APPROXIMATIONS AT LARGE DEPTH, $\tau_\nu \gg 1$:

- LOWER BOUNDARY CONDITION

DIFFUSION (ROSSELAND) APPROXIMATION:

AT SOME DEPTH, τ_ν:

$$S_\nu(\tau_\nu) \quad \& \quad \left.\frac{d^n S_\nu(t_\nu)}{dt_\nu^n}\right|_{t_\nu=\tau_\nu} \quad \text{KNOWN, ALL } n$$

THEN, AT ANY DEPTH, t_ν, TAYLOR SERIES FOR $S_\nu(t_\nu)$:

$$S_\nu(t_\nu) = \underbrace{S(\tau_\nu)}_{2^{nd}\text{ EDD.}} + (t_\nu-\tau_\nu)\overbrace{\left.\frac{dS_\nu(t_\nu)}{dt_\nu}\right|_{t_\nu=\tau_\nu}}^{\text{E-B}}$$

$$+ \frac{(t_\nu-\tau_\nu)^2}{2}\left.\frac{d^2S_\nu(t_\nu)}{dt_\nu^2}\right|_{t_\nu=\tau_\nu} + \cdots$$

FORMAL Soln FOR $I_\nu^+(\tau_\nu,\mu)$ & $I_\nu^-(\tau_\nu,\mu)$:

Eg. $$I_\nu^+(\tau_\nu,\mu) = \frac{1}{\mu}\int_{\tau_\nu}^{\infty} S_\nu(t_\nu)\, e^{-(t_\nu-\tau_\nu)/\mu}\, dt_\nu$$

$$I_\nu^-(\tau_\nu,\mu) = -\frac{1}{\mu}\int_{0}^{\tau_\nu} S_\nu(t_\nu)\, e^{-(t_\nu-\tau_\nu)/\mu}\, dt_\nu$$

IN LIMIT OF $\tau_\nu \to \infty$, FOR $-1 \le \mu \le 1$:

E-B

$$I_\nu(\tau_\nu,\mu) \approx S_\nu(\tau_\nu) + \mu \left.\frac{dS_\nu(t_\nu)}{dt_\nu}\right|_{\tau_\nu} + \mu^2 \left.\frac{d^2S(t_\nu)}{dt_\nu^2}\right|_{\tau_\nu} + \dots$$

2nd EDD.

-RUTTEN, p. 87-88

- FOR I_ν^+ & I_ν^-

ANGLE-MOMENTS:

$$J_\nu(\tau_\nu) \approx S_\nu(\tau_\nu) + \frac{1}{3} \left.\frac{d^2 S_\nu(t_\nu)}{dt_\nu^2}\right|_{\tau_\nu} + \cdots$$

- EVEN TERMS

$$F_\nu(\tau_\nu) \approx \frac{4\pi}{3} \left.\frac{dS_\nu(t_\nu)}{dt_\nu}\right|_{\tau_\nu} + \frac{4\pi}{5} \left.\frac{d^3 S_\nu(t_\nu)}{dt_\nu^3}\right|_{\tau_\nu} + \cdots$$

- ODD TERMS

$$K_\nu(\tau_\nu) \approx \frac{1}{3} S_\nu(\tau_\nu) + \frac{1}{5} \left.\frac{d^2 S_\nu(t_\nu)}{dt_\nu^2}\right|_{\tau_\nu} + \cdots$$

- EVEN TERMS

AT $\tau_\nu \to \infty$, $S_\nu(\tau_\nu) \approx \mathcal{O}(1)$:

$$S_\nu(t_\nu) \approx S_\nu(\tau_\nu) + (t_\nu - \tau_\nu) \left.\frac{dS_\nu(t_\nu)}{dt_\nu}\right|_{\tau_\nu}$$

$\therefore$ TO $\mathcal{O}(1)$:

$$I_\nu(\tau_\nu, \mu) \approx S_\nu(\tau_\nu) + \mu \left.\frac{dS_\nu(t_\nu)}{dt_\nu}\right|_{\tau_\nu}$$

$$J_\nu(\tau_\nu) \approx S_\nu(\tau_\nu) \quad \text{——} \quad (1)$$

$$F_\nu(\tau_\nu) \approx \frac{4\pi}{3} \left.\frac{dS_\nu(t_\nu)}{dt_\nu}\right|_{\tau_\nu}$$

$$K_\nu(\tau_\nu) \approx \frac{1}{3} S_\nu(\tau_\nu) \quad \text{——} \quad (2)$$

Eqs. (1) & (2) :

$$K_\nu(\tau_\nu) = \frac{1}{3} J_\nu(\tau_\nu)$$

- 1ST EDDINGTON APPROX.

$$\therefore \frac{c}{4\pi} P_\nu(\tau_\nu) = \frac{1}{3} \frac{c}{4\pi} \mu_\nu(\tau_\nu)$$

$$\therefore P_\nu(\tau_\nu) = \frac{1}{3} \mu_\nu(\tau_\nu)$$

- SENSE

LTE: $S_\nu(\tau_\nu) = B_\nu(T_{KIN}(\tau_\nu))$:

$$I_\nu(\tau_\nu, \mu) \approx B_\nu(\tau_\nu) + \mu \left.\frac{dB_\nu(t_\nu)}{dt_\nu}\right|_{\tau_\nu}$$

$$J_\nu(\tau_\nu) \approx B_\nu(\tau_\nu)$$ — SAME AS T.E.

$$F_\nu(\tau_\nu) \approx \frac{4\pi}{3} \frac{dB_\nu(\tau_\nu)}{d\tau_\nu}$$

RECALL:

IN 1D MODEL, RADIAL τ_ν-SCALE:

$$d\tau_\nu(z) = -\alpha_\nu(z)\, dz$$

$$F_\nu(z) \approx -\frac{4\pi}{3} \frac{1}{\alpha_\nu(z)} \frac{dB_\nu(z)}{dz}$$

- MONOCHROMATIC

BOLOMETRIC FLUX, $\mathcal{F}$:

$$\mathcal{F}(z) \approx -\frac{4\pi}{3}\int_{\nu=0}^{\infty} \frac{1}{\alpha_\nu(z)} \frac{dB_\nu(T(z))}{dz}\, d\nu$$

$$= -\frac{4\pi}{3}\left\langle \frac{1}{\alpha_\nu(z)} \right\rangle \frac{dT(z)}{dz} \int_{\nu=0}^{\infty} \frac{dB_\nu(T)}{dT}\, d\nu$$

STEFAN-BOLTZMANN VARIATION:

$$\int_0^{\infty} \frac{B_\nu(T)}{dT}\, d\nu = \frac{4\sigma}{\pi} T^3$$

$$\therefore\ \mathcal{F}(z) = -\frac{16\sigma}{3}\left\langle \frac{1}{\alpha_\nu(z)} \right\rangle T_{KIN}^3(z) \frac{dT_{KIN}(z)}{dz}$$

$$= -\beta(z)\, \frac{dT_{KIN}(z)}{dz}$$

- DIFFUSION Eq.

∴ DIFFUSION APPROX. $(\tau_\nu \to \infty)$:

$$\frac{dT_{KIN}(z)}{dz} = -\frac{\mathcal{F}(z)}{\beta(z)}$$

- DIFFUSION CONSTANT:

$$\beta(z) = \frac{16\sigma}{\pi} \left\langle \frac{1}{\alpha_\nu(z)} \right\rangle T_{KIN}(z)$$

- $\left\langle \frac{1}{\alpha_\nu} \right\rangle$ = ν-AVERAGED MEAN EXINCTION COEFFICIENT (cm^{-1})

ROSSELAND MEAN EXTINCTION COEFFICIENT, α_R (cm^{-1}):

$$\frac{1}{\alpha_R(z)} \equiv \frac{\int_0^\infty \frac{1}{\alpha_\nu(z)} \frac{dB_\nu(z)}{dT}\, d\nu}{\int_0^\infty \frac{dB_\nu(z)}{dT}\, d\nu}$$

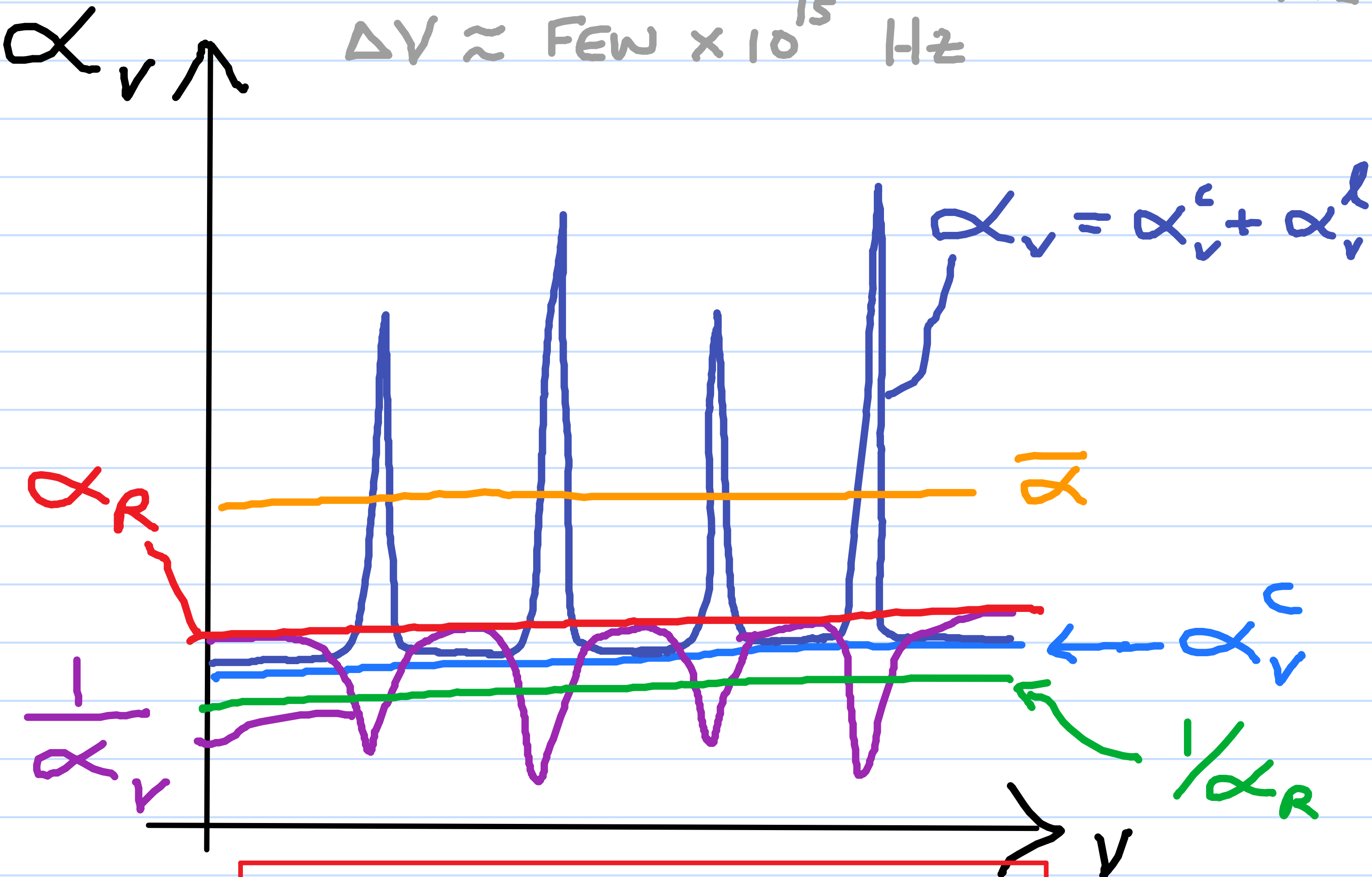
- HARMONIC MEAN

CONTRAST:

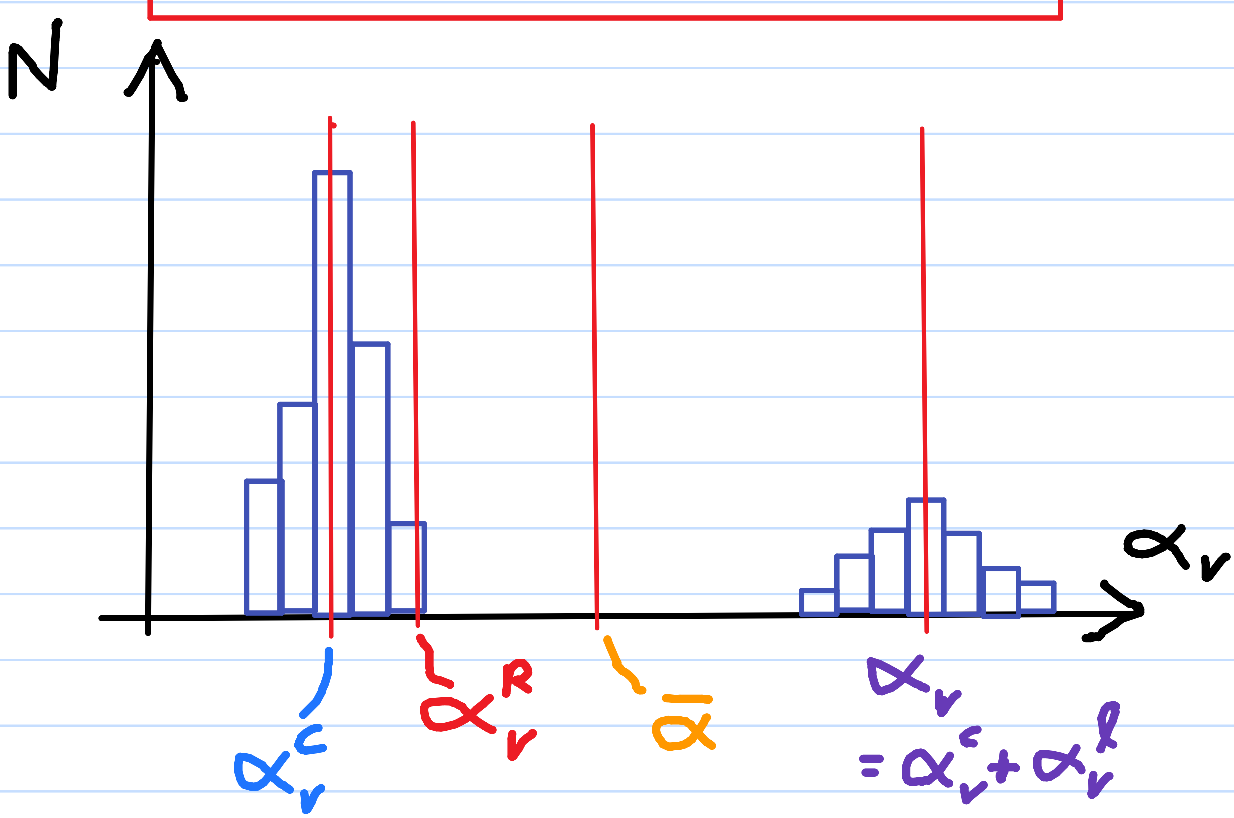
STRAIGHT MEAN: $\bar{\alpha} = \frac{\int_0^\infty w_\nu \alpha_\nu \, d\nu}{\int_0^\infty w_\nu \, d\nu}$

- w_ν = WEIGHT fn

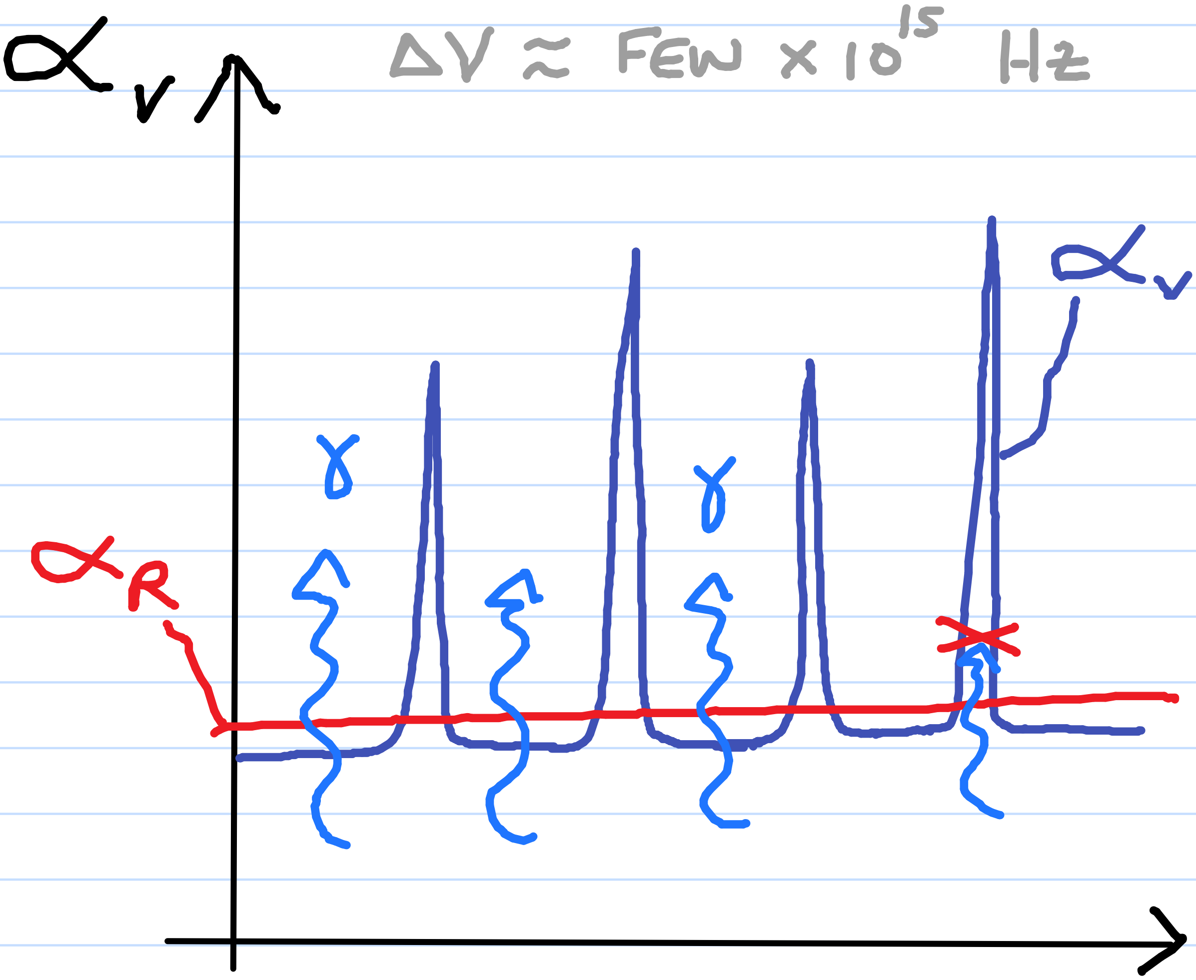
$\Delta\nu \approx$ FEW $\times 10^{15}$ Hz



$$\boxed{\alpha_R \gtrsim \alpha_\nu^c < \bar{\alpha}}$$



α_R IS A FLUX-WEIGHTED MEAN EXTINCTION



$$K_R(z) = \frac{\alpha_R(z)}{\rho} \quad (cm^2/g)$$

$$d\tau_R(z) = -\alpha_R(z)\,dz$$

- ROSSELAND τ-SCALE

THEN:

$$\mathcal{F}(z) \approx -\frac{16}{3}\sigma \frac{1}{\alpha_R(z)} T_{KIN}^3(z) \frac{dT_{KIN}(z)}{dz}$$

- IN LTE (ASSUMING $T_{RAD}(z) = T_{KIN}(z)$)