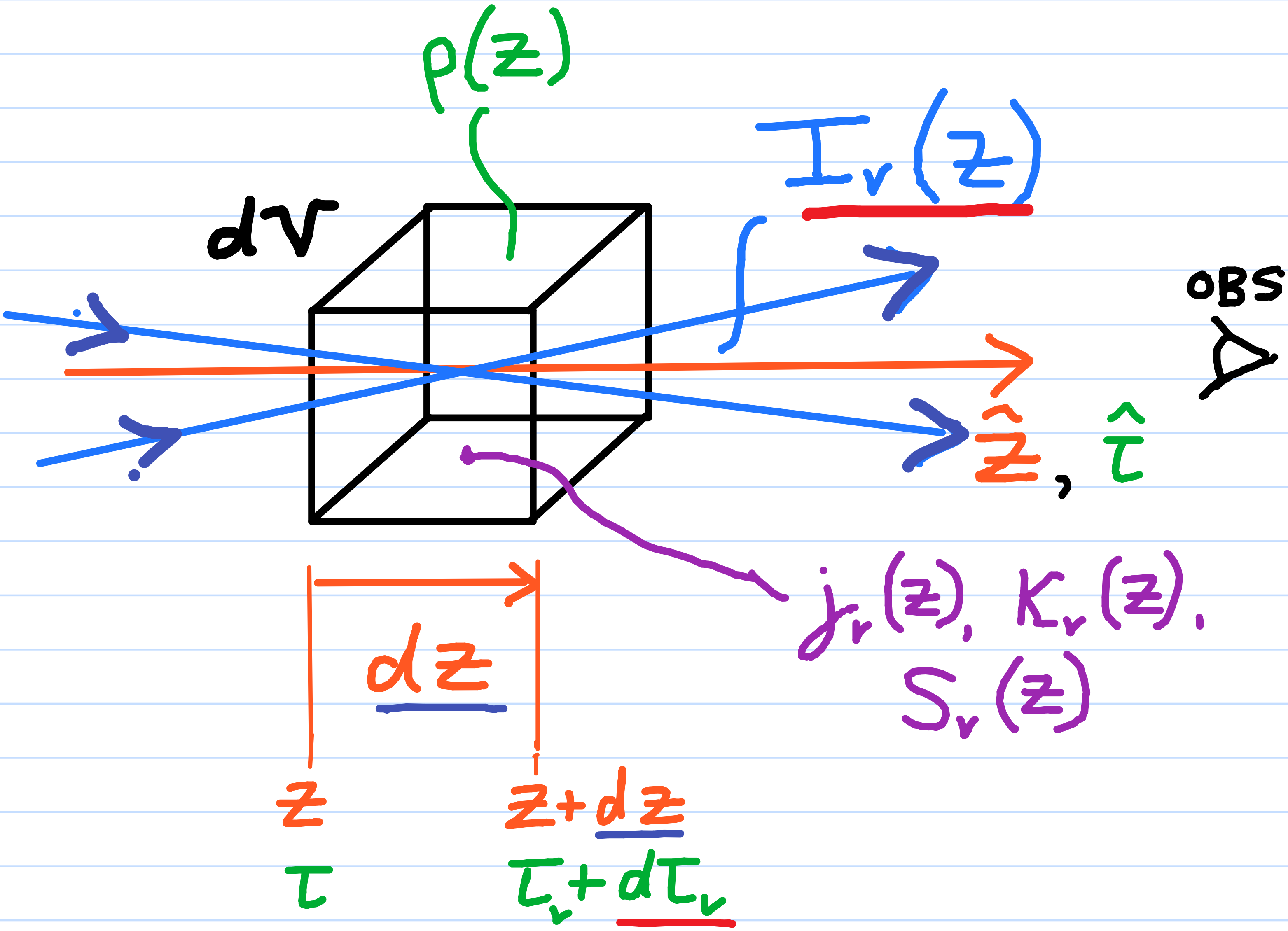
MONOCHROMATIC RADIATIVE TRANSFER:



ALONG BEAM:

$$dI_\nu(z) = I_\nu(z+dz) - I_\nu(z)$$

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$\therefore$ CONSERVATION OF E_ν:

$$dI_\nu(z) = \underline{j_\nu}(z)\,dz - \underline{\alpha_\nu}(z)\,\underline{I_\nu}(z)\,dz$$

$\therefore$ RADIATIVE TRANSFER EQ.:

$$\boxed{\frac{dI_\nu(z)}{dz} = j_\nu(z) - \alpha_\nu(z)\, I_\nu(z)}$$

γ-SOURCE γ-SINK

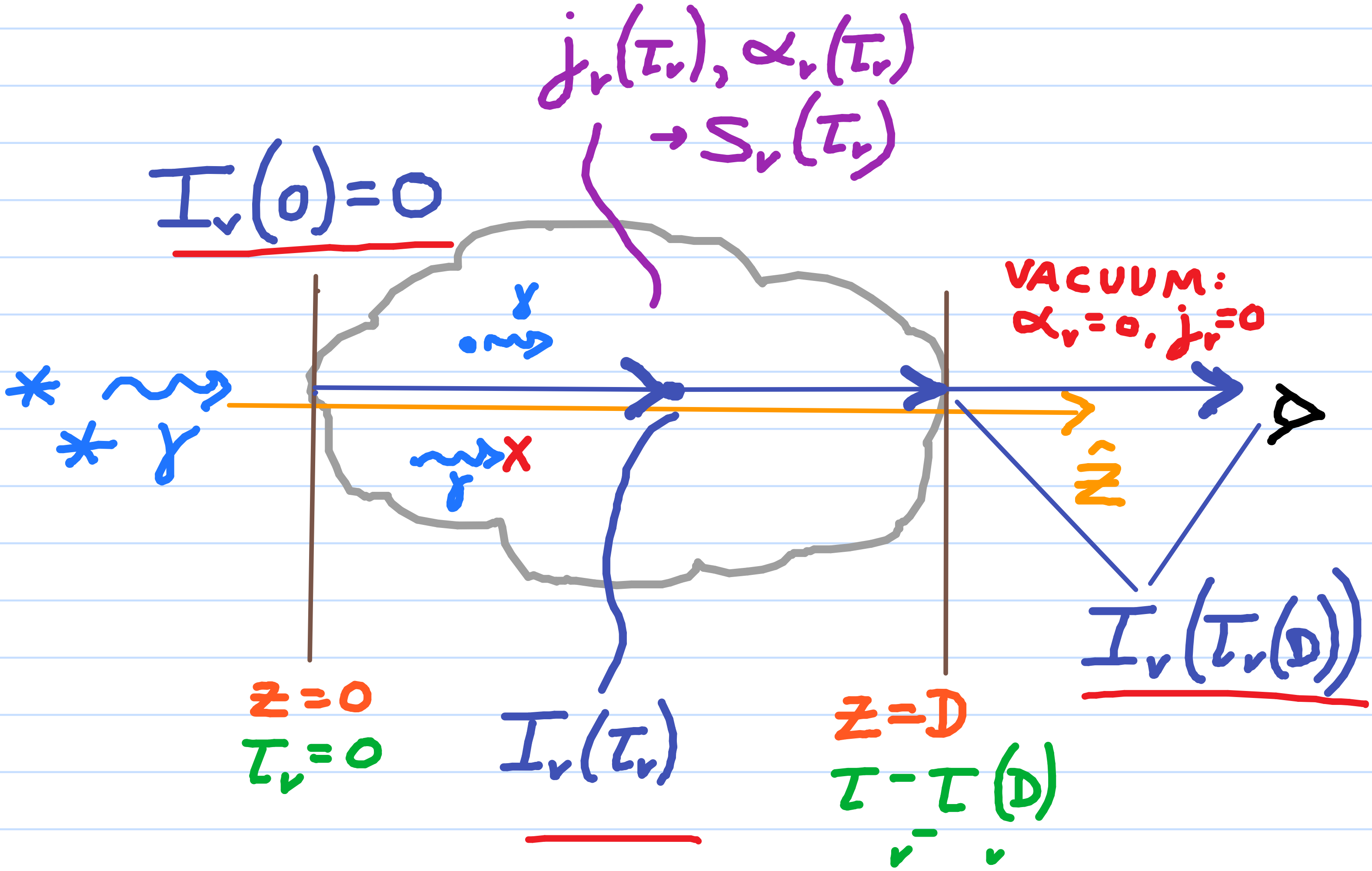
$$\frac{dI_\nu(z)}{\alpha_\nu(z)\,dz} = \frac{j_\nu(z)}{\alpha_\nu(z)} - I_\nu(z)$$

$\therefore$ RADIATIVE TRANSFER EQ.:

$$\frac{dI_\nu(I_\nu)}{dI_\nu} = \underbrace{S_\nu(I_\nu)}_{\gamma\text{-SOURCE}} - \underbrace{I_\nu(I_\nu)}_{\gamma\text{-SINK}}$$

1ST ORDER NON-LINEAR O. D. E.

FORMAL SOLUTION OF RAD. TRANS. Eq.:

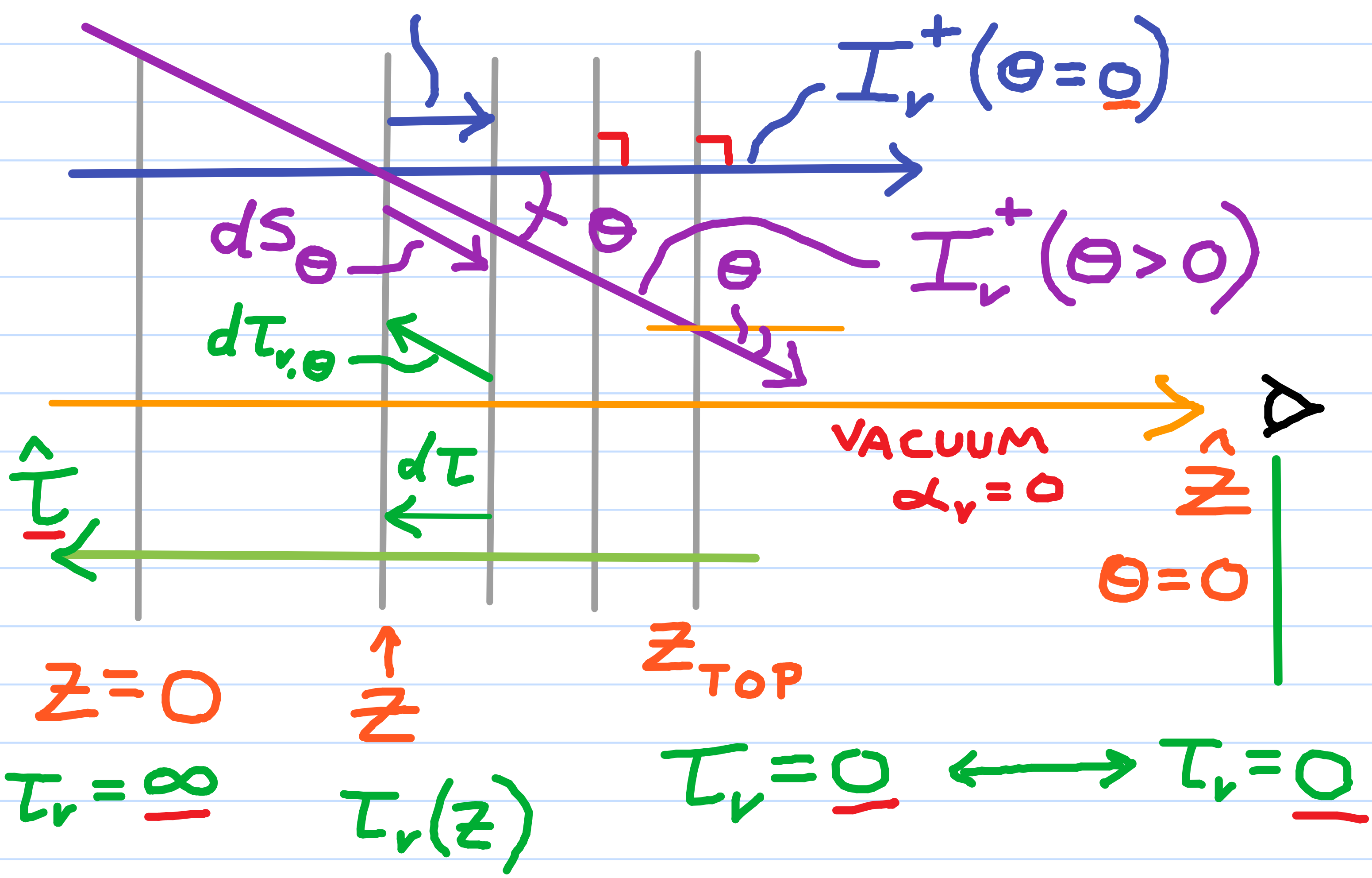


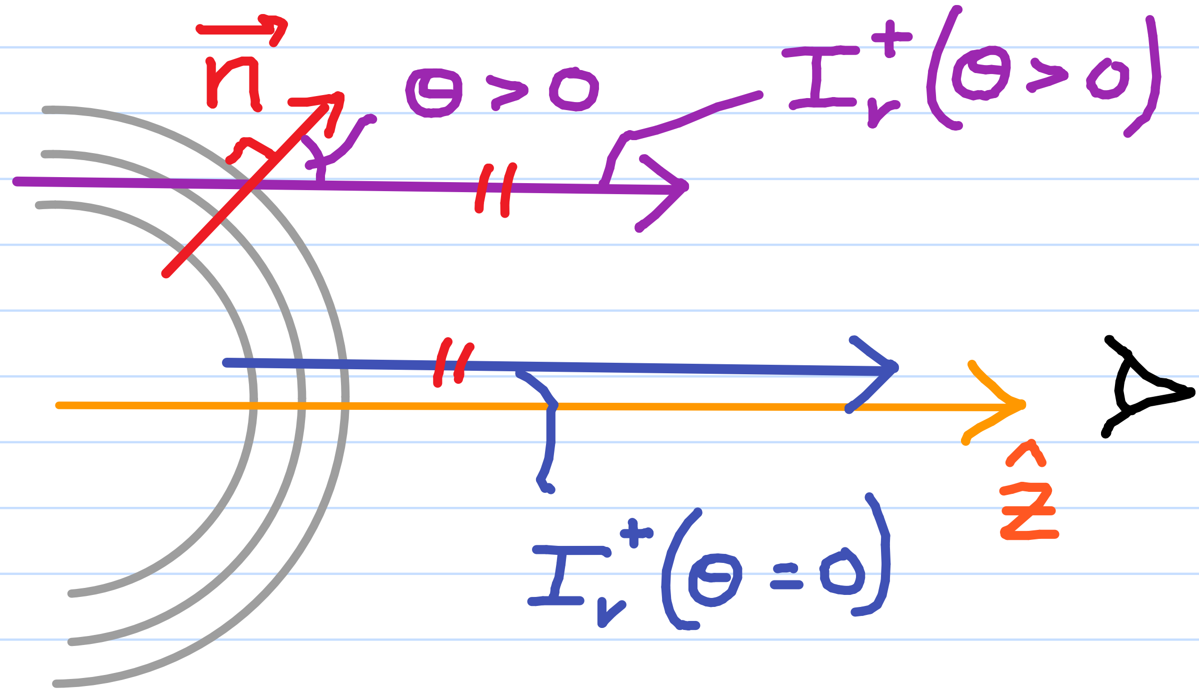
$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_{t_\nu=0}^{\tau_\nu} S_\nu(t_\nu)e^{-(\tau_\nu - t_\nu)}\, dt_\nu$$

- FORMAL SOLUTION

RAD. TRANS. Eq. IN 1D MODEL:

RADIAL BEAM





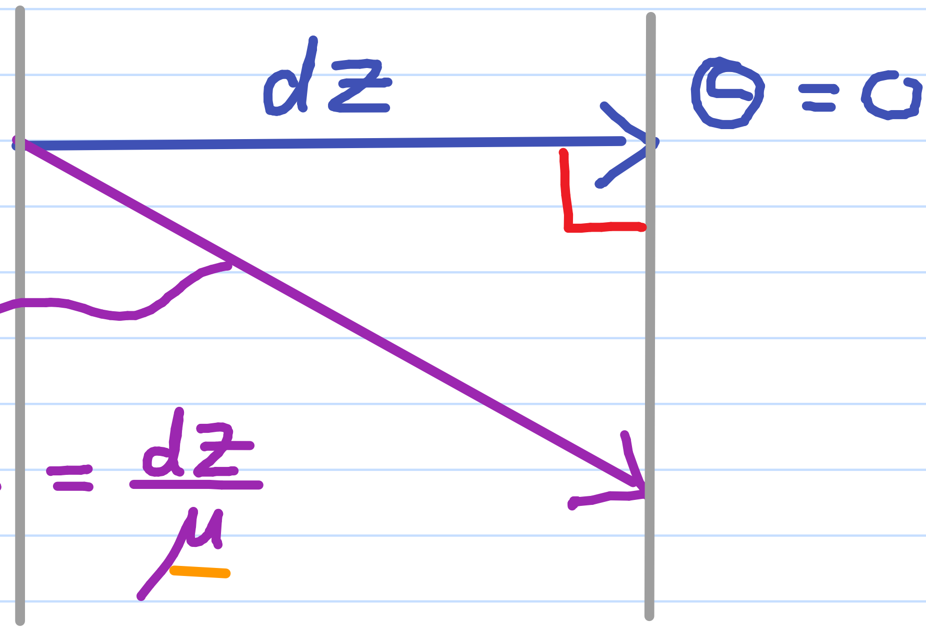
RADIAL BEAM, $\theta = 0$, $\mu = 1$:

$$d\tau_\nu(z) = \alpha_\nu(z)(-dz) = -\alpha_\nu(z)dz > 0$$

$$\tau_\nu(z) = -\int_{z_{TOP}}^{z} \alpha_\nu(z)\, dz = \int_{z}^{z_{TOP}} \alpha_\nu(z)\, dz$$

- RADIAL τ_ν-SCALE
- STANDARD DEPTH VARIABLE

BEAM IN ANY DIRECTION, θ (μ):



$$dS_\theta = \frac{dz}{\cos\theta} = \frac{dz}{\mu}$$

$$\frac{dI_\nu(S_\theta)}{dS_\theta} = j_\nu(S_\theta) - \alpha_\nu(S_\theta)\, I_\nu(S_\theta)$$

$$\therefore \frac{dI_\nu(S_\theta)}{-\alpha_\nu(S_\theta)\, dS_\theta} = \frac{-j_\nu(S_\theta)}{\alpha_\nu(S_\theta)} + I_\nu(S_\theta)$$

$$\therefore \frac{dI_\nu(\tau_{\nu,\theta})}{d\tau_{\nu,\theta}} = I_\nu(\tau_{\nu,\theta}) - S_\nu(\tau_{\nu,\theta})$$

BEAM τ_ν-SCALE, $\tau_{\nu,\theta} = \frac{\tau_\nu}{\mu}$

$\therefore$ RAD. TRANS. EQ. IN 1D ATMOSPHERE FOR ANY BEAM ON RADIAL τ_ν-SCALE:

$$\boxed{\mu \frac{dI_\nu(\tau_\nu, \mu)}{d\tau_\nu} = I_\nu(\tau_\nu, \mu) - S_\nu(\tau_\nu)}$$

ANGLE-AVERAGES OF RAD. TRANS. Eq. IN 1D AXI-SYMMETRIC ATMOSPHERE

(ON RADIAL τ_ν-SCALE)

ANGLE-AVERAGE OF $x(\vec{\underline{l}})$:

$$\langle x \rangle = \frac{1}{4\pi} \oint_0^{4\pi} x(\vec{l})\, d\omega$$

$$= \frac{1}{2} \oint_{\mu=-1}^{1} x(\mu)\, d\mu$$

RAD TRANS Eq. AT DEPTH τ_ν:

$$\frac{dI_\nu(\mu)}{d\tau_\nu} = I_\nu(\mu) - S_\nu$$

ANGLE-AVG:

$$\frac{1}{2}\int_{-1}^{1}\mu \frac{dI_\nu(\mu)}{d\tau_\nu}\,d\mu = \frac{1}{2}\int_{-1}^{1} I_\nu(\mu)\,d\mu - \frac{1}{2}\int_{-1}^{1} S_\nu\,d\mu$$

$$\frac{1}{2}\frac{1}{d\tau_\nu}\int_{-1}^{1}\mu\, dI_\nu(\mu)\,d\mu = \frac{1}{2}\int_{-1}^{1} I_\nu(\mu)\,d\mu - \frac{1}{2}\int_{-1}^{1} S_\nu\,d\mu$$

-ASSUME S_ν ISOTROPIC: $S_\nu(\mu) = S_\nu$

$$\boxed{\therefore \frac{dH_\nu(\tau_\nu)}{d\tau_\nu} = J_\nu(\tau_\nu) - S_\nu(\tau_\nu)}$$

RECALL:

$d\tau_\nu = -\alpha_\nu dz$, $S_\nu = j_\nu/\alpha_\nu$, $H_\nu \equiv \frac{\mathcal{F}_\nu}{4\pi}$:

$$\frac{d\mathcal{F}_\nu(z)}{dz} = 4\pi\left\{j_\nu(z) - \alpha_\nu(z)\,J_\nu(z)\right\}$$

BOLOMETRIC:

$$\frac{dF(z)}{dz} = 4\pi\int_{\nu=0}^{\infty}\left\{j_\nu(z) - \alpha_\nu(z)\,J_\nu(z)\right\}dz$$

$= 0$ IN FLUX-CONSTANT THERMAL Eq.

$$\frac{1}{2}\frac{1}{d\tau_\nu}\int_{-1}^{1}\mu^2 dI_\nu(\mu)\,d\mu = \frac{1}{2}\int_{-1}^{1}\mu I_\nu(\mu)\,d\mu - \frac{1}{2}\int_{-1}^{1}\mu S_\nu\,d\mu$$

IF S_ν ISOTROPIC: $\int_{-1}^{1}\mu S_\nu(\mu)\,d\mu = 0$

$$\therefore \frac{dK_\nu(\tau_\nu)}{d\tau_\nu} = H_\nu(\tau_\nu)$$

RECALL: $K_\nu = \frac{cP_\nu}{4\pi}$

$$\therefore c\frac{dP_\nu(\tau_\nu)}{d} = F_\nu(\tau_\nu)$$

$$= F_\nu^+(\tau_\nu) - F_\nu^-(\tau_\nu) \geq 0$$

$$\boxed{\frac{d^2K_\nu(\tau_\nu)}{d\tau_\nu^2} = \frac{dH_\nu(\tau_\nu)}{d\tau_\nu} = J_\nu(\tau_\nu) - S_\nu(\tau_\nu)}$$

- 2^{nd} ORDER TRANSPORT Eq.