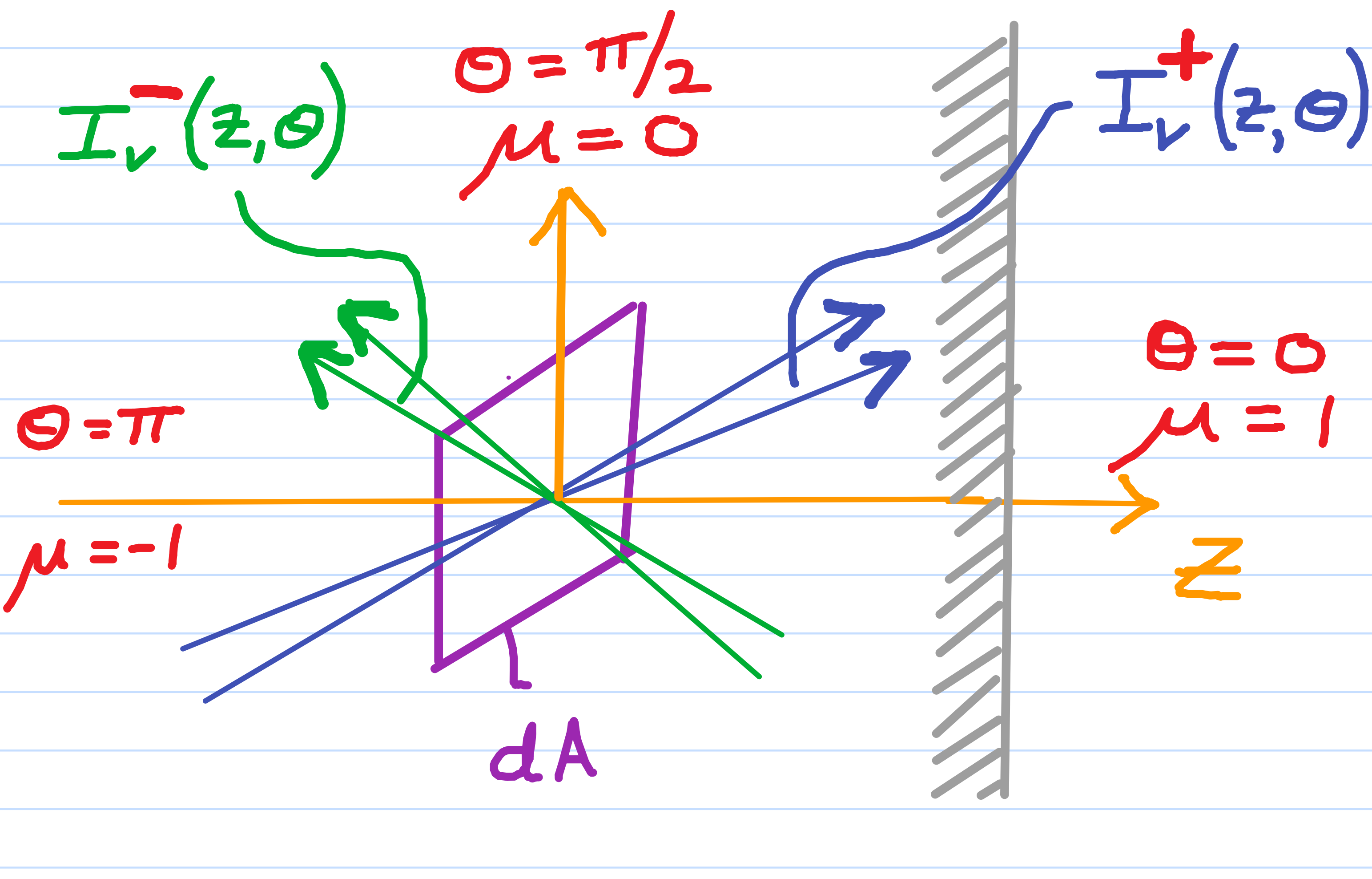
MONOCHROMATIC PHYSICAL FLUX

$\mathcal{F}_\nu(\vec{r}, t, \nu)$ $(\mathrm{erg/s/cm^2/Hz})$

- ANGLE-WEIGTED ANGLE-SUM OF $I_\nu(\vec{l})$

$\mathcal{F}_\nu$ IS NET E_ν PROPAGATING $\perp$ TO dA



I_ν^{+} = OUT-GOING I_ν

$\left(0 < \theta < \pi/2 \, , \; 0 < \mu < 1\right)$

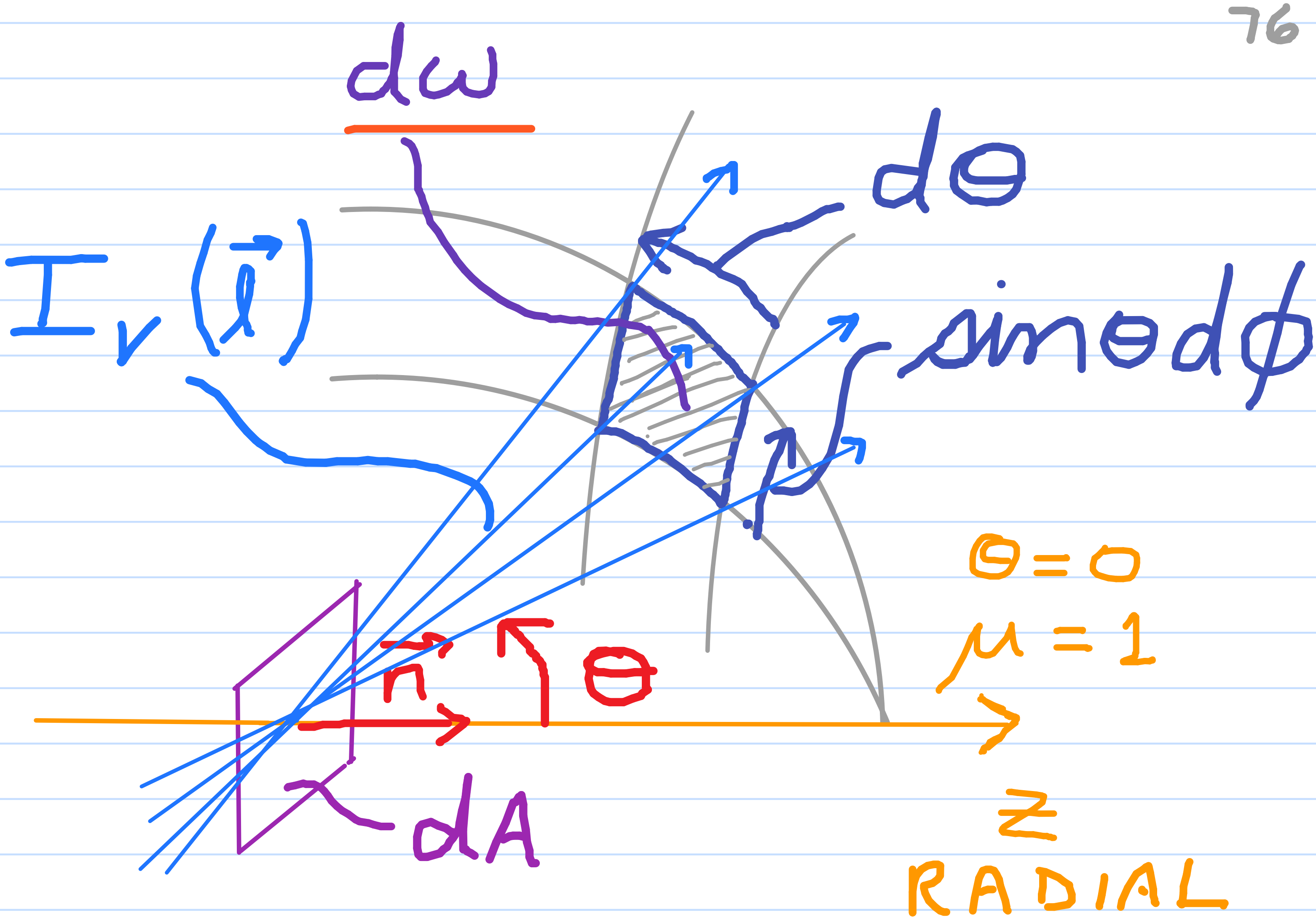
I_ν^{-} = IN-COMING I_ν

$\left(\pi/2 < \theta < \pi \, , \; -1 < \mu < 0\right)$

WEIGHT = $\cos\theta \equiv \mu$

$$F_\nu(\vec{r}) = \oint_0^{4\pi} I_\nu(\vec{r}, \vec{\ell}) \cdot \underline{\cos\theta} \, d\omega$$

(erg/s/cm^2/Hz)



$$d\omega = \sin\theta \, d\theta \, d\phi$$

$$F_\nu(\vec{r}) =$$

$$\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} I_\nu(\vec{r}, \theta, \phi) \cos\theta \sin\theta \, d\theta \, d\phi$$

1D AXI-SYMMETRIC MODEL:

$$I_\nu(\vec{r}, \theta, \cancel{\phi}) = I_\nu(\underline{z}, \theta):$$

$$F_\nu(\underline{z}) = \underline{2\pi} \int_{\theta=0}^{\pi} I_\nu(\underline{z}, \theta) \cos\theta \sin\theta \, d\theta$$

$$= 2\pi \int_0^{\underline{\pi/2}} I_\nu^+(z, \theta) \cos\theta \sin\theta \, d\theta$$

$$+ 2\pi \int_{\underline{\pi/2}}^{\pi} I_\nu^-(z, \theta) \cos\theta \sin\theta \, d\theta$$

IN 2^{nd} INTEGRAL (I_ν^-):

POLAR DIRECTION $\underline{\pi-\theta}$ wrt $-$ve $\hat{z}$

$$\cos(\underline{\pi-\theta}) = -\cos\theta$$

$$\sin(\underline{\pi-\theta}) = \sin\theta$$

$$d(\underline{\pi-\theta}) = -d\theta$$

$$\theta = \pi/2 \rightarrow \underline{\pi-\theta} = \pi/2$$

$$\theta = \pi \rightarrow \underline{\pi-\theta} = 0$$

$$\tilde{F}_\nu(z) = 2\pi \int_{\theta=0}^{\pi/2} I_\nu^+(z,\theta)\cos\theta\sin\theta\, d\theta$$

$$- 2\pi \int_{\underline{\pi-\theta} = \underline{0}}^{\underline{\pi/2}} I_\nu^-(z, \underline{\pi-\theta})\cos\theta\sin\theta\, d\theta$$

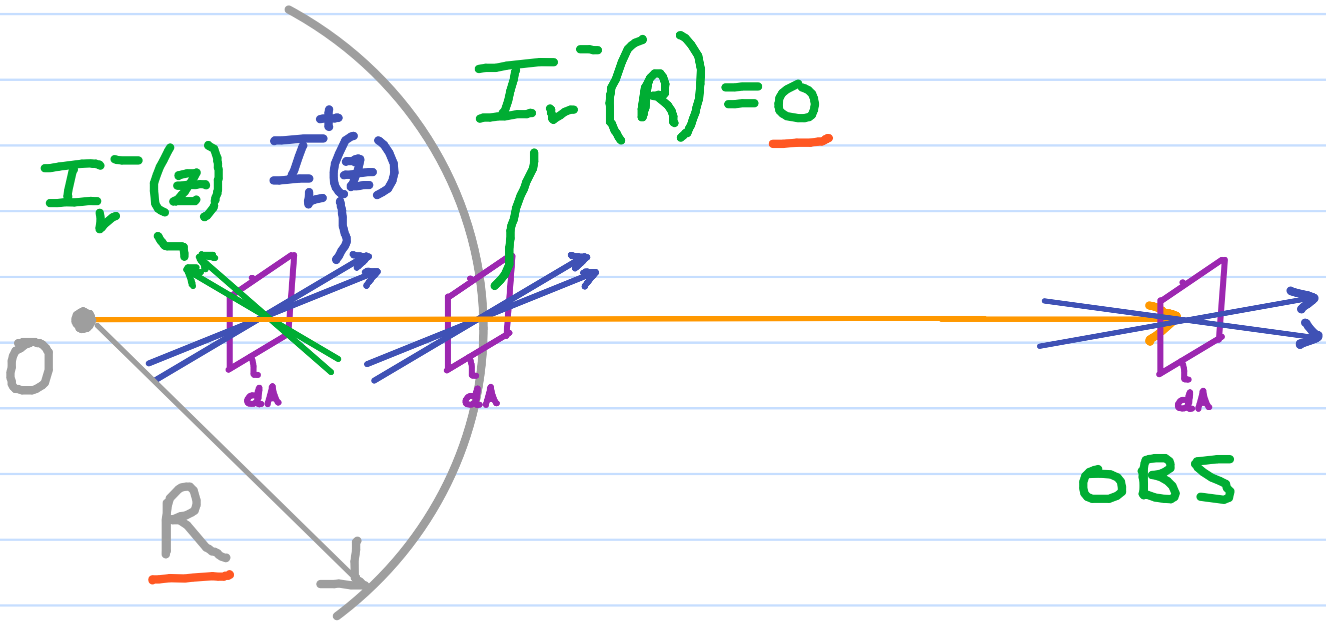
$$= \tilde{F}_\nu^+(z) - \tilde{F}_\nu^-(z)$$

$\mu \equiv \cos\theta \; ; \; d\mu = -\sin\theta \, d\theta$

$$\mathcal{F}_\nu(z) = 2\pi \int_0^1 \mu \, I_\nu^+(z,\mu) \, d\mu$$

$$- 2\pi \int_0^{-1} \mu \, I_\nu^-(z,\mu) \, d\mu$$

SPECIAL CASES:



1) SURFACE: $z = R$:

$$I_\nu^-(R) = 0 \quad \therefore \quad \mathcal{F}_\nu(R) = \mathcal{F}_\nu^+(R)$$

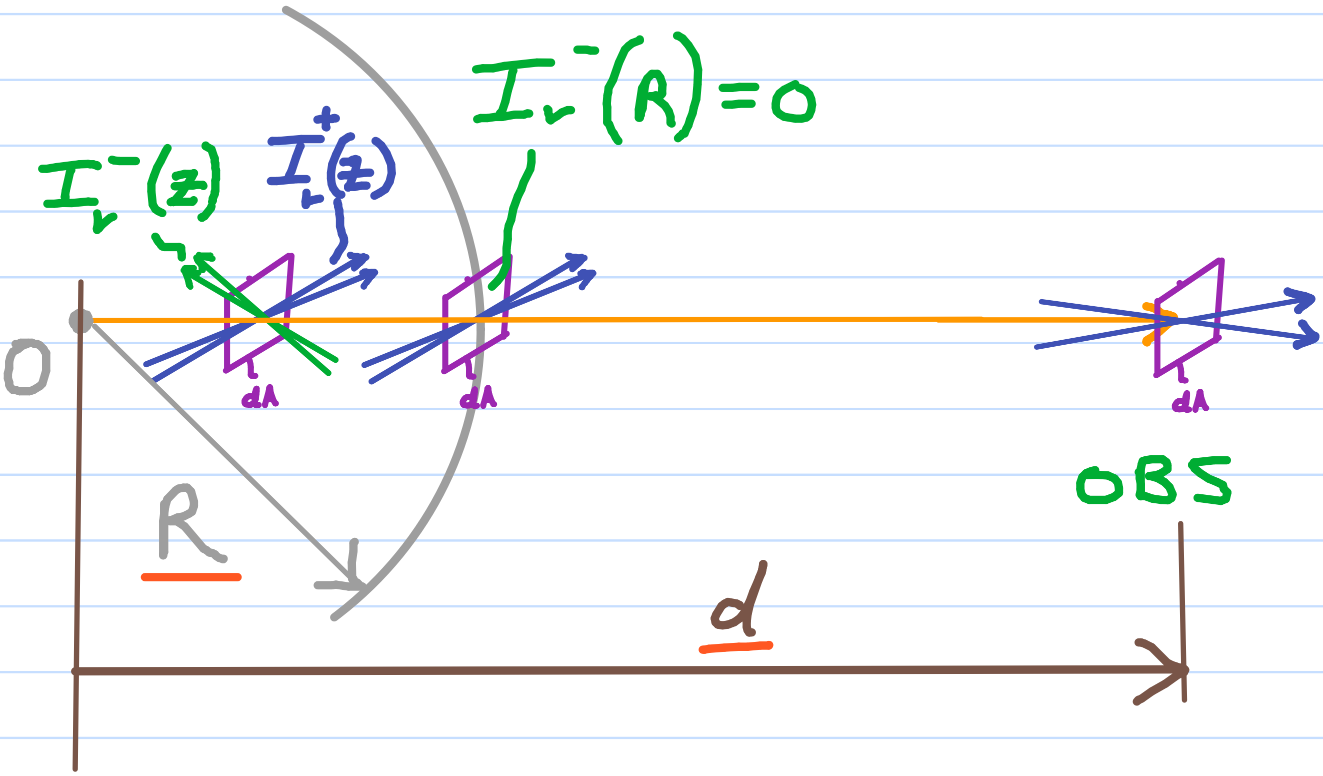
$$= 2\pi \int_0^1 \mu \, I_\nu^+(z,\mu) \, d\mu$$

BOLOMETRIC SURFACE FLUX, $\mathcal{F}$:

$$\mathcal{F} \equiv \mathcal{F}(R) = \int_{\nu=0}^{\infty} \mathcal{F}_\nu(R) \, d\nu = \sigma T_{eff}^4$$

INPUT

SPECIAL CASES:



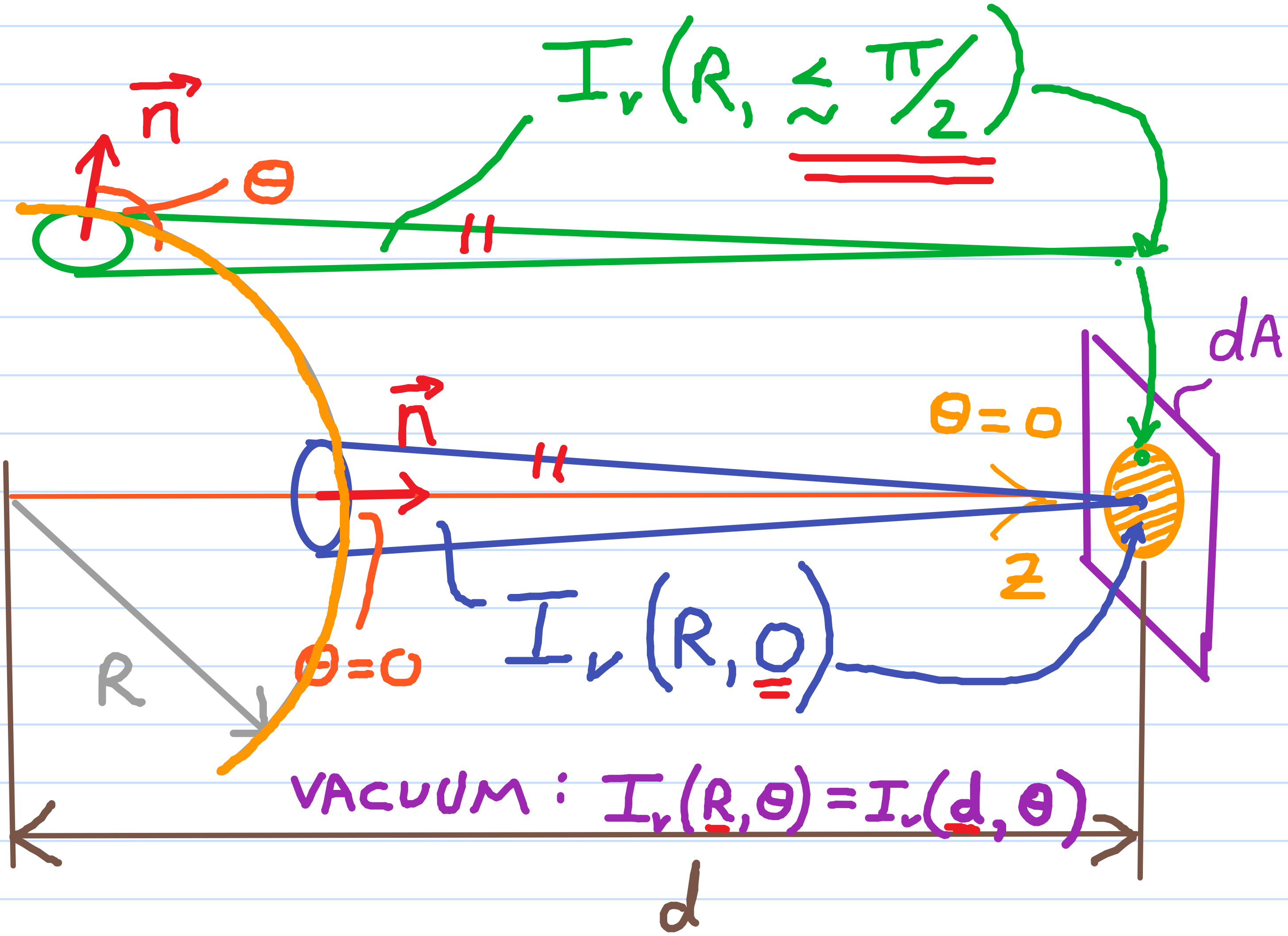
2) FLUX AT EARTH, f_ν : $z=d$:

$$f_\nu \equiv \mathcal{F}_\nu^+(d) = \frac{R^2}{d^2} F_\nu(R)$$

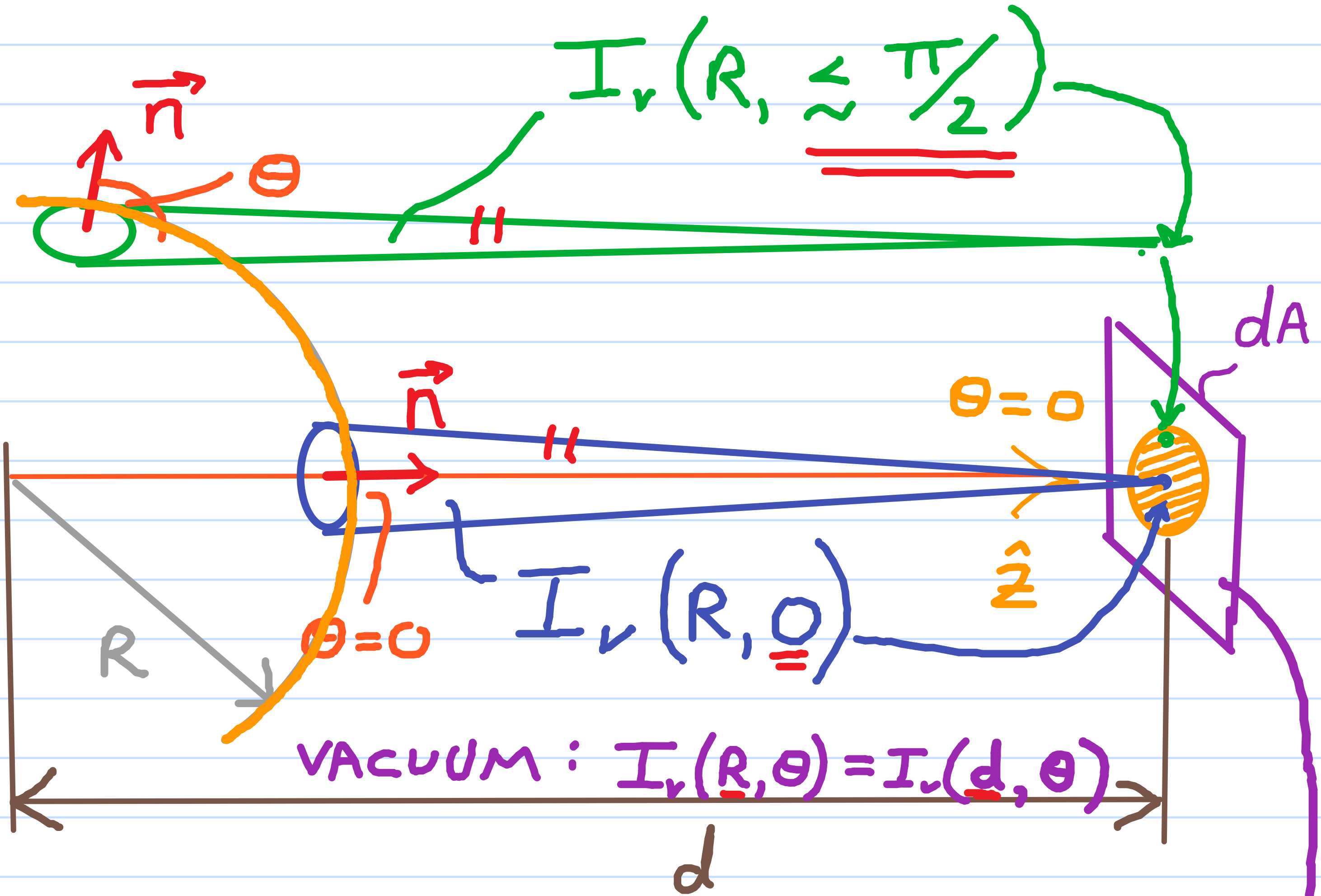
I_ν vs f_ν :

Eg. SUN $(R \approx 10^{-2} d)$

SPATIALLY RESOLVED



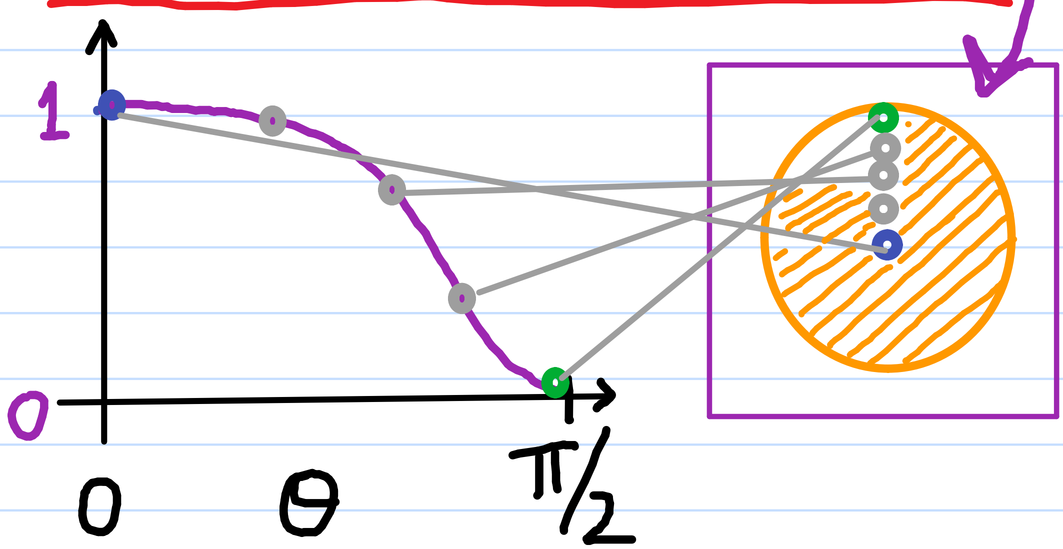
I_ν vs f_ν : Eg. SUN



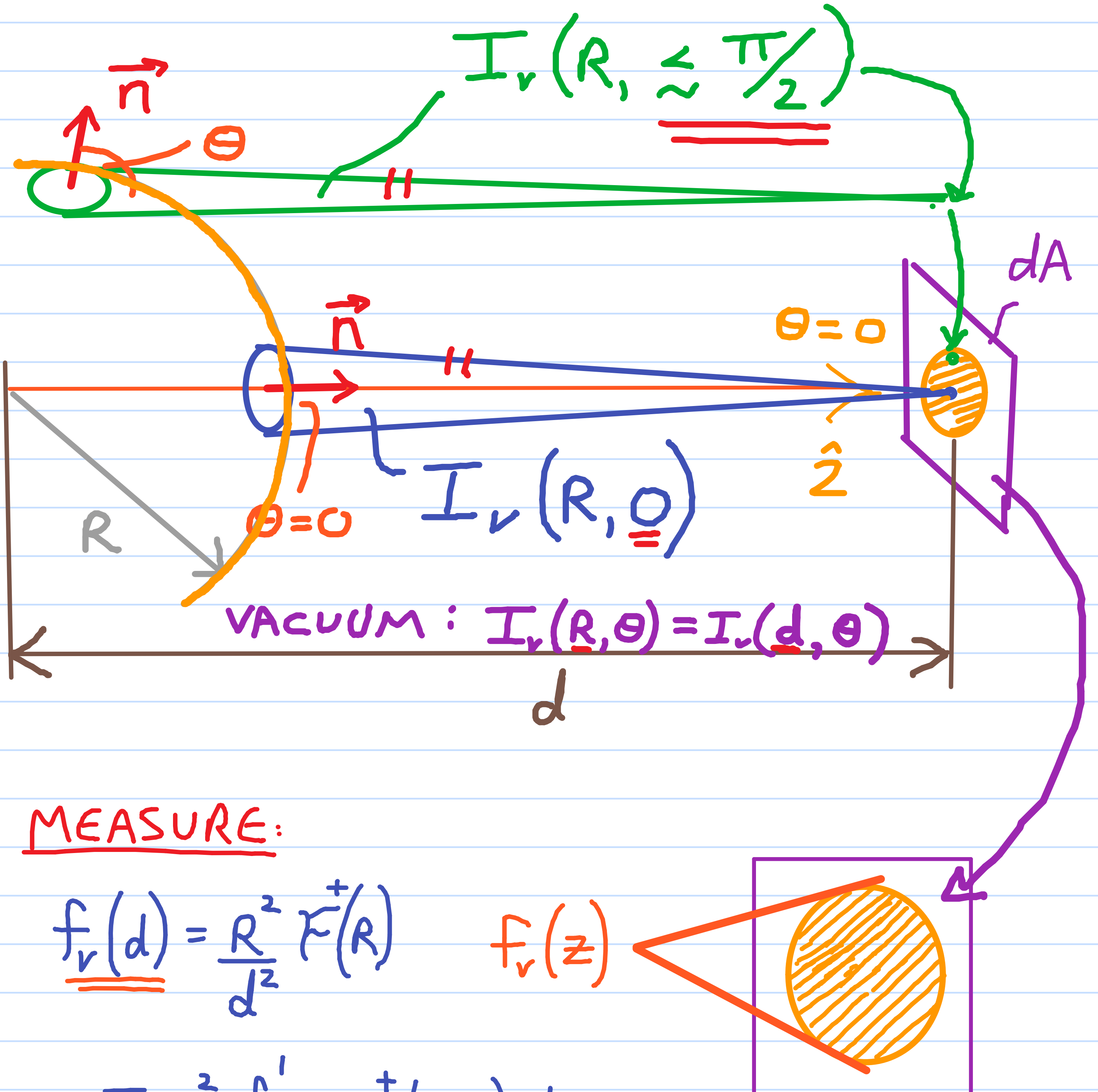
LIMB DARKENING CURVE:

MEASURE:

$$\frac{I_\nu(R,\theta)}{I_\nu(R,0)}$$



I_ν vs f_ν : Eg. SUN



MEASURE:

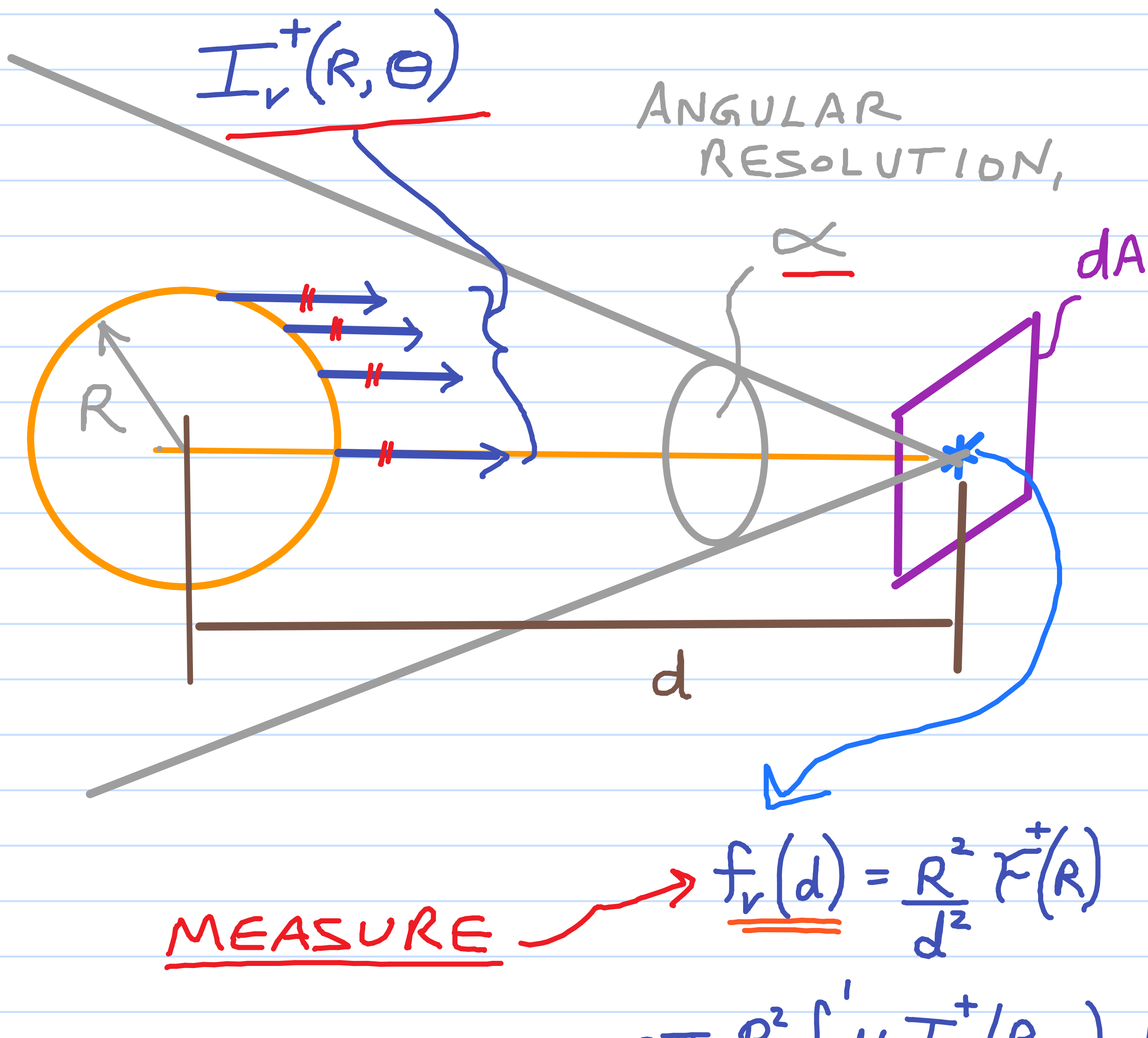
$$f_\nu(d) = \frac{R^2}{d^2} F^+(R)$$

$$= 2\pi \frac{R^2}{d^2} \int_{\mu=0}^{1} \mu I_\nu^+(R,\mu)\, d\mu$$

I_ν vs f_ν :

Eg. ANY OTHER STAR $(R \ll d)$

SPATIALLY UNRESOLVED POINT SOURCE



MEASURE $\rightarrow$ $f_\nu(d) = \dfrac{R^2}{d^2} F^+(R)$

$$= 2\pi \frac{R^2}{d^2} \int_{\mu=0}^{1} \mu\, I_\nu^+(R,\mu)\, d\mu$$