

## ATMOSPHERIC APPROXIMATIONS:

### SURFACE APPROXIMATIONS ( $\tau_v = 0$ ):

- SIGNIFICANT:  $I_\nu(0, \mu)$ ,  $\mathcal{F}_\nu(0)$   
OBSERVABLE

1) 2<sup>nd</sup> EDDINGTON APPROX.:

- 3D HOMOGENEITY  
 $\therefore$  ISOTHERMAL ( $T_{KIN}(\tau_v) = C$ )  
 (i.e. "LAMBERT RADIATOR")

$$\therefore \Sigma_\nu(\tau_v) = B_\nu(T_{KIN} = C) = \underline{a_\nu}$$

146

FORMAL Sol'n AT SURFACE ( $T_r = 0$ ):

$$I_r^-(0, \mu) = 0, \quad -1 < \mu < 0$$

$$I_r^+(0, \mu) = \frac{1}{\mu} \int_{t_r=0}^{\infty} S_v(t_r) e^{-t_r/\mu} dt_r$$

$$= a_0 \int_0^{\infty} \frac{e^{-t_r/\mu}}{\mu} dt_r = -a_0 \left( e^{-t_r/\mu} \right) \Big|_{t_r=0}^{\infty}$$

$$= a_0, \quad 0 < \mu < 1$$

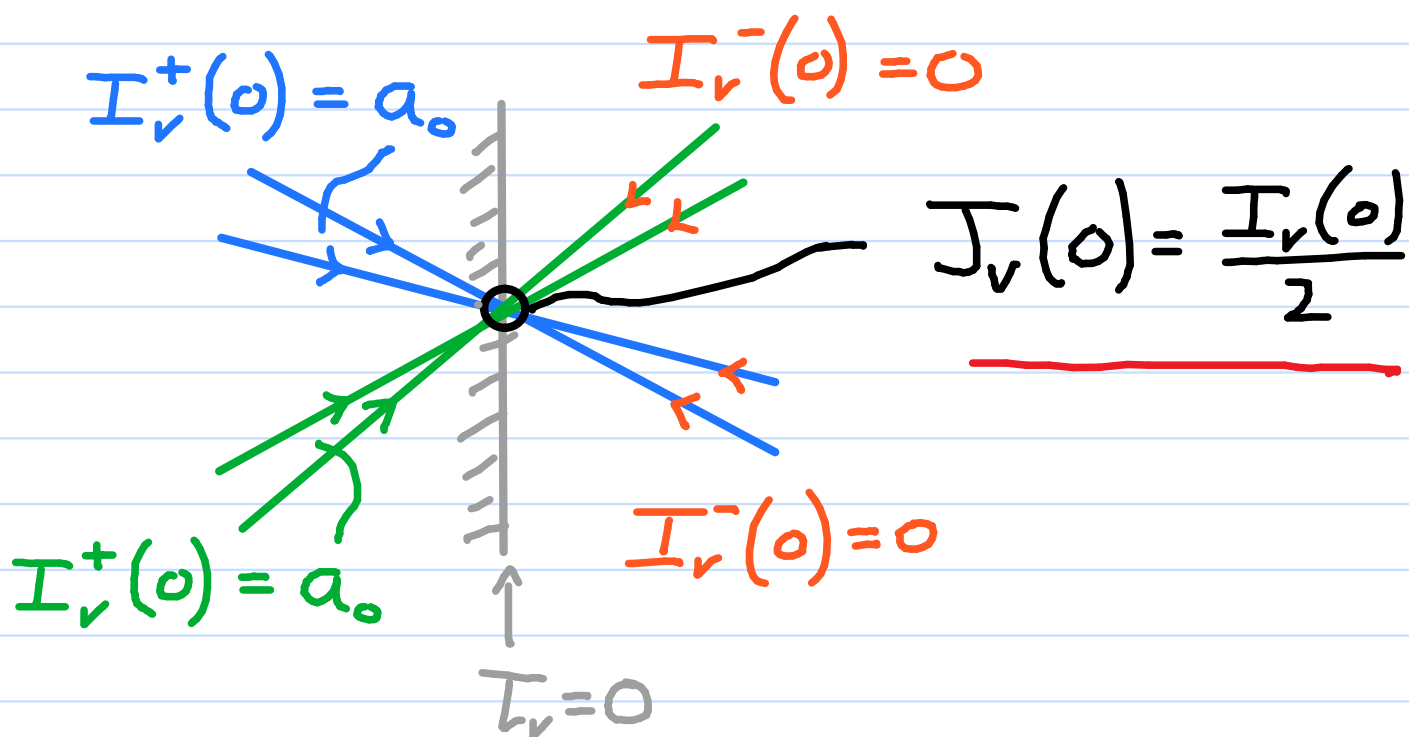
$$\therefore I_r^+(0, \mu) = a_0 = \underline{S_v}, \quad 0 < \mu < 1$$

THEN:

$$J_v(o) = \frac{1}{2} \int_{-1}^1 I_v(o, \mu) d\mu$$

$$= \frac{1}{2} \int_{-1}^0 I_v^-(o, \mu) d\mu + \frac{1}{2} \int_0^1 I_v^+(o, \mu) d\mu$$

$$= \frac{a_o}{2} \int_0^1 d\mu = \frac{a_o}{2} = \frac{I_v^+(o)}{2}$$



SIMILARLY:

$$F_{\nu}^{+}(0) = 2\pi \int_0^1 \mu I_{\nu}^{+}(0, \mu) d\mu = \pi a_0$$

$$= \pi I_{\nu}^{+}(0) = 2\pi J_{\nu}(0)$$

2<sup>nd</sup> EDDINGTON APPROXIMATION:

ANY REAL STAR :

$$F_{\nu}(0) \approx 2\pi J_{\nu}(0)$$

## 2) EDDINGTON - BARBIER (E-B)

### APPROXIMATION

ASSUME:

$$S_{\nu}(T_{\nu}) = a_0 + a_1 T_{\nu} ; a_0 \& a_1 > 0$$

- IMPOSSIBLE FOR  $B_{\nu}(T_{\nu})$  AT ALL  
V VALUES

FORMAL Sol'n AT  $\tau_v = 0$ :

FOR  $0 < \mu < 1$ :

$$I_v^+(0, \mu) = \frac{1}{\mu} \int_0^{\infty} (a_0 + a_1 t_v) e^{-t_v/\mu} dt_v$$

$$= a_0 \int_0^{\infty} \frac{e^{-t_v}}{\mu} dt_v + a_1 \int_0^{\infty} \frac{t_v e^{-t_v/\mu}}{\mu} dt_v$$

$$I_v^+(0, \mu) = a_0 + a_1 \mu$$

$$\therefore I_v^+(0, \mu) = S_v(\tau_v = \mu)$$

EDDINGTON-BARBIER (E-B)  
RELATION

AND:

$$J_{\nu}(0) = a_0/2 + a_1/4 \\ = 1/2 S_{\nu}(\tau_{\nu} = 1/2)$$

$$K_{\nu}^{+}(0) = \pi a_0 + \frac{2\pi}{3} a_1 \\ = \pi S_{\nu}(\tau_{\nu} = 2/3)$$

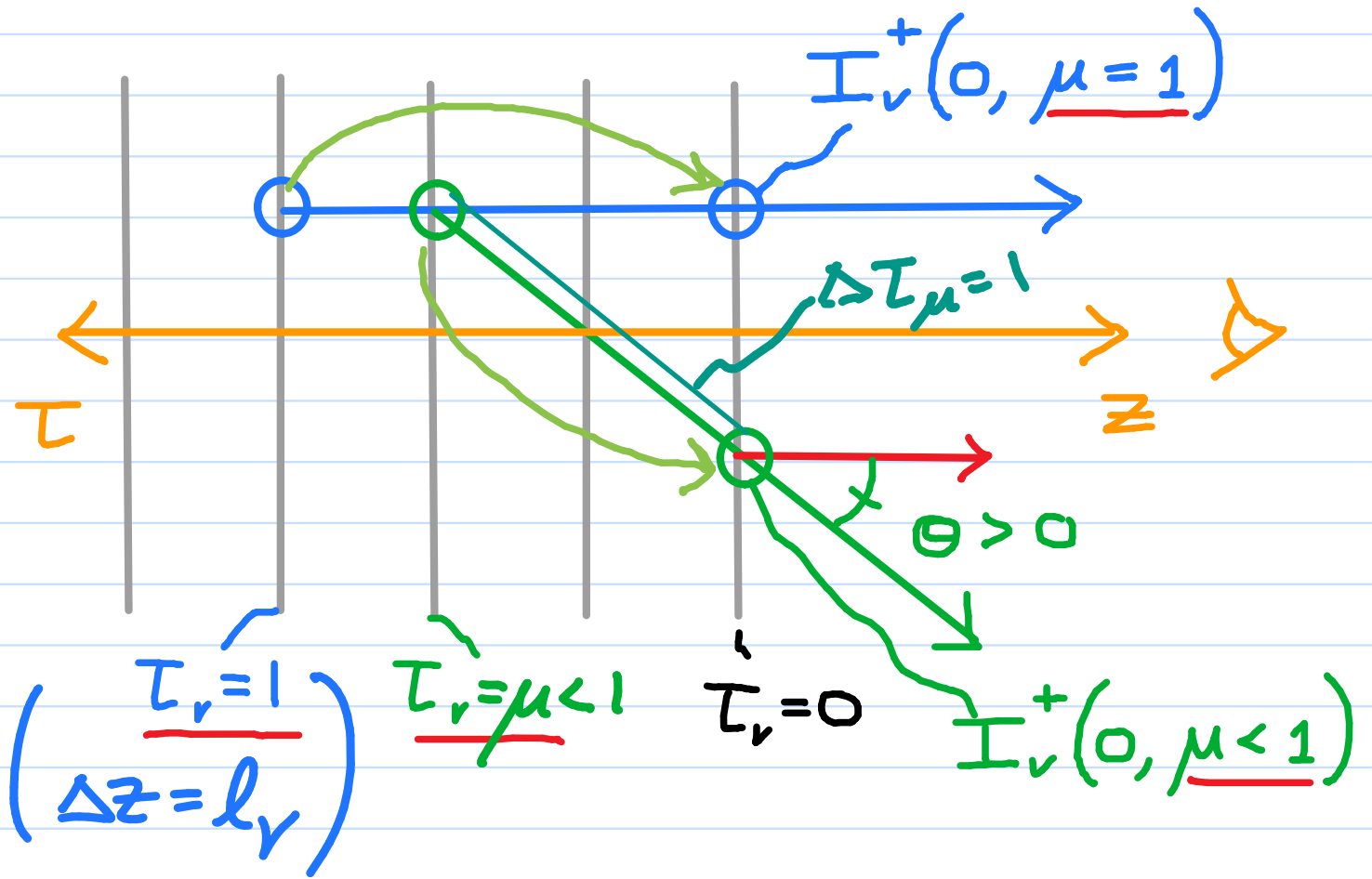
EDDINGTON - BARBIER (E-B)  
APPROXIMATION:

ANY REAL STAR :

$$\therefore I_{\nu}^{+}(0, \mu) \approx S_{\nu}(\tau_{\nu} = \mu)$$

EDDINGTON - BARBIER (E-B)  
APPROXIMATION

$$\therefore I_v^+(0, \mu) \approx S_v(\tau, \mu) :$$



FOR  $\mu=1$  (RADIAL BEAM):

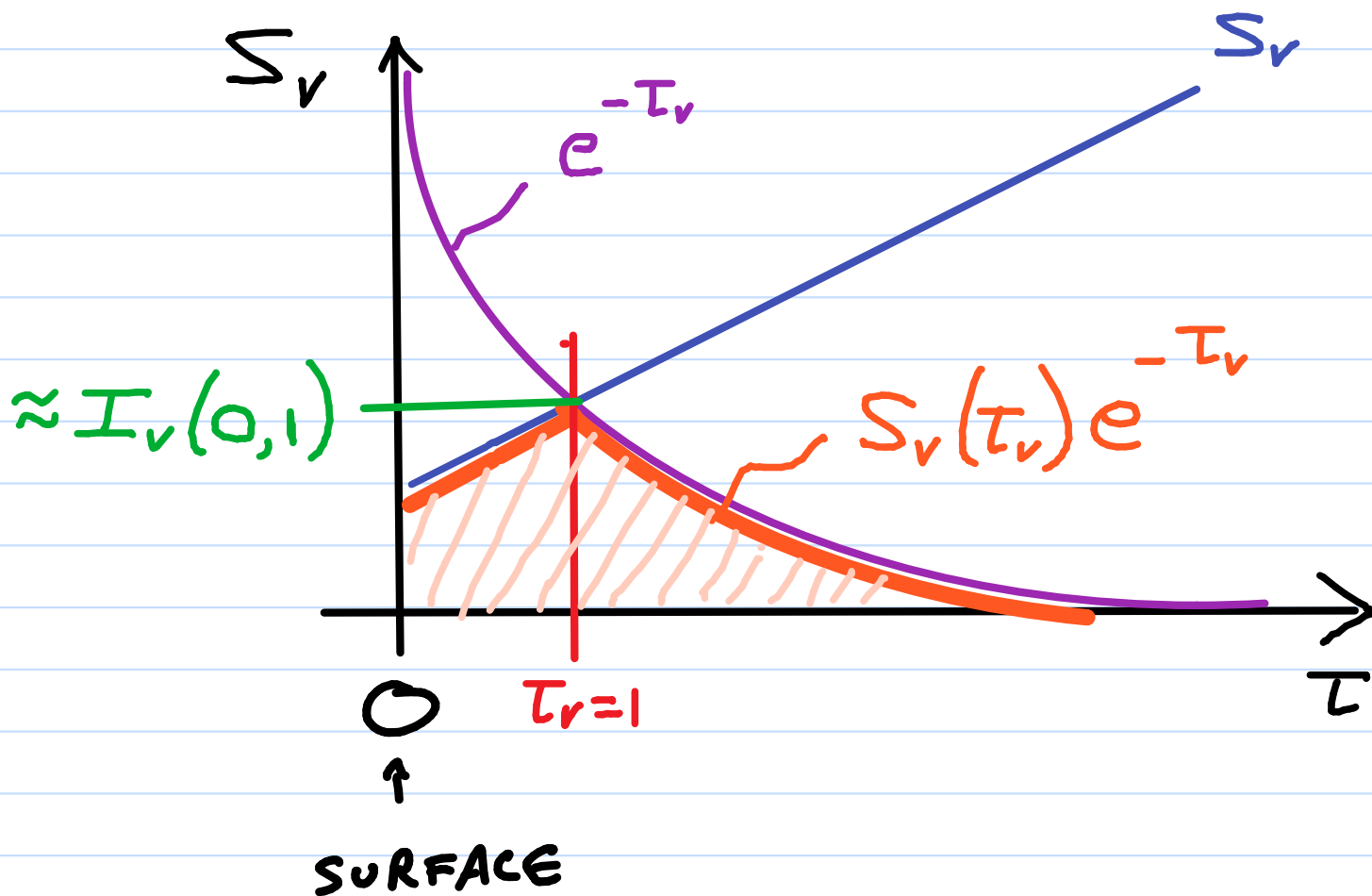
$$I_v^+(0, 1) \approx S_v(1)$$

FOR  $\mu=1$  (RADIAL BEAM):

$$I_v^+(0, 1) \approx S_v(1)$$

FORMAL Soln:

$$I_v^+(0, 1) = \int_0^{\infty} S_v(t_v) e^{-t_v} dt_v \approx S_v(\tau_v=1)$$



FOR  $0 < \mu < 1$ :

$$I_{\nu}^{+}(0, \mu) \approx S_{\nu}(\mu)$$

FORMAL Soln.

$$I_{\nu}^{+}(0, \mu) = \int_0^{\infty} S_{\nu}(t_{\nu}) e^{-t_{\nu}/\mu} \frac{dt_{\nu}}{\mu} \approx S_{\nu}(T_{\nu}/\mu = 1) = S_{\nu}(T_{\nu} = \mu)$$

LTE E-B RELATION:

$$\text{LTE: } S_{\nu}(T_{\nu}) = \underline{B_{\nu}}(T_{\text{KIN}}(T_{\nu})) = "B_{\nu}(T_{\nu})"$$

$\therefore$  E-B RELATION:

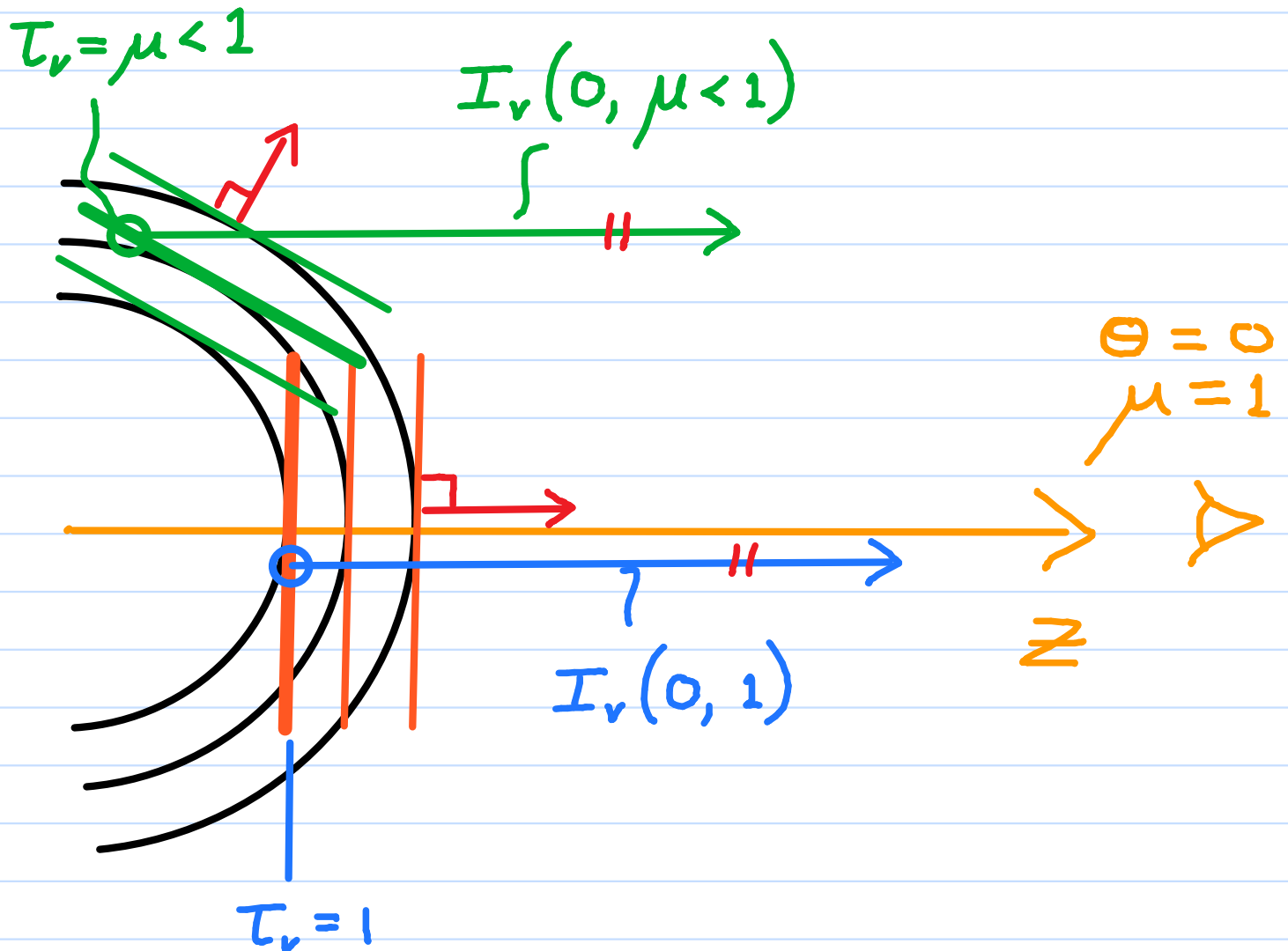
$$I_{\nu}^{+}(0, \mu) = B_{\nu}(T_{\text{KIN}}(T_{\nu} = \mu)) = B_{\nu}(\mu)$$

CAN MEASURE  $T_{\text{KIN}}(T_{\nu})$

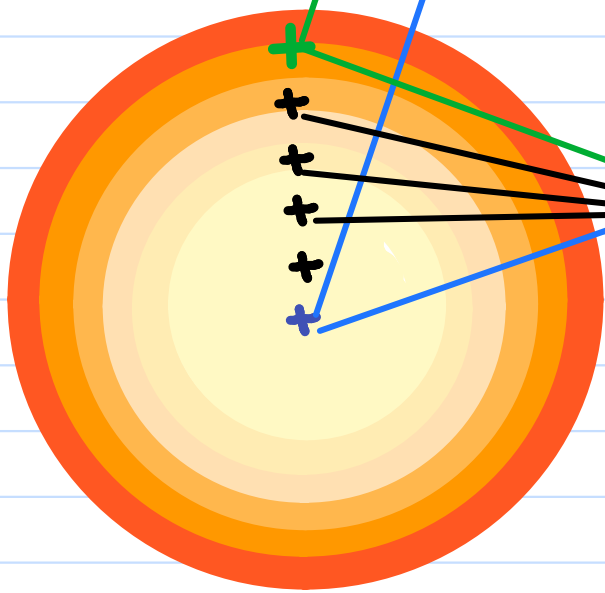
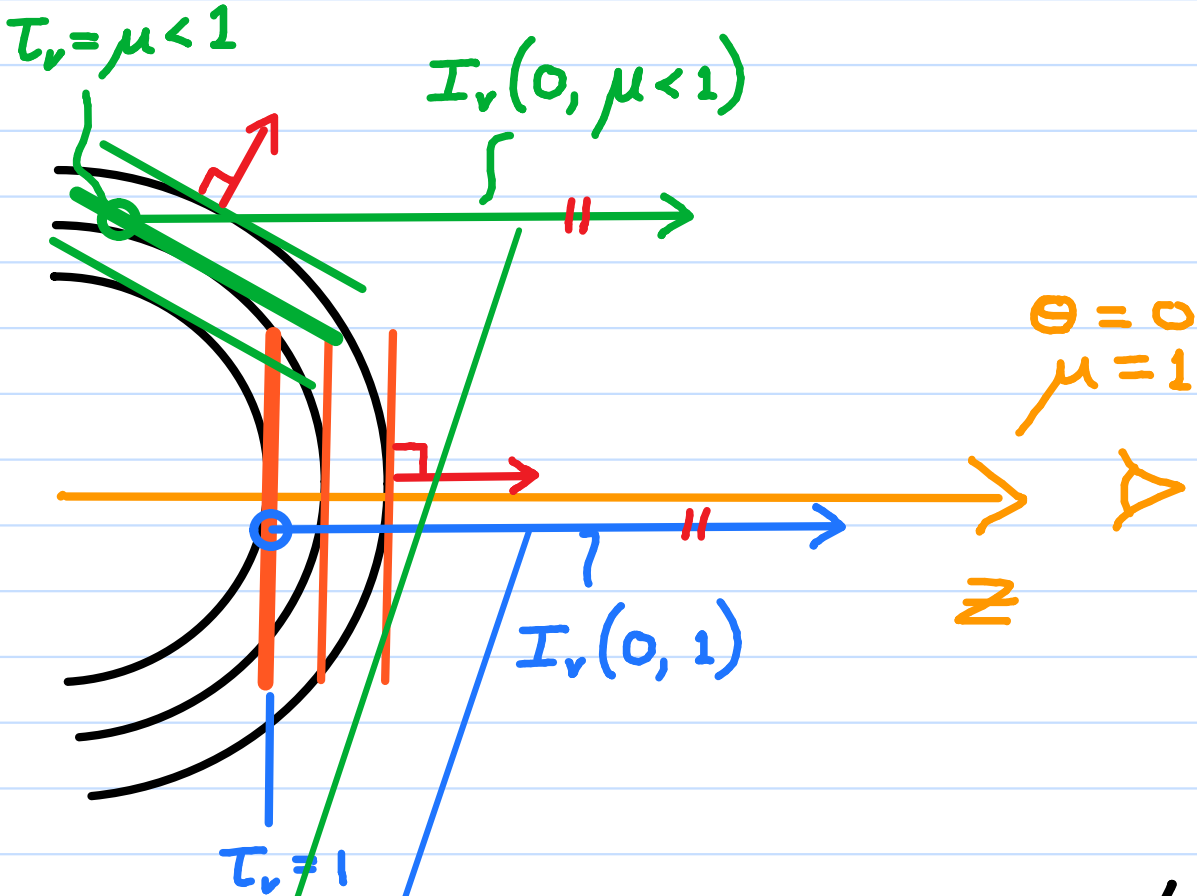
# APPLICATIONS OF THE LTE E-B RELATION:

## 1) SOLAR LIMB DARKENING:

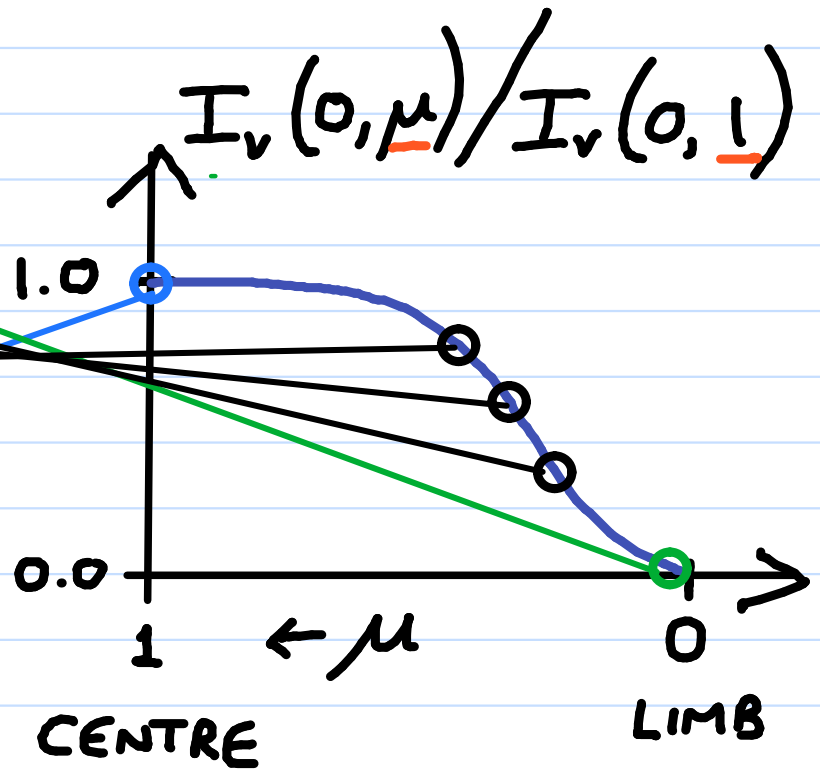
SUN IS SPATIALLY RESOLVED:



1) SOLAR LIMB DARKENING:



ON DETECTOR



LIMB-DARKENING CURVE (LDC)

LTE E-B RELATION:

$$I_{\nu}^{+}(0, \underline{\mu}) \approx B_{\nu}(\tau_{\nu} = \underline{\mu})$$

IN SOLAR ATMOSPHERE:

$$\frac{dT_{\text{KIN}}}{d\tau_{\nu}} > 0$$

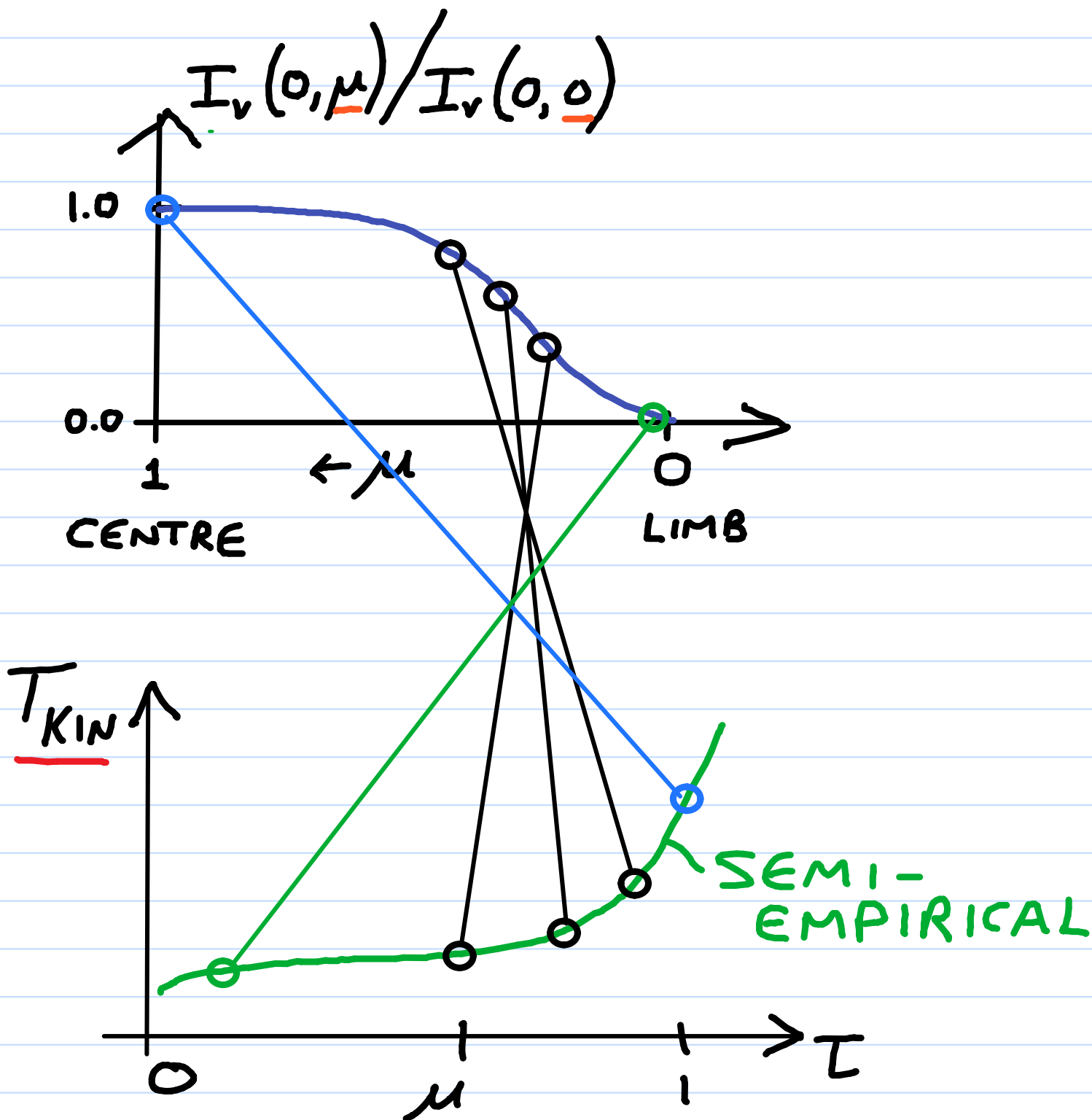
$$\therefore T_{\text{KIN}}(\tau_{\nu} = 1) > T_{\text{KIN}}(\tau_{\nu} = \mu < 1)$$

$$\therefore B_{\nu}(\tau_{\nu} = 1) > B_{\nu}(\tau_{\nu} = \mu < 1)$$

$$\therefore \underline{I_{\nu}^{+}(0, \tau_{\nu} = 1) > I_{\nu}^{+}(0, \mu < 1)}$$

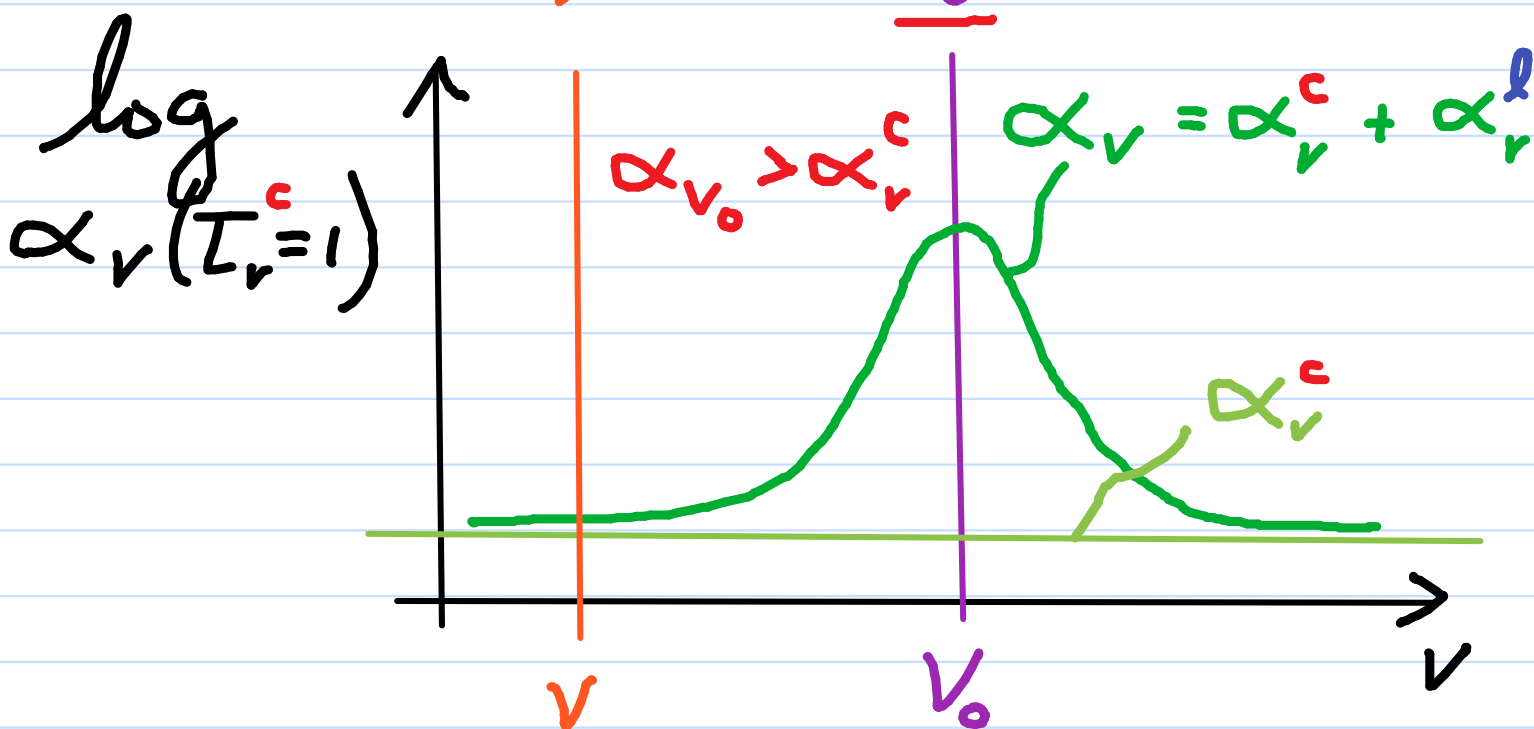
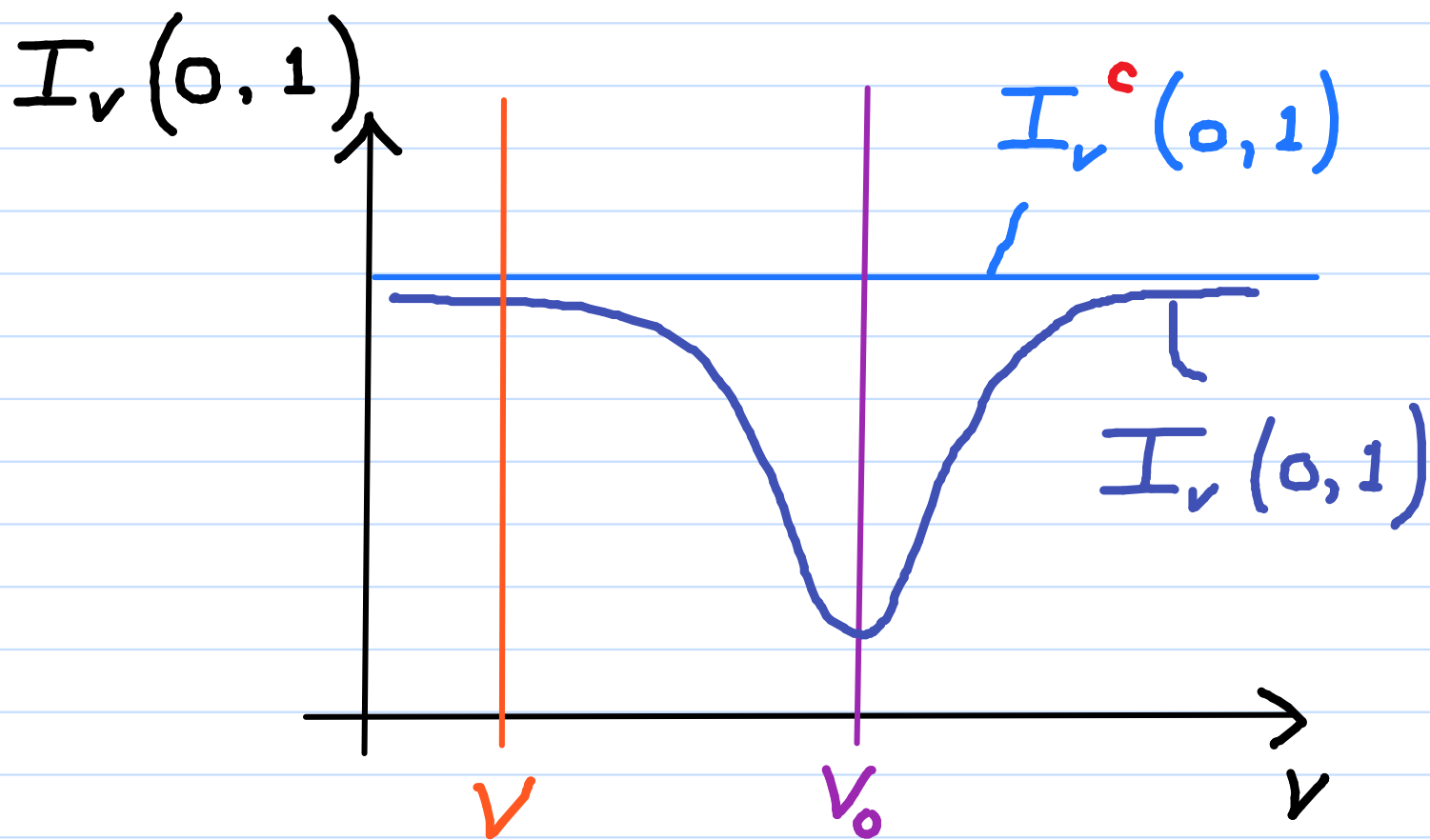
USEFUL: USE LDC TO MEASURE  
 $T_{KIN}(L_v)$  STRUCTURE

$$I_v^+(0, \mu) \approx B_v(T_{KIN}(L_v = \mu))$$



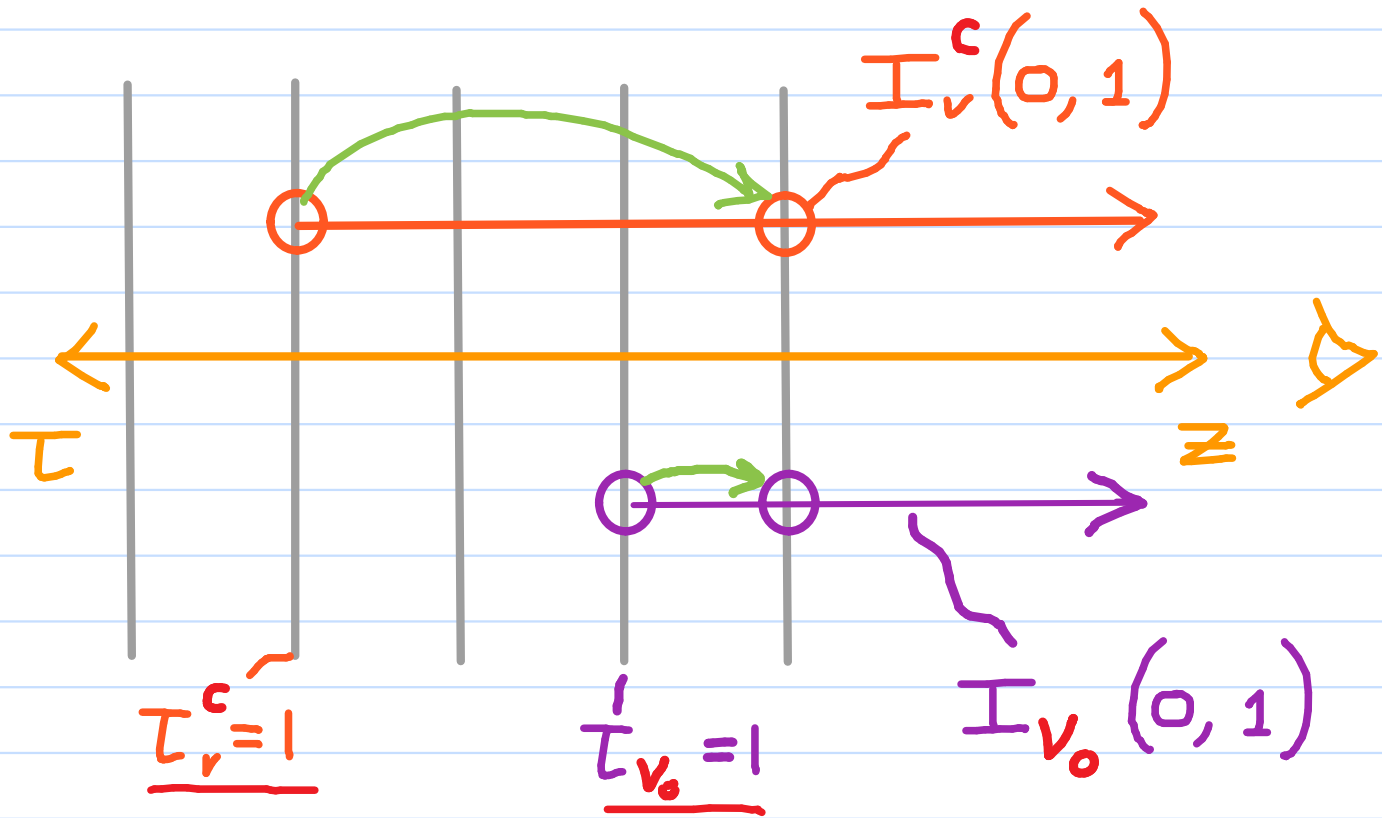
## 2) SPECTRAL ABSORPTION LINES:

$\mu = 1$  (RADIAL):



FOR  $\mu=1$  (RADIAL BEAM):

$$I_v^+(0, 1) \approx S_v(1)$$



MONOCHROMATIC  $T_v$ -SCALES AT

$v = v_0$  &  $v \approx v_0$ :

$$|dT_{v_0}(z)| = \alpha_{v_0}(z) dz$$

$$|dT_v^c(z)| = \alpha_v^c(z) dz$$

RECALL:  $\alpha_{\nu_0}(z) > \alpha_{\nu}^c(z)$ , ALL  $z$

$$\therefore z(\tau_{\nu_0}=1) > z(\tau_{\nu}^c=1)$$

$$\frac{dT_{KIN}(z)}{dz} < 0$$

$$\therefore \frac{dB_{\nu}(z)}{dz} < 0$$

LTE E-B RELATION:

$$I_{\nu}^+(0,1) \approx B_{\nu}(z(\tau_{\nu}=1))$$

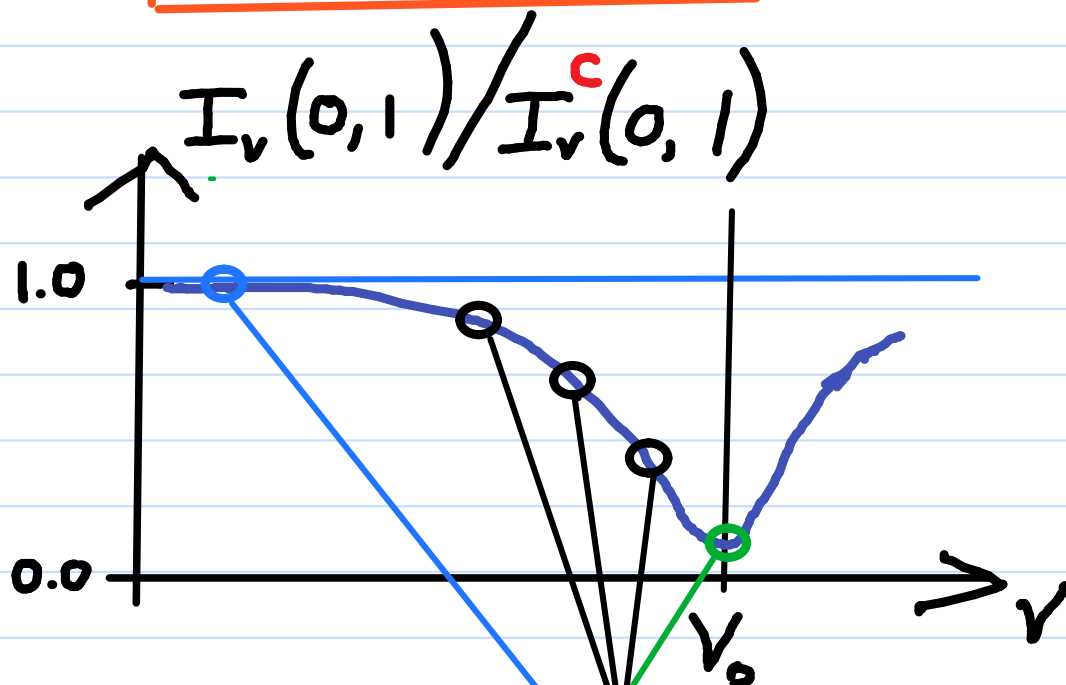
$$\therefore B_{\nu}(z(\tau_{\nu_0}=1)) < B_{\nu}(z(\tau_{\nu}^c)=1)$$

$$\therefore I_{\nu_0}(0,1) < I_{\nu}^c(0,1)$$

USEFUL: USE BROAD LINE TO

MEASURE  $T_{KIN}(\tau_\nu)$  STRUCTURE

$$I_\nu^+(0, 1) \approx B_\nu(T_{KIN}(z(\tau_\nu=1)))$$



Eg. CaII K  
LINE IN SUN

