$\therefore$ STRÖMGREN Eq.:

$$\int_0^\infty \alpha_\nu(z)\,S_\nu(z)\,d\nu = \int_0^\infty \alpha_\nu(z)\,\Lambda_z[S_\nu(z)]\,d\nu$$

LTE: $S_\nu(z) = B_\nu(T_{KIN}(z))$

$\therefore$ LTE STRÖMGREN Eq.:

$$\int_0^\infty \alpha_\nu(z) B_\nu(T_{KIN}(z))\, d\nu = \int_0^\infty \alpha_\nu(z) \Lambda_z [B_\nu(T_{KIN}(z))]\, d\nu$$

- SOLVE FOR $T_{KIN}(z) \rightarrow T_{KIN}(\tau_\nu)$

SIMPLE COMPUTATIONAL PROCEDURE:

INITIAL GUESS: $T_{KIN}^{(0)}(z)$

CORRECTION:

$$\underline{\delta T^{(n+1)}(z)} = \int_0^\infty \alpha_\nu^{(n)}(z)\,\Lambda_z\left[B_\nu\left(\underline{T_{KIN}^{(n)}(z)}\right)\right] d\nu - \int_0^\infty \alpha_\nu^{(n)}(z)\, B_\nu\left(T_{KIN}^{(n)}(z)\right) d\nu$$

$$T^{(n+1)}(z) = T^{(n)}(z) + \delta T^{(n+1)}(z)$$

UNTIL:

$$\frac{\delta T}{T} = \frac{T^{(n+1)} - T^{(n)}}{T^{(n)}} < \varepsilon \ll 1,$$

ALL z

THE GRAY APPROXIMATION: $T_{KIN}(\tau)$

USEFUL:

- INITIAL GUESS $T_{KIN}^{(0)}(\tau)$
- TEST CASE

ASSUME GRAY EXTINCTION:

$$\alpha_\nu(z, \nu) = \alpha(z)$$

$$\left(\text{AND } \kappa(z) = \alpha(z)/\rho(z)\right)$$

Eg. $\alpha_R(z)$

VERY CRUDE!

- ONLY THOMSON SCATTERING IS GRAY

⇒ LEAST BAD FOR O STARS

THEN: GRAY RADIAL τ-SCALE:

$$d\tau(z) = -\alpha(z)\, dz$$

Eg. $\tau_R(z)$

BOLOMETRIC GRAY RAD. TRANS. Eq.:

IN PLANE-|| ATMOSPHERE:

$$\mu \frac{dI_\nu(z,\mu)}{dz} = \underbrace{j_\nu(z)}_{\alpha(z) S_\nu(z)} - \alpha(z) I_\nu(z)$$

$$\mu \frac{d}{dz} \int_0^\infty I_\nu(z,\mu)\, d\nu = \alpha(z) \int_0^\infty S_\nu(z)\, d\nu - \alpha(z) \int_0^\infty I_\nu(z)\, d\nu$$

$$\therefore \mu \frac{dI(z)}{dz} = \alpha(z)\left(S(z) - I(z)\right)$$

RECALL: $d\tau = -\alpha dz$

$$\therefore \mu \frac{dI(\tau)}{d\tau} = \left(I(\tau) - S(\tau)\right) \quad \text{— } 1$$

- BUT: $I_\nu = I_\nu(\nu)$ & $S_\nu = S_\nu(\nu)$

GRAY STRÖMGREN Eq. (R.E.):

$$\underline{\alpha} \int_0^\infty S_\nu(z)\, d\nu = \underline{\alpha} \int_0^\infty J_\nu(z)\, d\nu$$

$$\therefore S(z) = J(z)$$

$$\text{OR} \quad S(\tau) = J(\tau) \quad \text{— } (2)$$

- **BUT**: $S_\nu(\tau, \nu) \neq J_\nu(\tau, \nu)$

RECALL: 1ST ANGLE-AVG. OF 1D RAD. TRANS. Eq.

- ON GRAY τ-SCALE

$$\frac{dK_\nu(\tau)}{d\tau} = \frac{F_\nu(\tau)}{4\pi}$$

$\therefore$ BOLOMETRIC:

$$\frac{d}{d\tau}\int_0^\infty K_\nu(\tau)\,d\nu = \frac{1}{4\pi}\int_0^\infty F_\nu(\tau)\,d\nu$$

$$\therefore \frac{dK(\tau)}{d\tau} = \frac{F(\tau)}{4\pi}$$

BOLOMETRIC FLUX-CONSTANCY (R.E.):

$$\mathcal{F}(\tau) = \mathcal{F}:$$

$$\frac{dK(\tau)}{d\tau} = \frac{\mathcal{F}}{4\pi}$$

Sol'n: $K(\tau) = \frac{\mathcal{F}\tau}{4\pi} + C$ – 3

AND $C = K(0)$

BOLOMETRIC, ON GRAY τ-SCALE:

RECALL: 1ST EDD. APPROX:

$$K(\tau) = \tfrac{1}{3} J(\tau)$$

$$\therefore K(0) = \tfrac{1}{3} J(0)$$

RECALL: 2nd EDD. APPROX:

$$J(0) = \frac{\mathcal{F}(0)}{2\pi} = \frac{\mathcal{F}}{2\pi}$$

$$\therefore G = \frac{\mathcal{F}}{6\pi}$$

$\therefore$ Eq. 3 BECOMES:

$$\frac{1}{3} J(\tau) = \frac{F}{4\pi}\tau + \frac{F}{6\pi}$$

OR $$J(\tau) = \frac{3}{4\pi} F\left(\tau + \frac{2}{3}\right)$$

Eq. 2 : GRAY R.E.: $J(\tau) = S(\tau)$

LTE: $$S(\tau) = \int_0^\infty B_\nu(\tau)\, d\nu \equiv B(\tau)$$

$\therefore$ IN GRAY R.E.:

$$B(\tau) = \frac{3}{4\pi} F\left(\tau + \frac{2}{3}\right)$$

$$B(\tau) = \frac{3}{4\pi} \mathcal{F} \left(\tau + \frac{2}{3} \right)$$

STEFAN-BOLTZMANN LAW:

$$B(\tau) = \frac{\sigma}{\pi} T_{KIN}^4(\tau)$$

Def'n OF T_{eff} : $\mathcal{F} = \mathcal{F}(0) \equiv \sigma T_{eff}^4$

$\therefore$ IN GRAY R.E.:

$$\frac{\sigma}{\pi} T_{KIN}^4(\tau) = \frac{3}{4\pi} \sigma T_{eff}^4 \left(\tau + \frac{2}{3} \right)$$

$$\boxed{\therefore T_{KIN}(\tau) = T_{eff} \left(\frac{3}{4}\tau + \frac{1}{2} \right)^{1/4}}$$

GRAY $T_{KIN}(\tau)$ STRUCTURE

GRAY $T_{KIN}(\tau)$ MODEL:

RECALL ROSSELAND τ-SCALE:

$d\tau_R = -\alpha_R(z)\,dz$ IS GRAY

$$T_{KIN}(\tau_R) = T_{eff}\left(\frac{3}{4}\tau_R + \frac{1}{2}\right)^{1/4}$$

- T_{eff} = INPUT PARAMETER

$\therefore T_{KIN}(0) = \frac{T_{eff}}{16}$; $T_{KIN}(2/3) = T_{eff}$

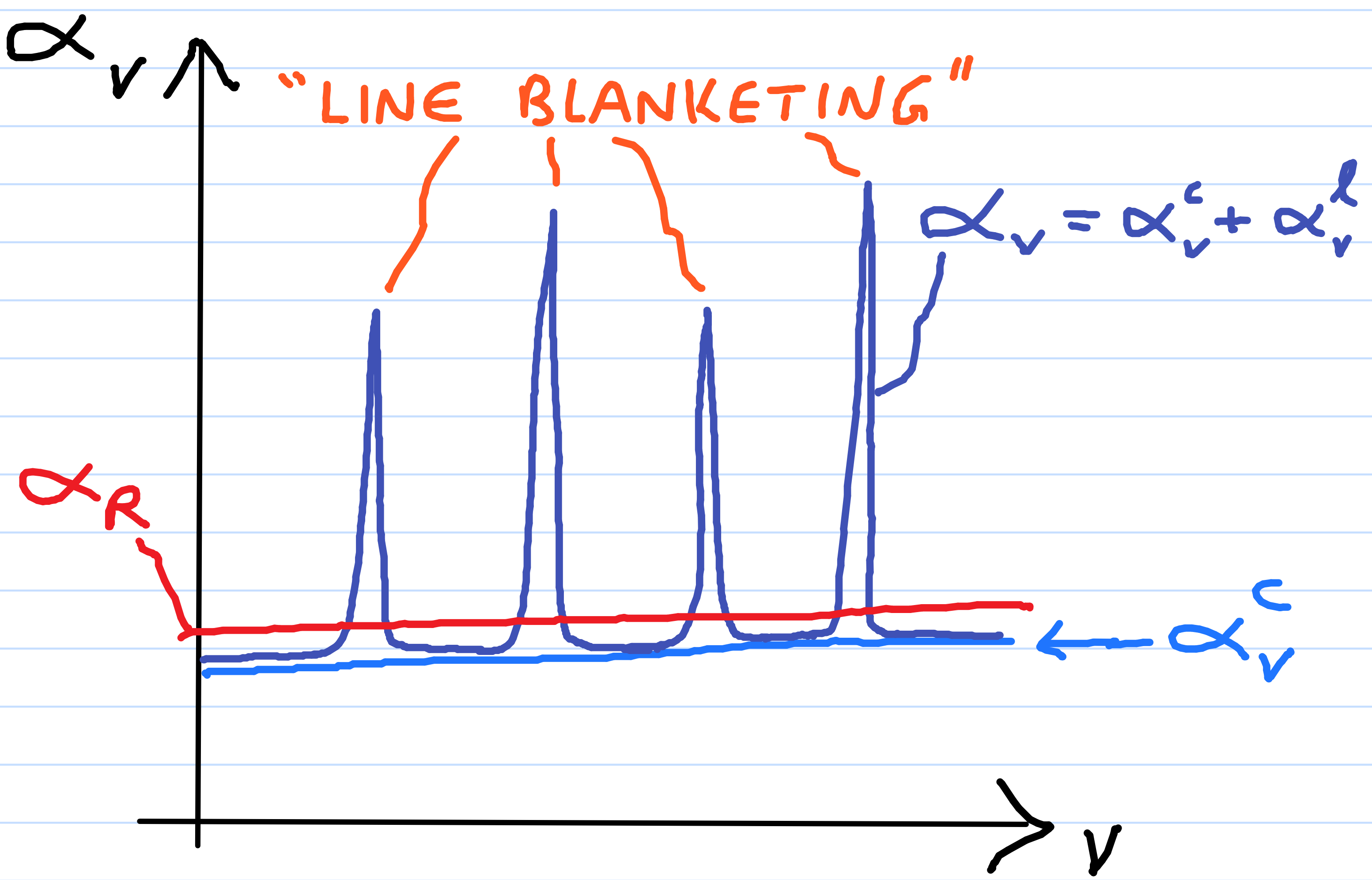
LINE-BLANKETED R.E. MODELS

- NON-GRAY: $\alpha_\nu(\nu) = \alpha_\nu^c + \alpha_\nu^\ell$

α_ν^c IS APPROXIMATELY GRAY

RECALL:

$\Delta\nu \approx$ FEW $\times 10^{15}$ Hz



$$\alpha_R > \alpha_\nu^c$$

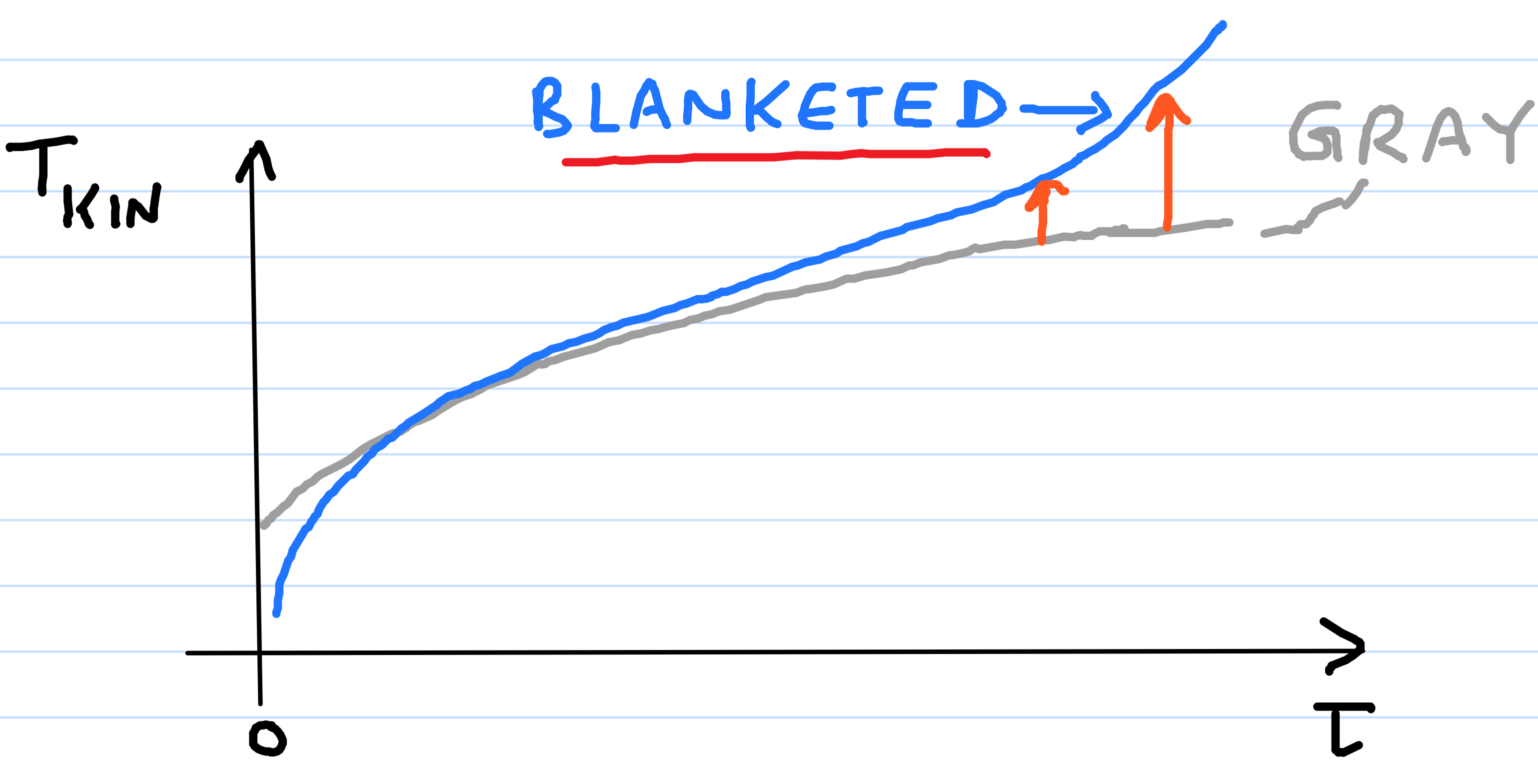
RECALL: DIFFUSION APPROX:

$$\tilde{F}(z) \approx -\frac{16}{3}\sigma \frac{1}{\alpha(z)} T_{KIN}^{3}(z) \frac{dT_{KIN}(z)}{dz}$$

AND $\alpha_R > \alpha_\nu^c$

$$\therefore \left|\frac{dT}{dz}\right|_{BLANKETED} > \left|\frac{dT}{dz}\right|_{GRAY}$$

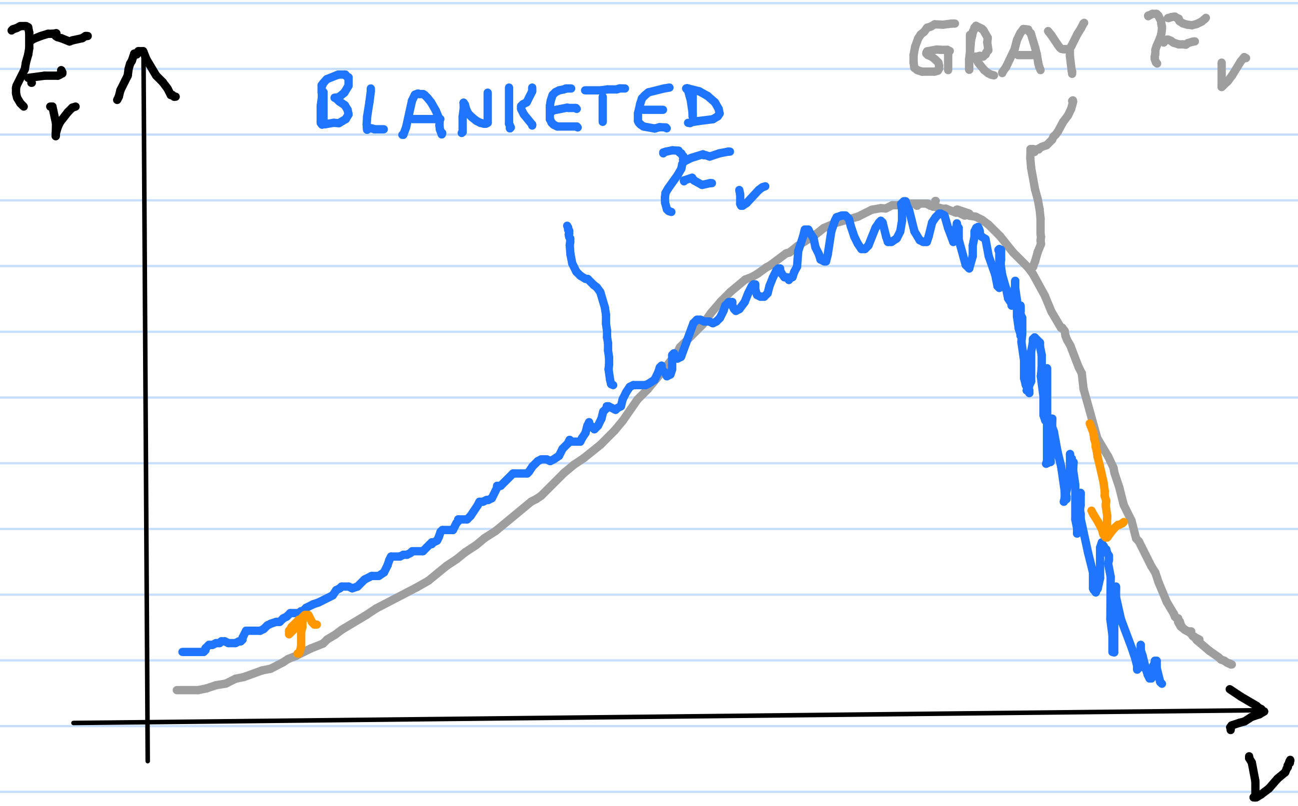
- BACK WARMING



FLUX RE-DISTRIBUTION:

SPECTRAL LINE DENSITY, $\frac{(\text{NO. LINES})}{\Delta\nu}$

INCREASES W. INCREASING ν



$$\mathcal{F}_{\text{BLANKETED}} = \mathcal{F}_{\text{GRAY}} = \sigma T_{\text{eff}}^4$$